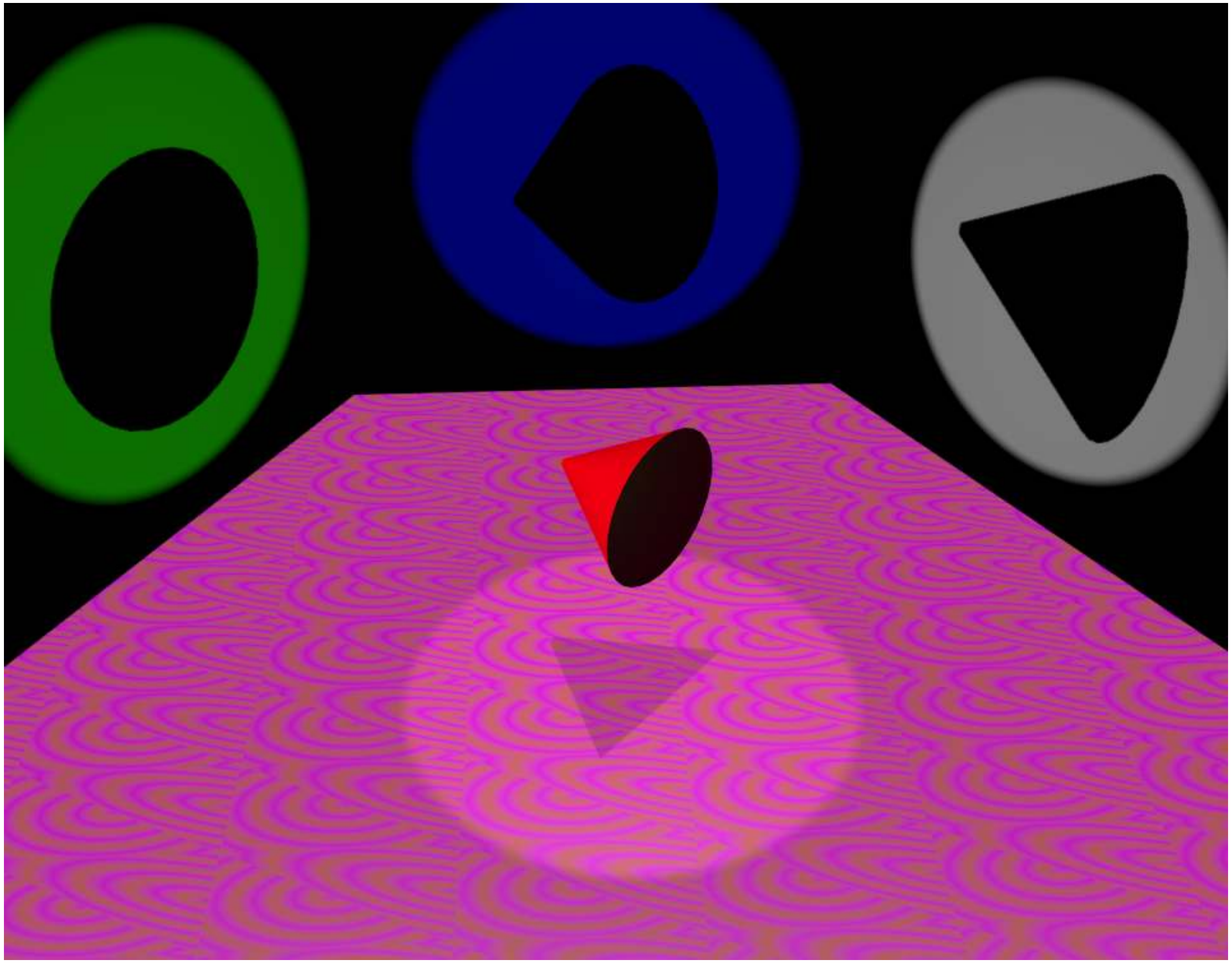
Topological pumping in quasiperiodic Hamiltonians

ETH Zürich

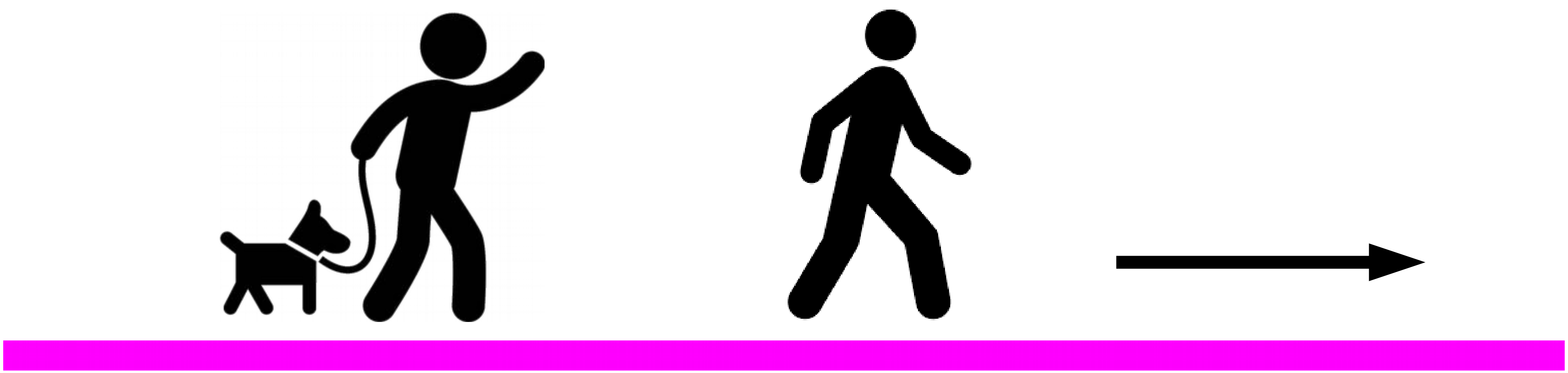
Jose Lado and Oded Zilberberg



8th February 2018

Welcome to Straightland

Straightland, a one dimensional universe



$$\vec{r} = (x, 0, 0) \qquad \nabla^2 = \frac{\partial^2}{\partial x^2}$$

$$i \frac{\partial}{\partial t} |\Psi\rangle = \left(-\frac{\partial^2}{\partial x^2} + V(x) \right) |\Psi\rangle$$

2D physics in Straightland

Straightland is a 1D universe

Is there magnetic field in a 1D universe?

$$\vec{F} = \vec{v} \times \vec{B}$$

Not in Straightland

Is there transverse conductivity in a 1D universe?

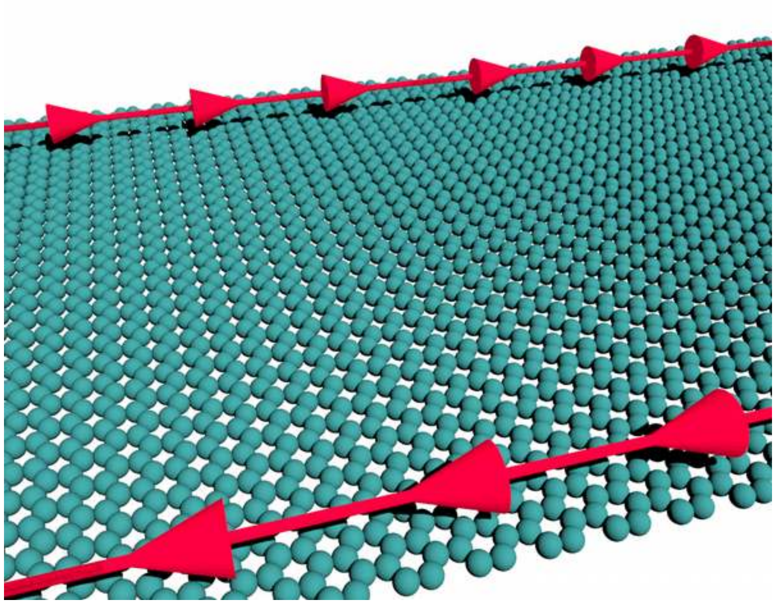
$$J_x = \sigma_{xy} E_y$$

Not in Straightland

Could we experiment 2D physics in a 1D universe?

The 2D Quantum Hall effect

The quantum 2D electron gas in a magnetic field



No backscattering channel

Perfect conductance

Topological protection

Protected surface modes appear due to non-trivial topology

Nobel prize in Physics (2016)

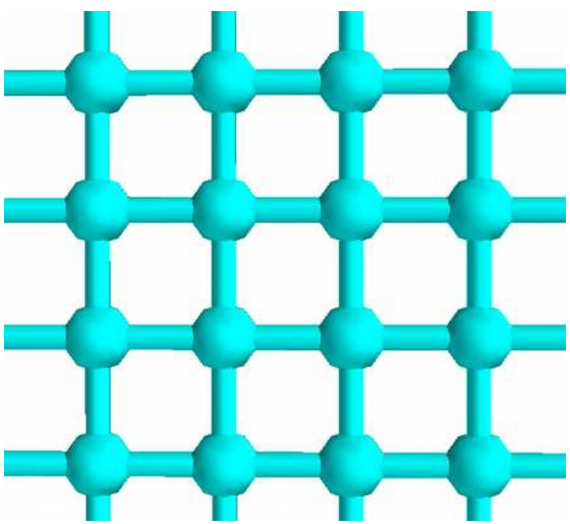
Phys. Rev. Lett. 49, 405 (1982)

From the 2D quantum Hall to a 1D quasiperiodic model

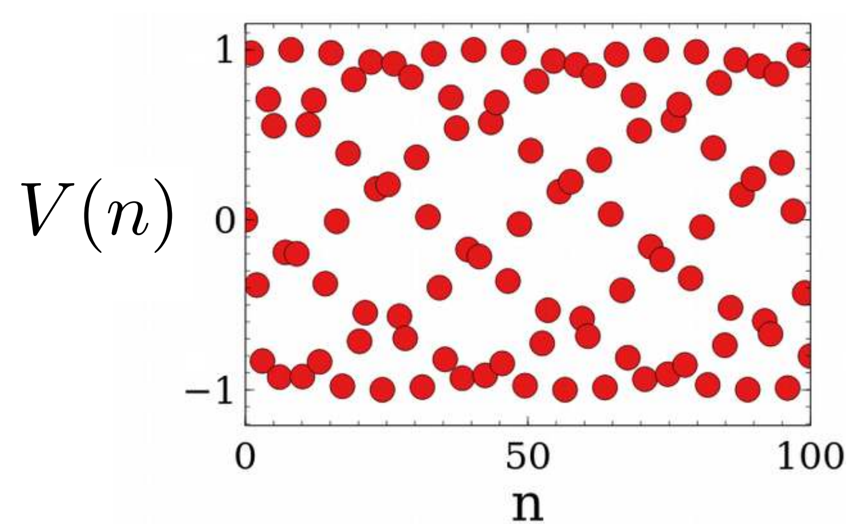
2D electron gas in a magnetic field

1D electron gas in a background potential

$$H|\Psi\rangle = \frac{(\vec{p}+\vec{A})^2}{2m}|\Psi\rangle \equiv \mathcal{H}|\Psi\rangle = V|\Psi\rangle + \frac{p^2}{2m}|\Psi\rangle$$



n=0 n=1 n=2 n=3

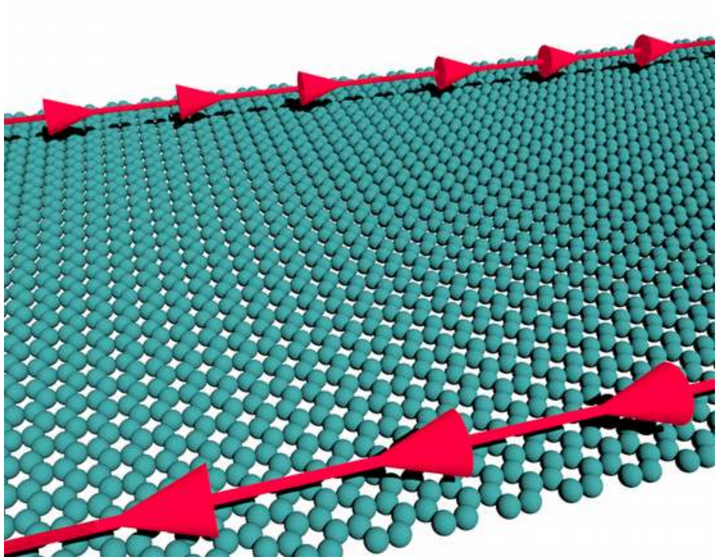


Phys. Rev. Lett. 49, 405 (1982)

From 2D to 1D

2D quantum Hall state

1D non periodic system



$\equiv$

2 dimensions

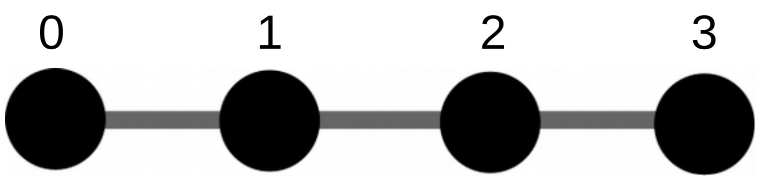
1 dimension + complexity

Quasiperiodic 1D systems are equivalent to 2D quantum Hall states

Nature Phys. 12, 624 (2016).

1D Hamiltonian for the Quantum Hall effect

Electrons in a magnetic field are studied by solving



$n = 0, 1, 2, 3, 4.....$ Site in a 1D lattice

$$\mathcal{H}_k = \boxed{2t\cos(Bn+k)c_n^\dagger c_n} + \boxed{t[c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n]}$$

Potential energy

Kinetic energy

$$\mathcal{H}|\Psi\rangle = \boxed{V(x)|\Psi\rangle} + \boxed{\frac{p^2}{2m}|\Psi\rangle}$$

The potential is in general in-commensurate with the lattice

Shifting and pumping

2D Electron in a magnetic field behave according to this 1D potential

Magnetic field **Site in the lattice** **Bloch momentum**

$$V(n) = 2t\cos(Bn + k)$$

The Hamiltonian is invariant under a transformation

$$\boxed{n \rightarrow n + n_0}$$

Shifting the lattice

$$\boxed{k \rightarrow k - Bn_0}$$

Shifting the Bloch momenta
(Pumping)

Moving a 1D system pumps in a 2D Hamiltonian

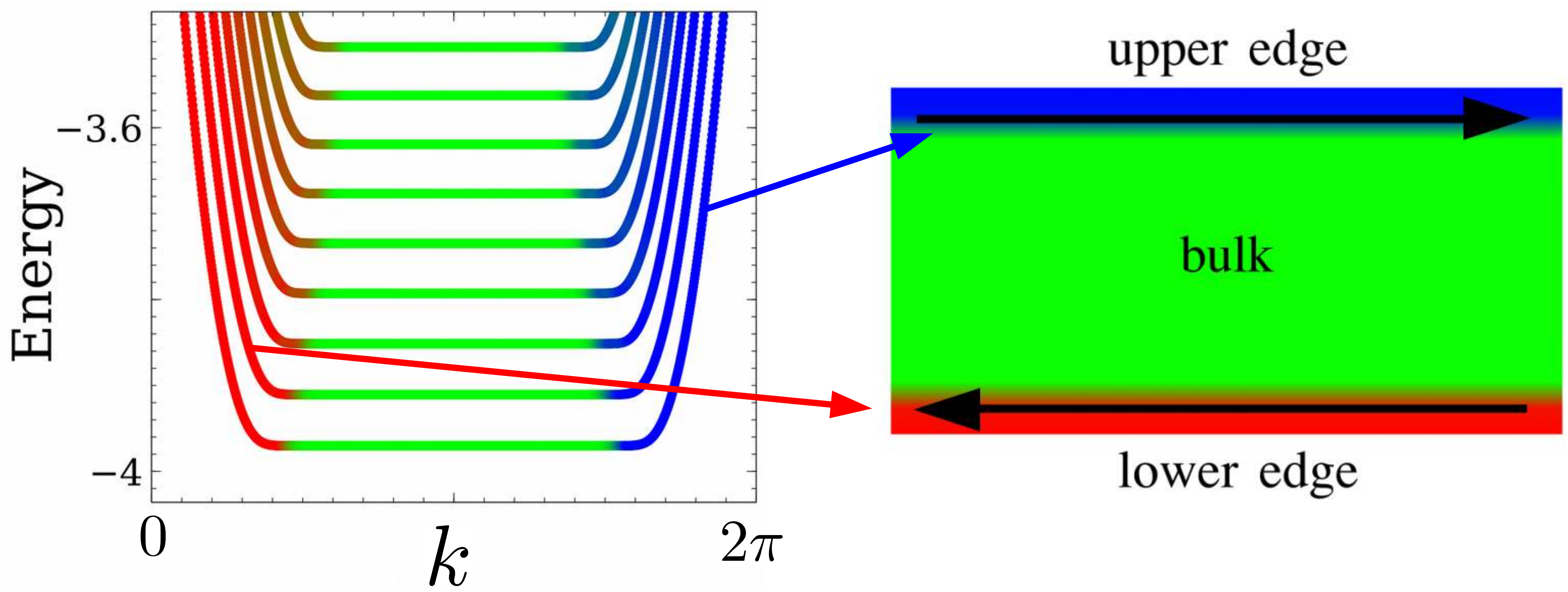
Phys. Rev. B 27, 6083 (1983).

Nature Phys. 12, 350 (2016)

Edge modes in the 2D Quantum Hall state

Spectra of the Quantum Hall effect

$$\mathcal{H}_k = 2t\cos(Bn + k)c_n^\dagger c_n + t[c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n]$$

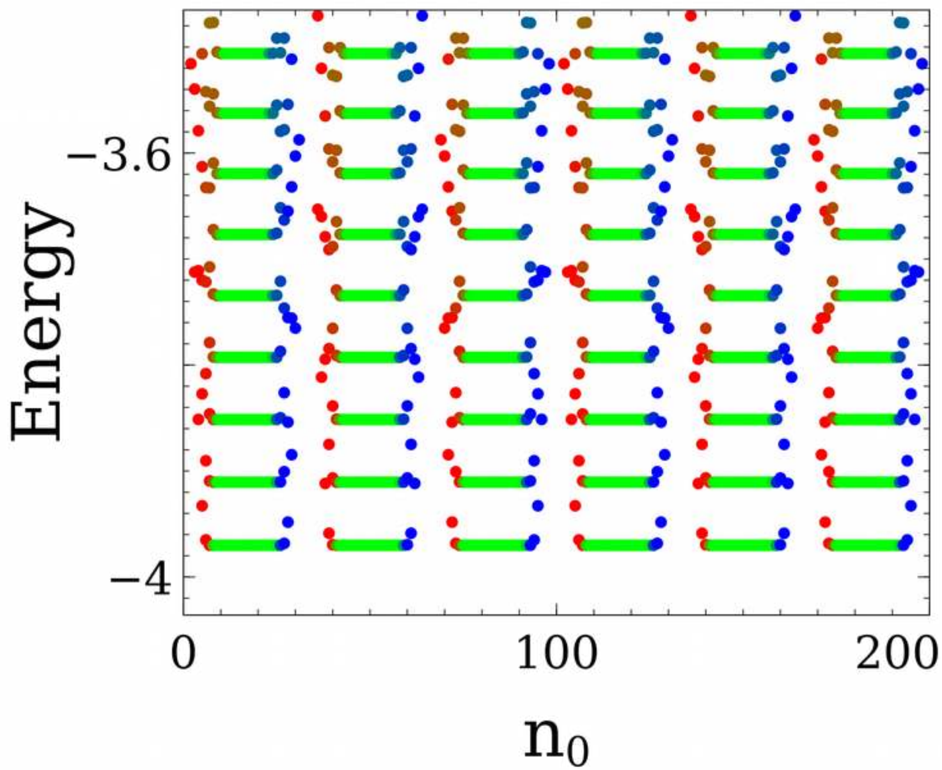


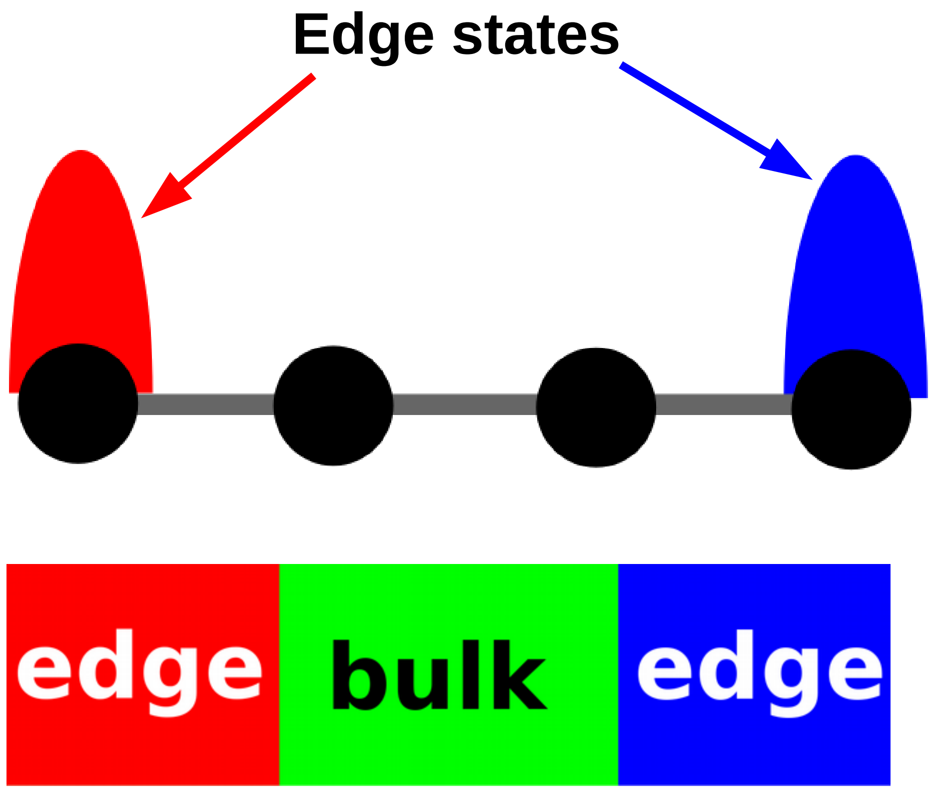
The band structure of the 2D quantum Hall effect shows chiral edge modes

Edge modes in the 1D incommensurate potential

$$\mathcal{H}_k = 2t\cos(Bn+k)c_n^\dagger c_n + tc_n^\dagger c_{n+1} + c.c.$$

$$n \to n + n_0$$





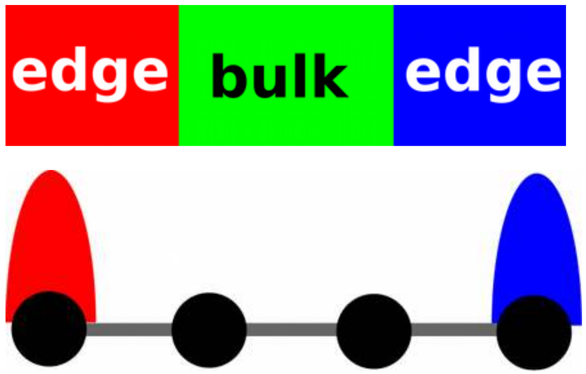
Each n_0 is a different quasiperiodic 1D system

Nature Phys. 12, 624 (2016).

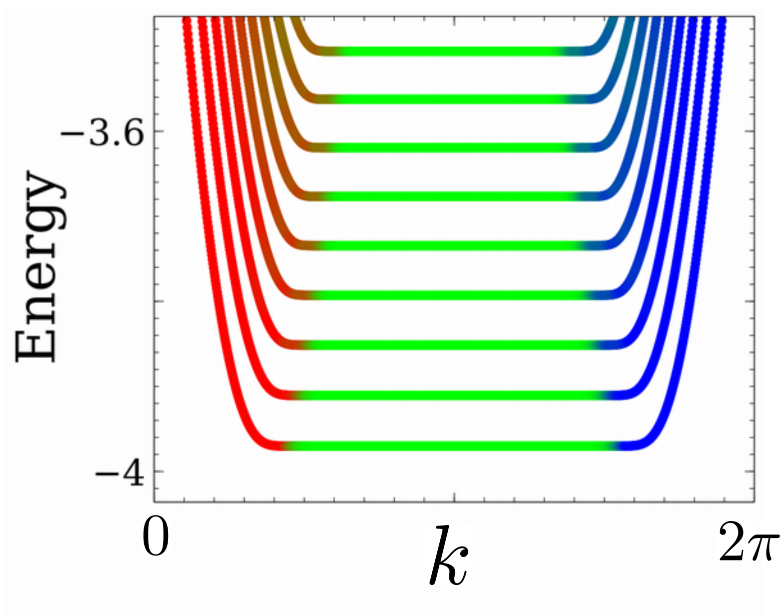
Phys. Rev. Lett. 110, 180403 (2013)

The quantum Hall state in Straightland

Quasiperiodicity in Straightland reflects the Hall state

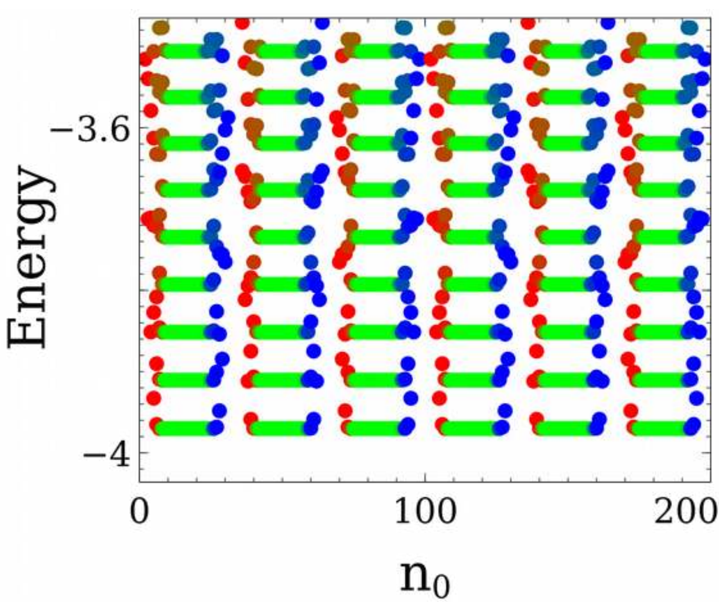


2D quantum Hall

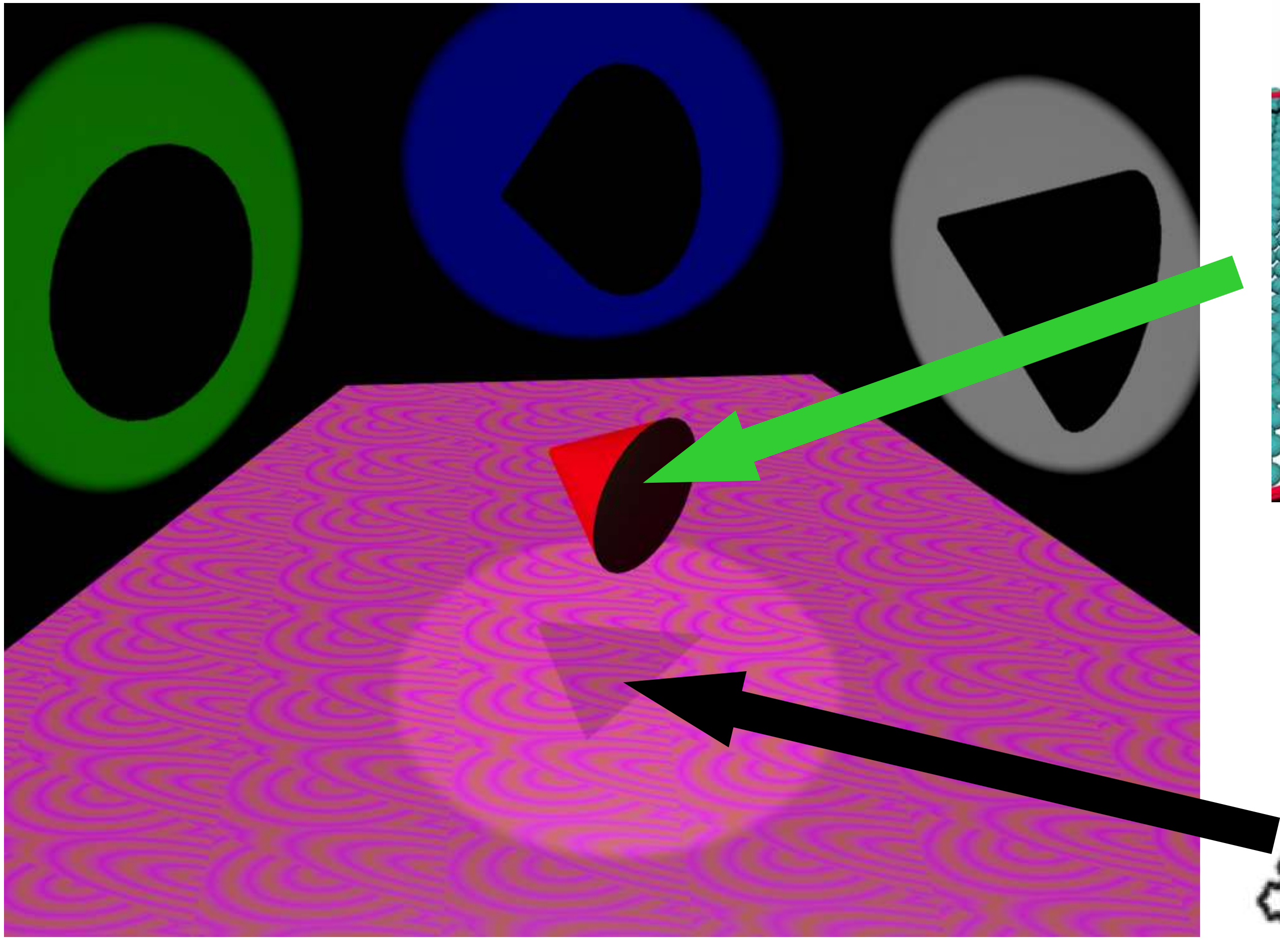


$\equiv$

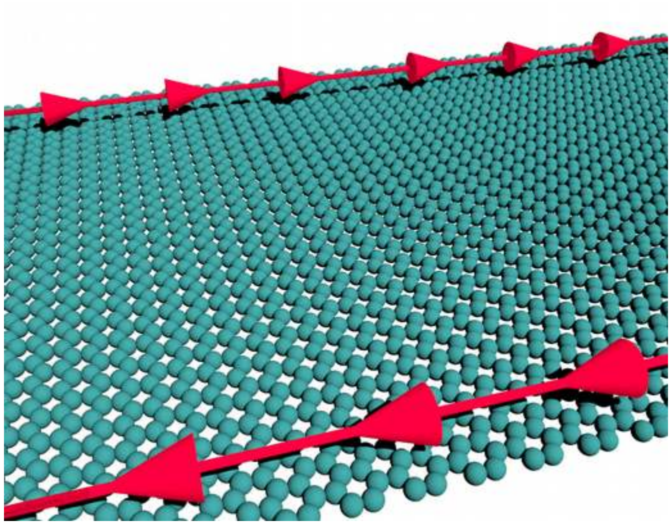
1D quasiperiodic



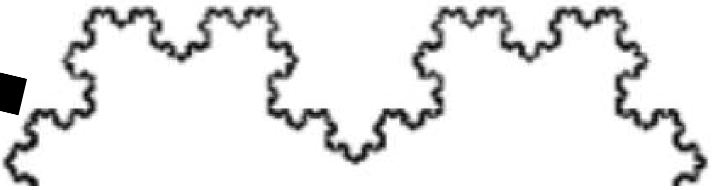
Quasiperiodicity as a shadow of the Quantum Hall state



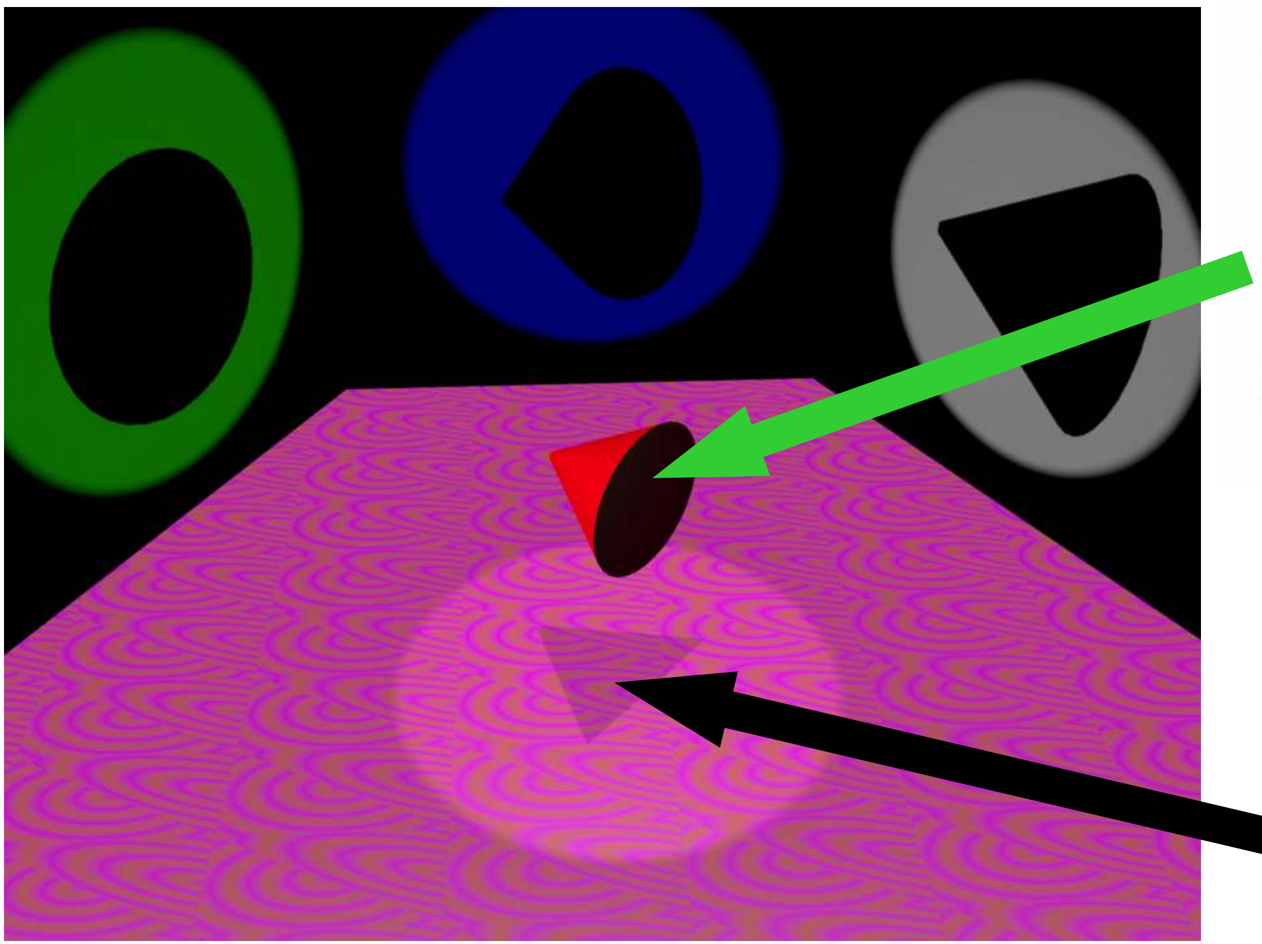
2D quantum Hall



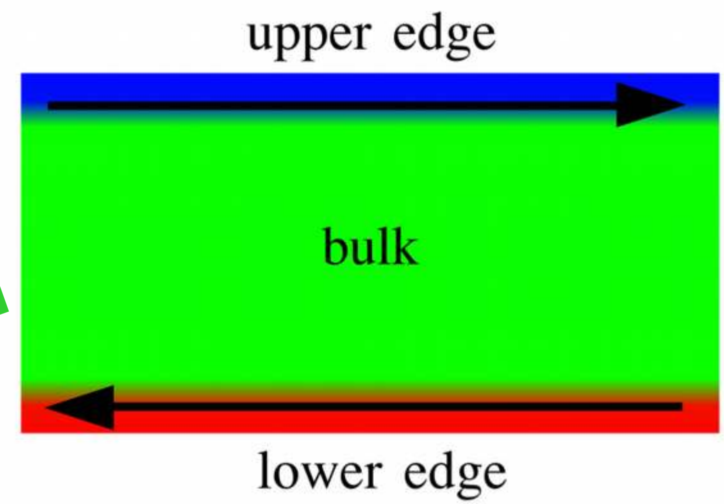
1D quasiperiodic Hamiltonian



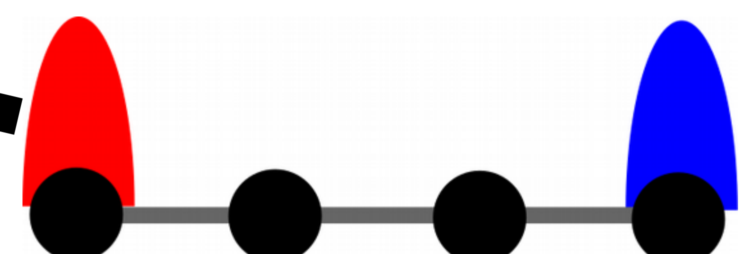
Quasiperiodicity as a shadow of the Quantum Hall state



2D quantum Hall



1D quasiperiodic Hamiltonian

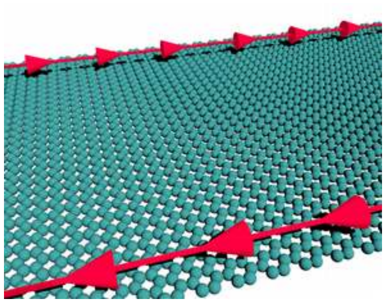
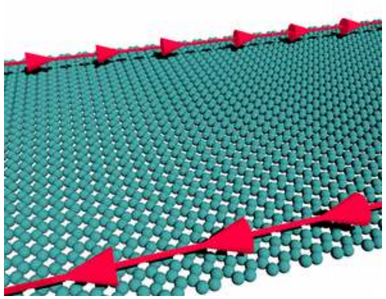


What’s next? Realizing high dimensional quantum Hall effect

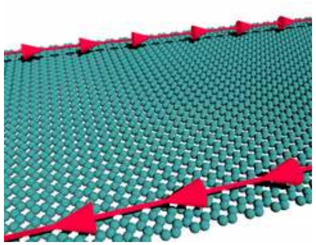
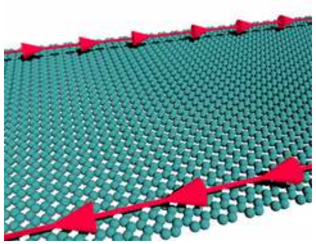
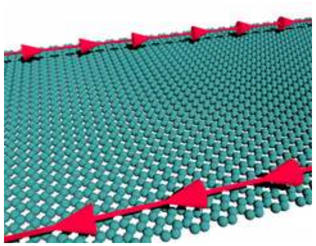
A 4D quantum Hall state in the lab

Nature 553, 55–58 (2018)

Nature 553, 59–62 (2018)

$H_{4D} =$  $\otimes$ 

<u>Future</u>: a 6D quantum Hall state in the lab

$H_{6D} =$  $\otimes$  $\otimes$ 

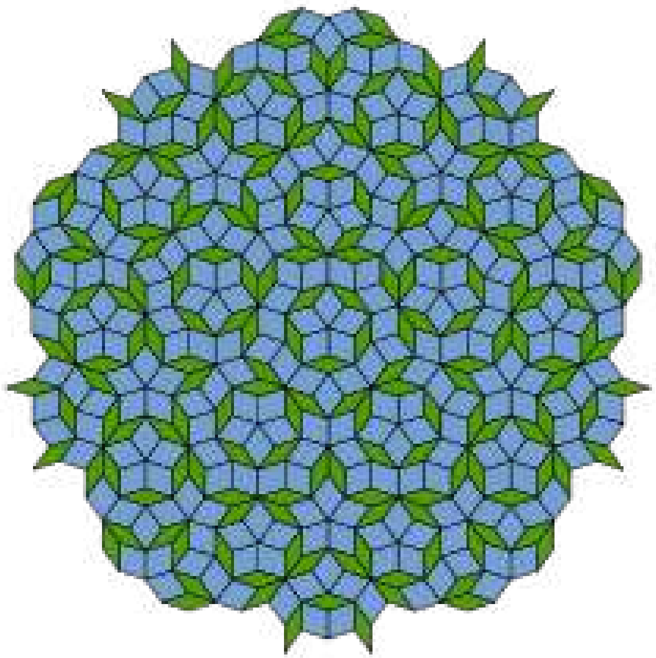
Quasiperiodic Hamiltonians realize high dimensional physics

Quasiperiodicity and quasicrystals

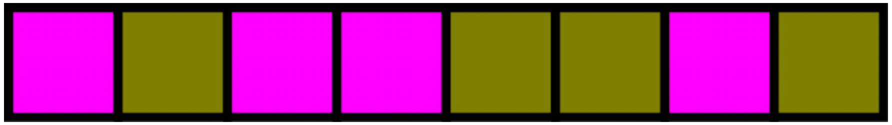
Neither periodic nor random

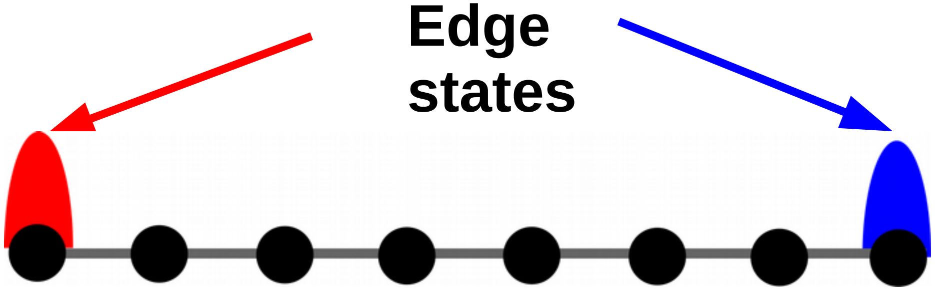
Nobel prize in Chemistry (2011)

2D quasicrystal



1D quasicrystal





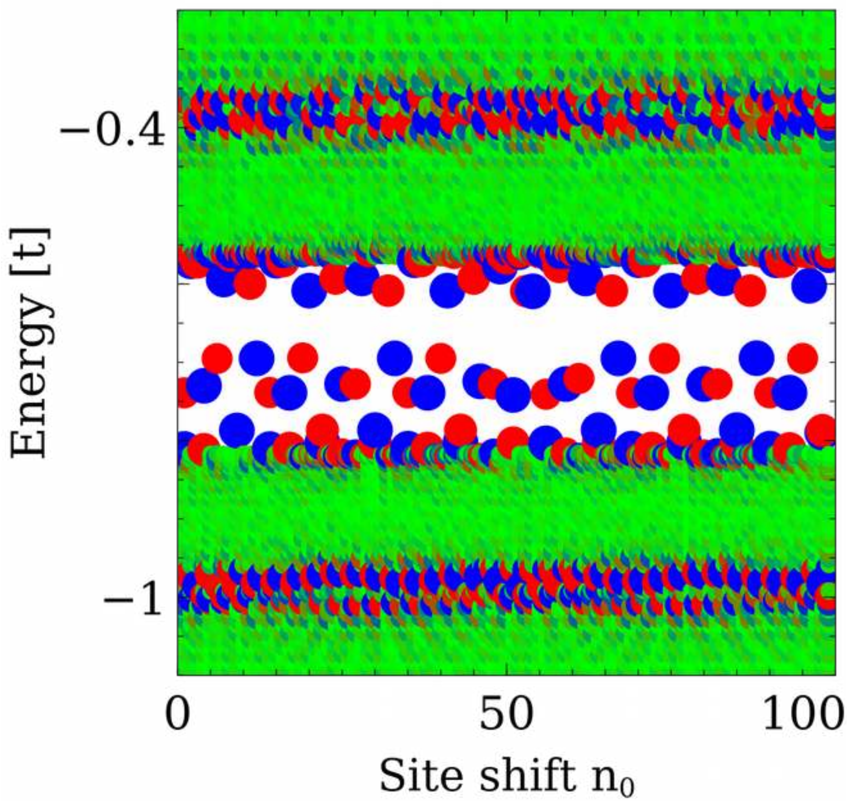
Could we have an analogy of the quantum Hall state in a 1D quasicrystal?

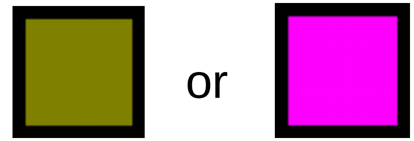
Edge states in a 1D quasicrystal

$$\mathcal{H} = \lambda V(n) c_n^\dagger c_n + t c_n^\dagger c_{n+1} + c.c.$$

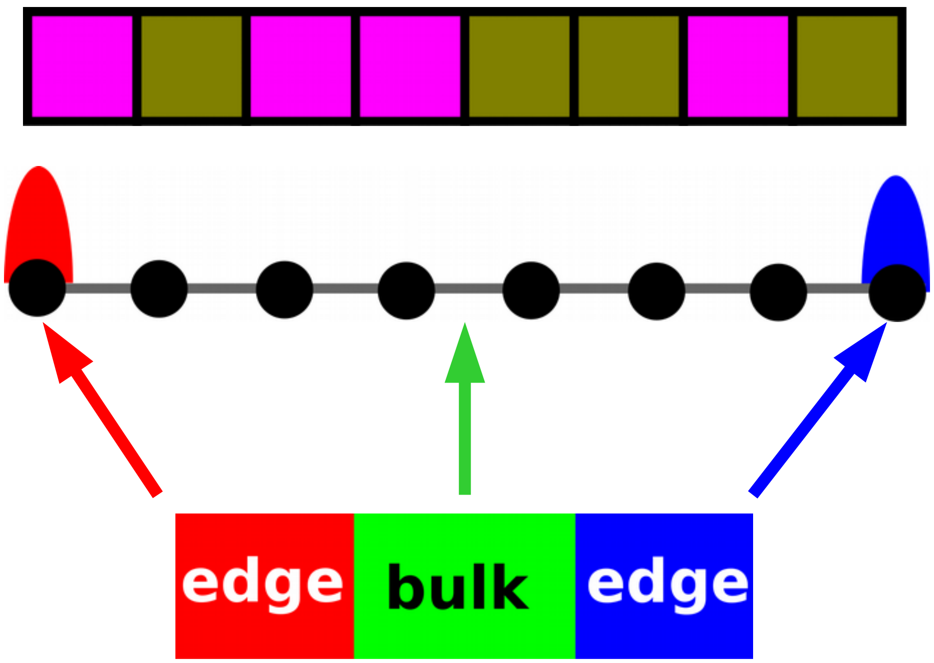
Phys. Rev. Lett. 109, 106402 (2012)
Phys. Rev. B 95, 161114 (2017)

Quasicrystal spectra



$V(n) =$  or

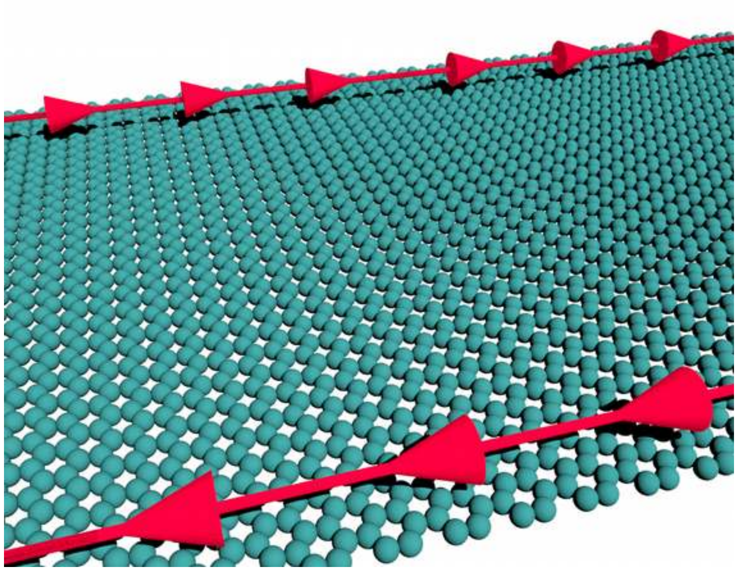
Hamiltonian



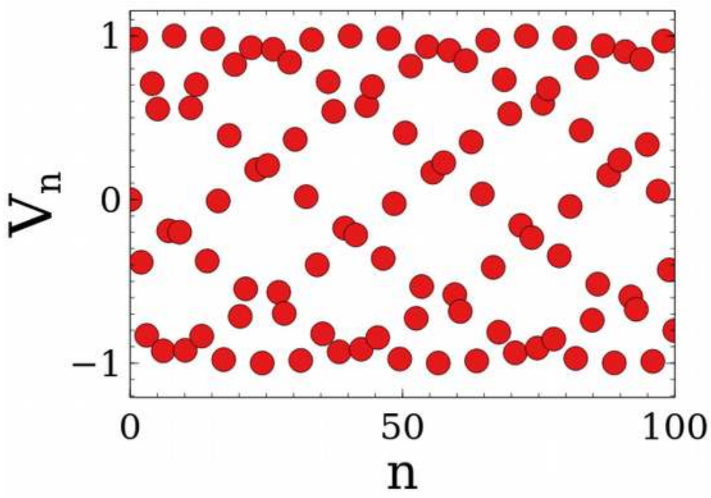
Edge states in a 1D quasiperiodic Hamiltonian are analogous to edge states in the 2D quantum Hall effect

Quantum Hall and quasicrystals

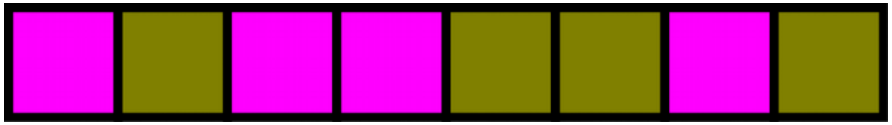
2D Quantum Hall



1D $\cos Bx$ potential



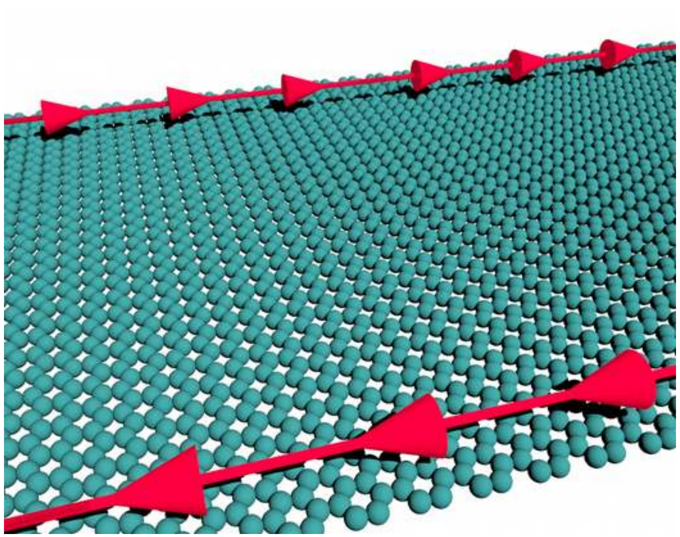
1D quasicrystal Hamiltonian



The spectra of a 1D quasicrystal can be understood with a 2D quantum Hall state

Summary

- Take home
  - Quasiperiodic Hamiltonians reflect high dimensional Quantum Hall states



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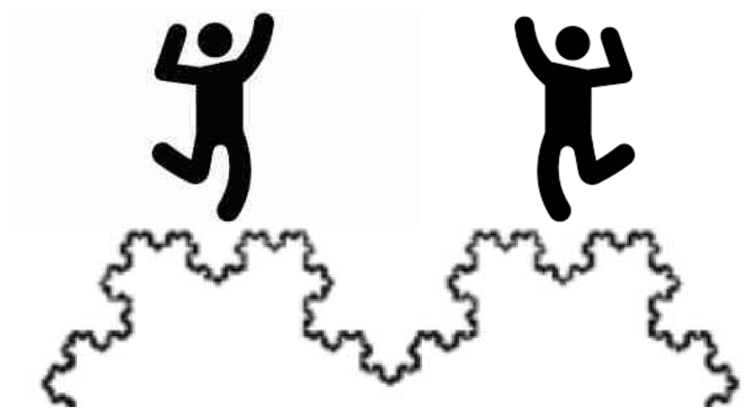

Thank you!