Topological superconductivity with antiferromagnetic insulators

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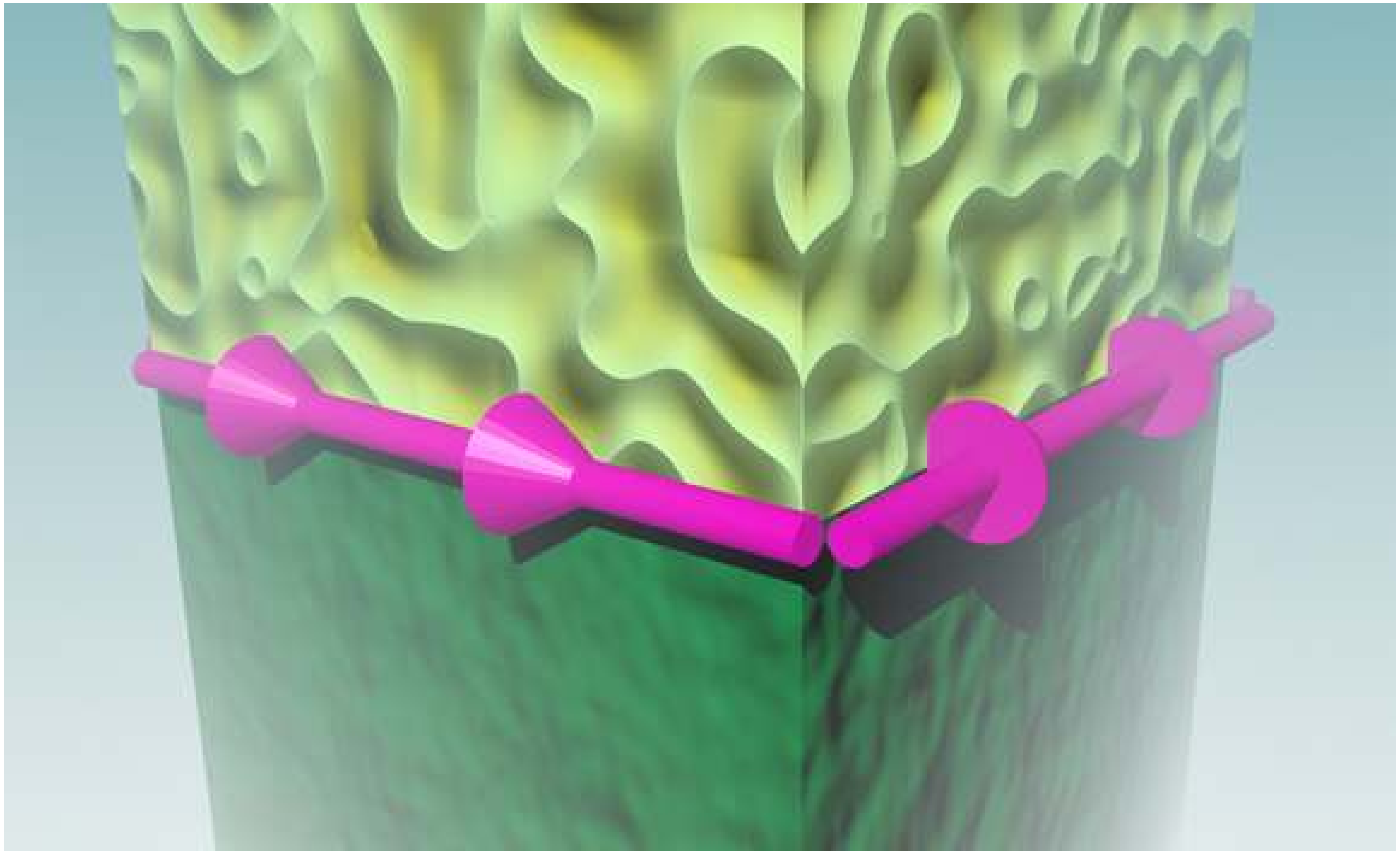


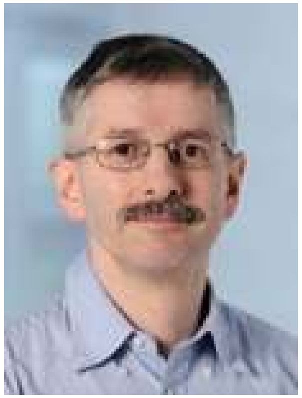
May 11th 2018, University of Alicante, Spain

About

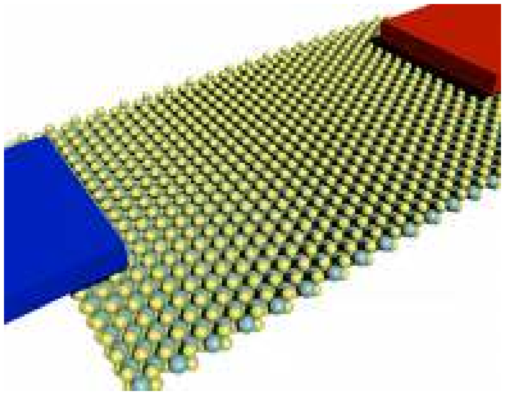
ETH Zurich, Switzerland

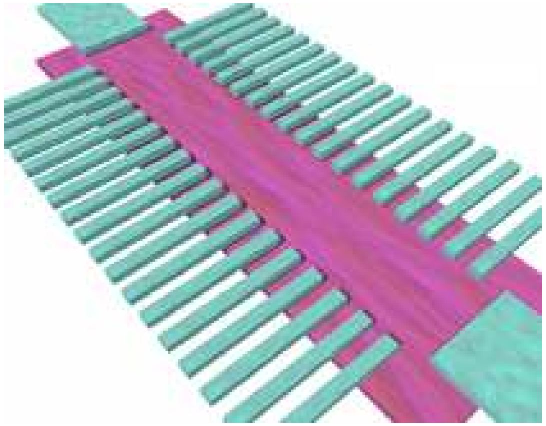
Institute for Theoretical Physics

Prof. Manfred Sigrist's group & Prof. Oded Zilberberg's group



Losing quantumness





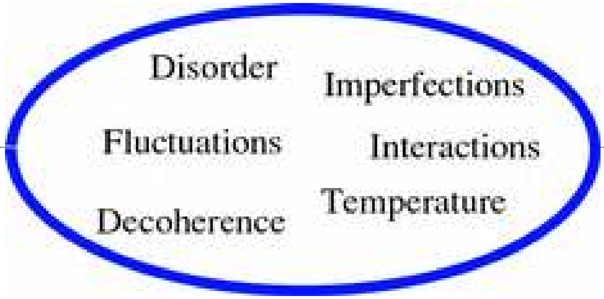
Size increase

But at some point

- Quantization is blurred away
- Coherence is lost
- Classical dynamics arises
- Planck's constant “disappears”

$$i\hbar\frac{\partial}{\partial t}\Psi = \mathcal{H}\Psi$$

Quantum



Classical

Are quantum phenomena restricted to the nanoscale?

Quantum-ness in the macroscale

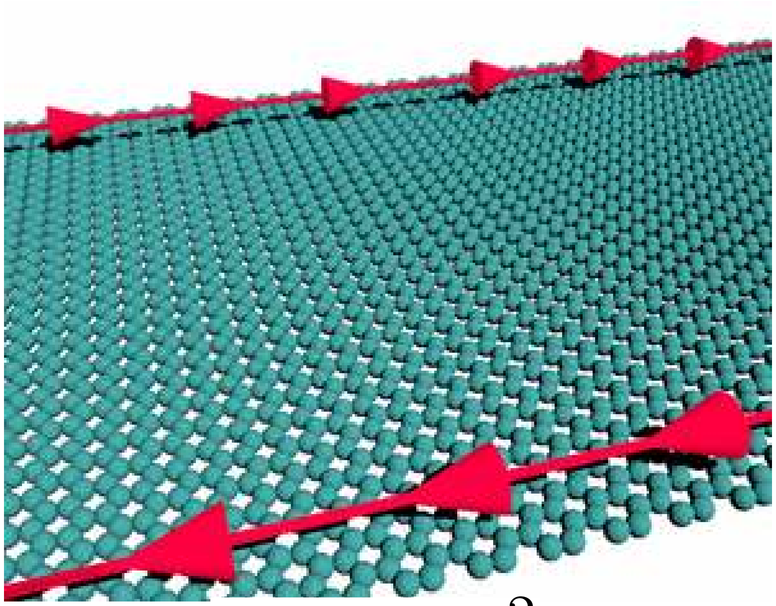
Superconductivity

$$\Phi = \frac{h}{2e}$$

Many-body state

<u>Quantization</u> of flux

Quantum Hall effect



$$\sigma_{xy} = \nu \frac{e^2}{h}$$

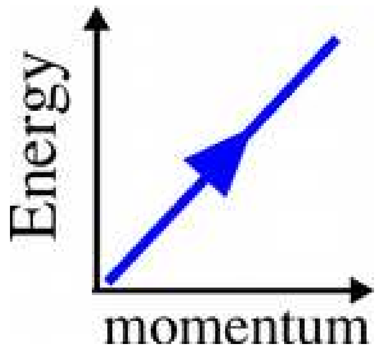
Single-particle state

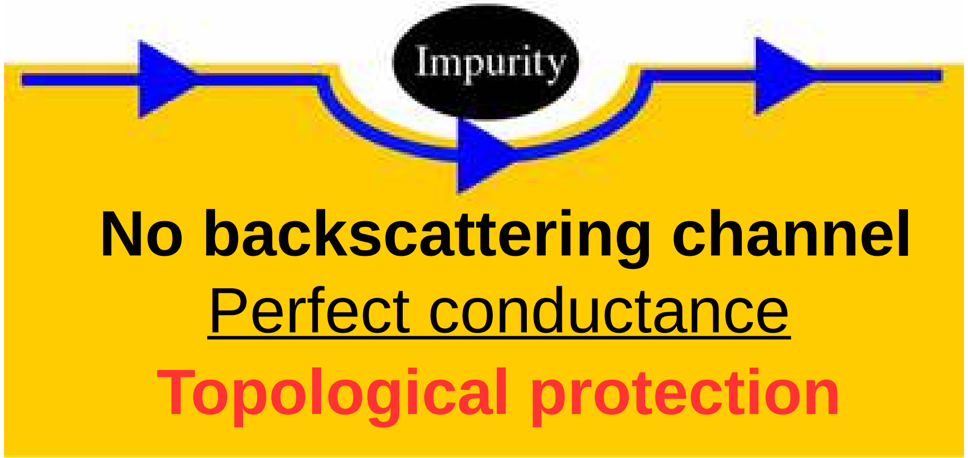
<u>Quantization</u> of conductance

Macroscopic quantum phenomena determine fundamental quantities with the highest precision

Topology: When defects no longer matter

Surface of Hall insulator





$$\sigma_{xy} = \frac{e^2}{h}\mathcal{C}$$

Hall conductance

Physical observable

$$\mathcal{C} = \frac{1}{2\pi}\int \Omega d^2 k$$

Chern number, **integer**

Topological invariant

$$\Omega = \partial_{k_x} A_y - \partial_{k_y} A_x$$

Berry curvature

$$\vec{A} = i\langle\Psi|\partial_{\vec{k}}|\Psi\rangle$$

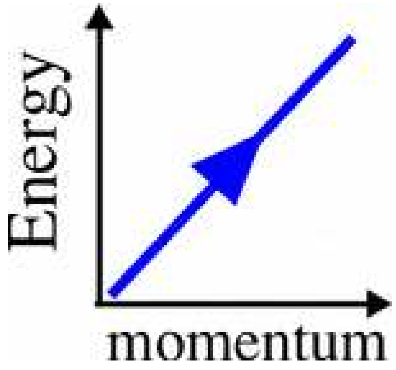
Berry connection

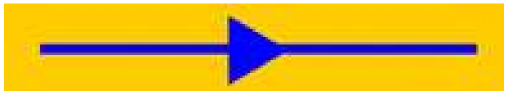
Geometrical properties of the Hamiltonian

Topology driven states

Hall insulator

2D system





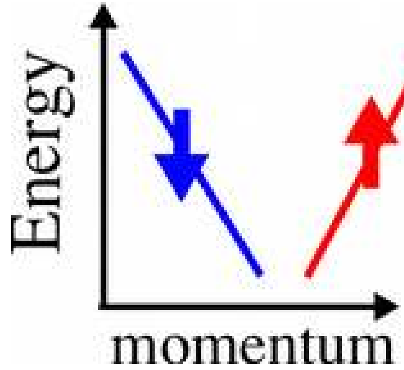
1D Chiral metal

Absence of longitudinal resistance

Electronics

Spin Hall insulator

2D system



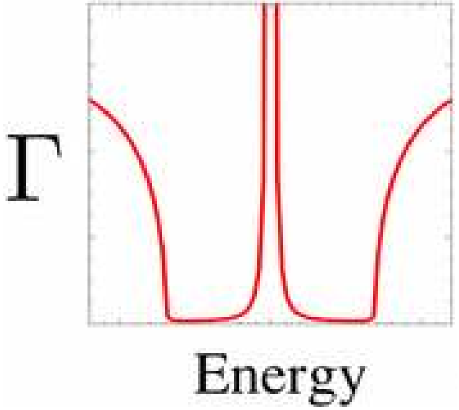
1D Helical metal

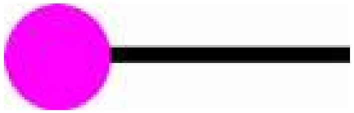
Pure spin flow without charge flow

Spintronics

Topological superconductors

1D system





0D Majorana bound states

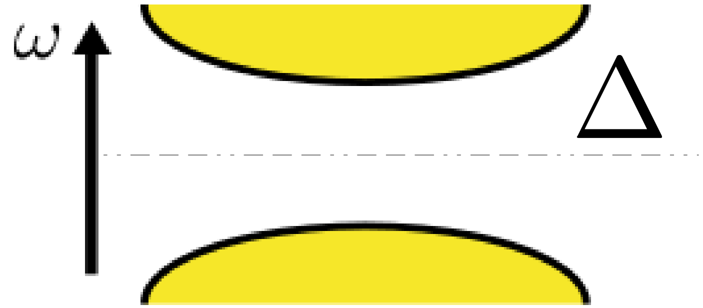
Anyonic braiding statistics

Topological quantum computing

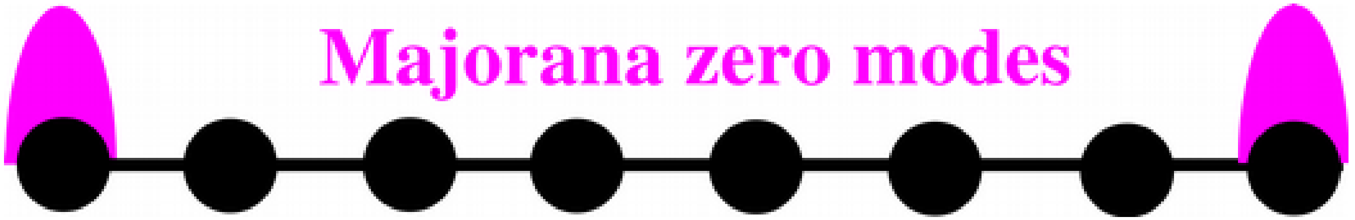
Topological superconductors

Topological superconductors with broken time reversal symmetry

Gapped bulk excitations

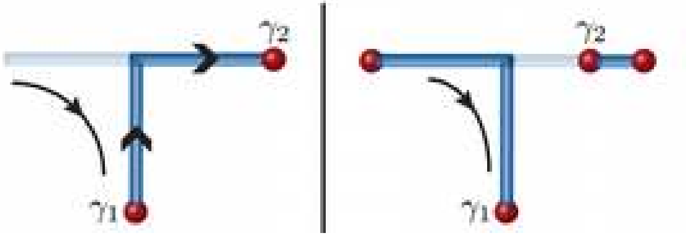


Gapless surface modes, single Majorana mode per edge



A. Y. Kitaev. Physics-Uspekhi, 44:131, 2001

Anyonic statistics, suitable for quantum computing



Nature Physics 7, 412-417 (2011)

Engineering topological phases of matter

Topological superconductors offer extraordinary opportunities both for fundamental and applied science

But!

Topological superconductors are extremely rare in nature

Can we create an artificial topological superconductor using conventional materials?

C.W.J. Beenakker, Ann. Rev. Condens. Matter Phys. 4, 113-136 (2013)

J. Alicea, Rep. Prog. Phys. 75, 076501 (2012)

The conventional way to engineer a topological superconductor

Step #1: Start with a helical electron gas

Step #2: Open a gap by proximity effect to a conventional superconductor

Step #3: Determine the conditions to be in a topological phase

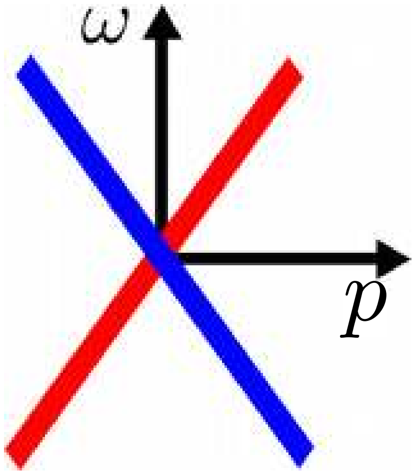
C.W.J. Beenakker, Ann. Rev. Condens. Matter Phys. 4, 113-136 (2013)

J. Alicea, Rep. Prog. Phys. 75, 076501 (2012)

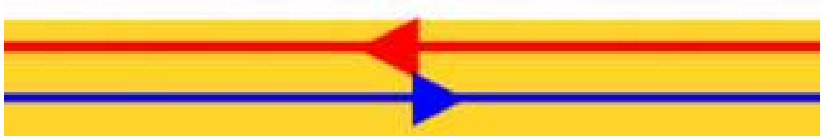
How to do your own topological superconductor

Ingredients

- s-wave pairing
- Helical states



Why helical states?



→ We would like a single channel
→ We need paring between +k and -k
→ s-wave only couples opposite spins

Objective: realize a spin-less 1D superconductor

The minimal model for a 1D topological superconductor

One dimensional spinless p-wave superconductor (Kitaev model)

$$H = \sum_n t c^\dagger_{n+1} c_n + \Delta c_n c_{n+1} + c.c.$$

p-wave superconductivity

A. Y. Kitaev. Physics-Uspekhi, 44:131, 2001

$$c_k = \sum_n e^{ikn} c_n$$

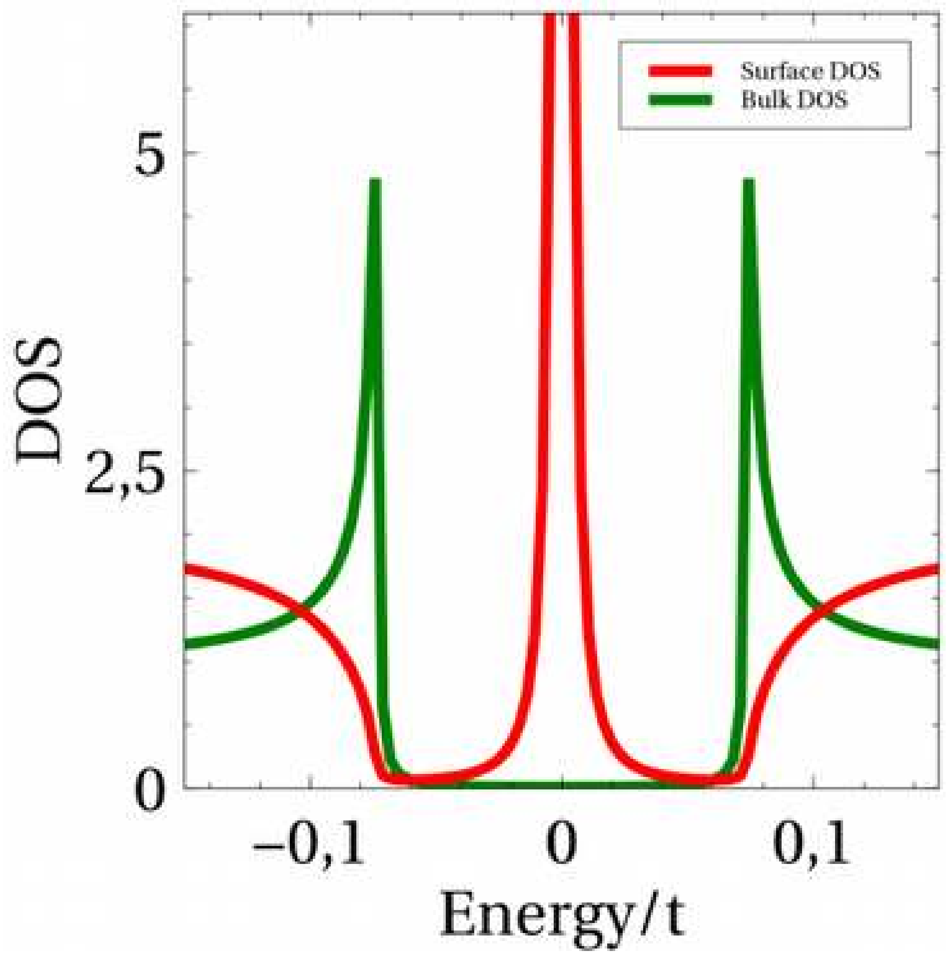
Change of basis

$$H = \sum_k \epsilon_k c^\dagger_k c_k + i\Delta \left[c_{-k} c_k \sin k - c^\dagger_{-k} c^\dagger_k \sin k \right]$$

$$\Delta(k) = -\Delta(-k)$$

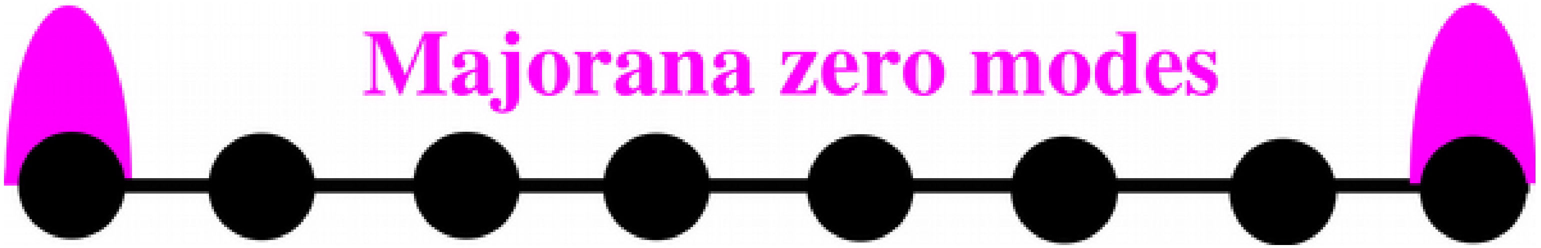
How is the spectrum of this model?

Spectrum of a topological superconductor



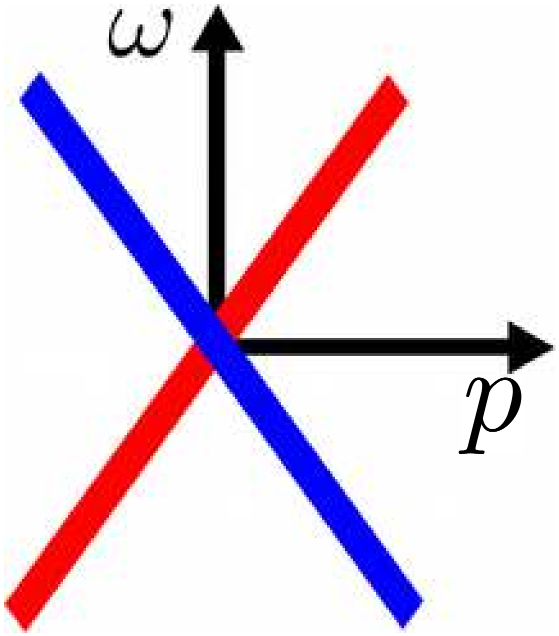
Bulk is gapped

Edge has a zero Majorana mode



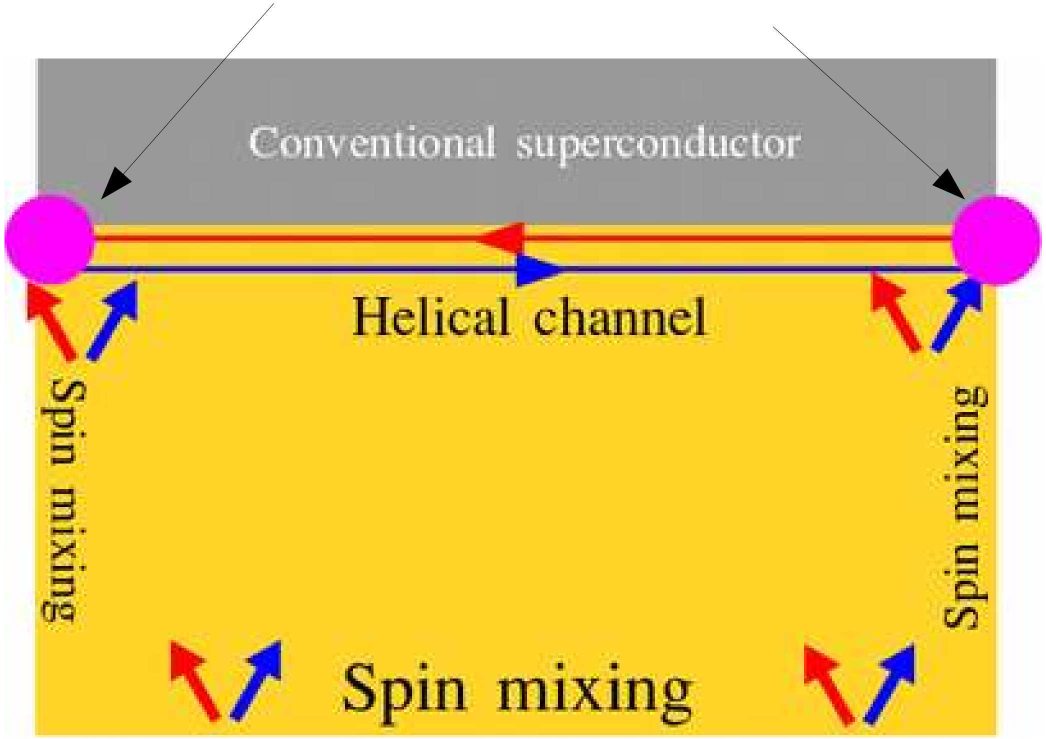
Majoranas with a topological insulator

Induce superconductivity in a helical channel



Surface a of a quantum spin Hall insulator

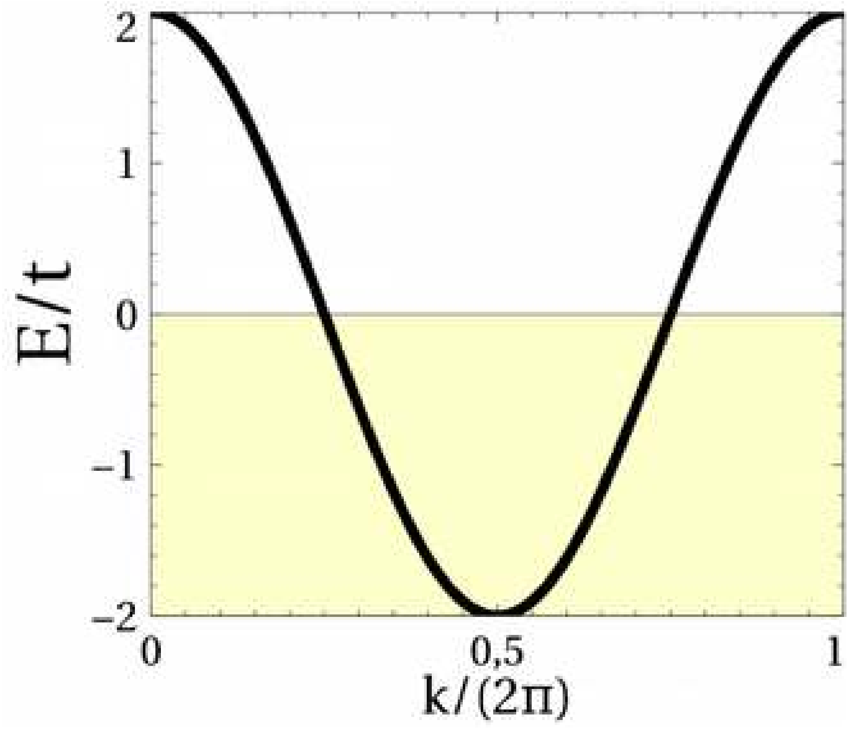
Majorana bound states



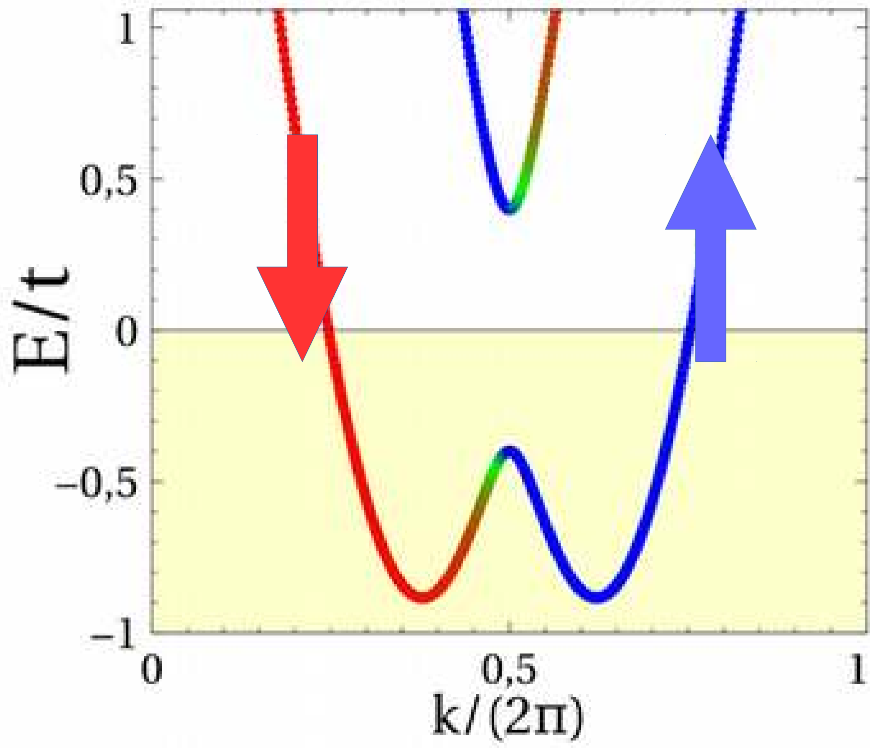
Liang Fu and C. L. Kane
Phys. Rev. Lett. 100, 096407 (2008)

Helical channels in Rashba wires

1D electron gas



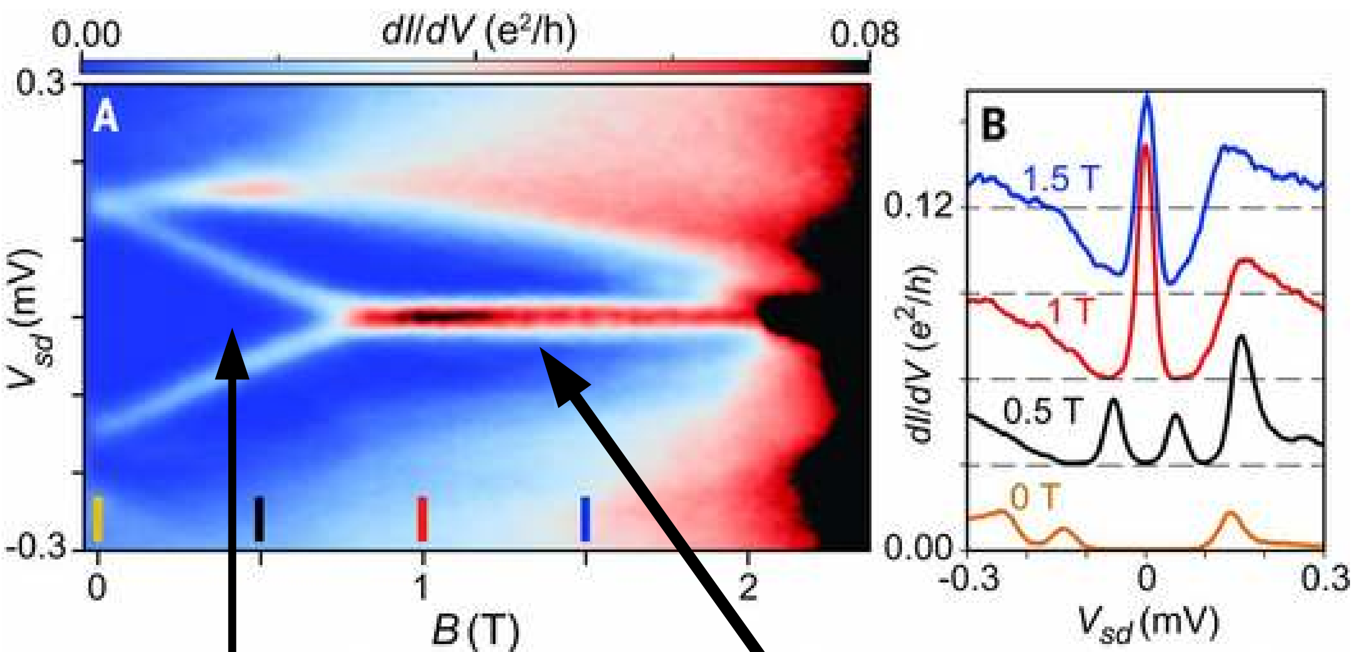
1D electron gas with Rashba and exchange



Phys. Rev. Lett. 104, 040502 (2010)
Phys. Rev. Lett. 105, 077001 (2011)

Helical channels triggered by Rashba coupling

Majorana in 1D Rashba wires

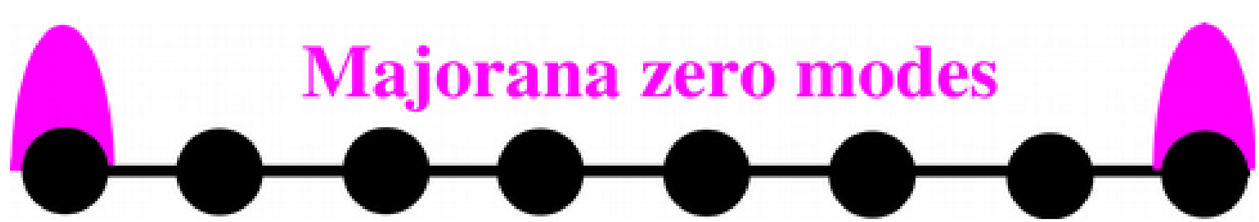


Nanowires & atomic chains

Science 336.6084 (2012)
Science 346.6209 (2014)
Science 354.6319 (2016)

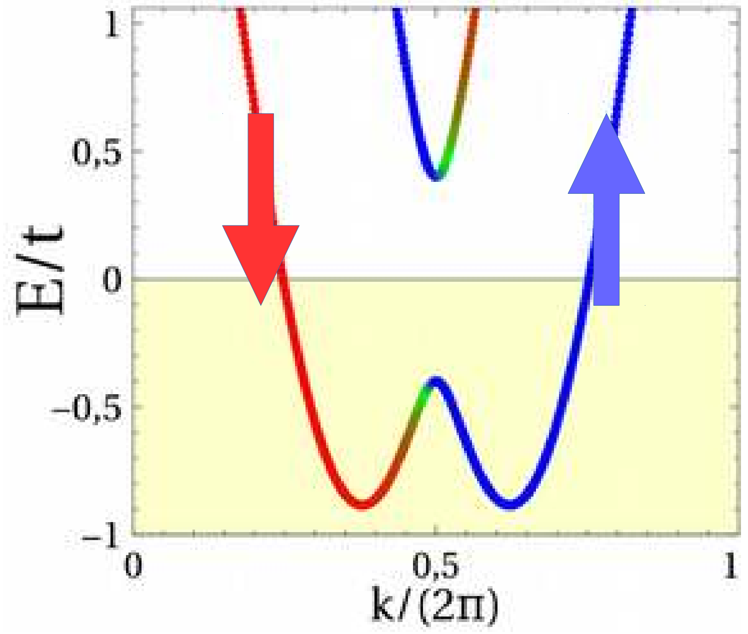
Topological superconductor

Trivial superconductor



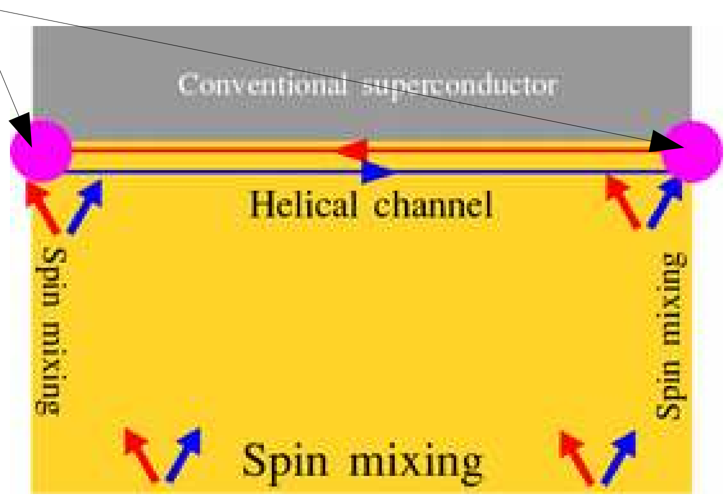
Building an artificial topological superconductor

Electron gas with
Exchange + Rashba SOC



Using helical states of a
topological insulator

Majorana states



Phys. Rev. Lett. 100, 096407 (2008)

These two schemes use an initial metallic state

Can we generate an artificial topological superconductor starting from an insulator?

Our objective

Build a 2D topological superconductor using as building blocks

- A conventional (s-wave) 3D superconductor
- **A 3D antiferromagnetic insulator**

The solid state platform *Oxide heterostructures*

The prize

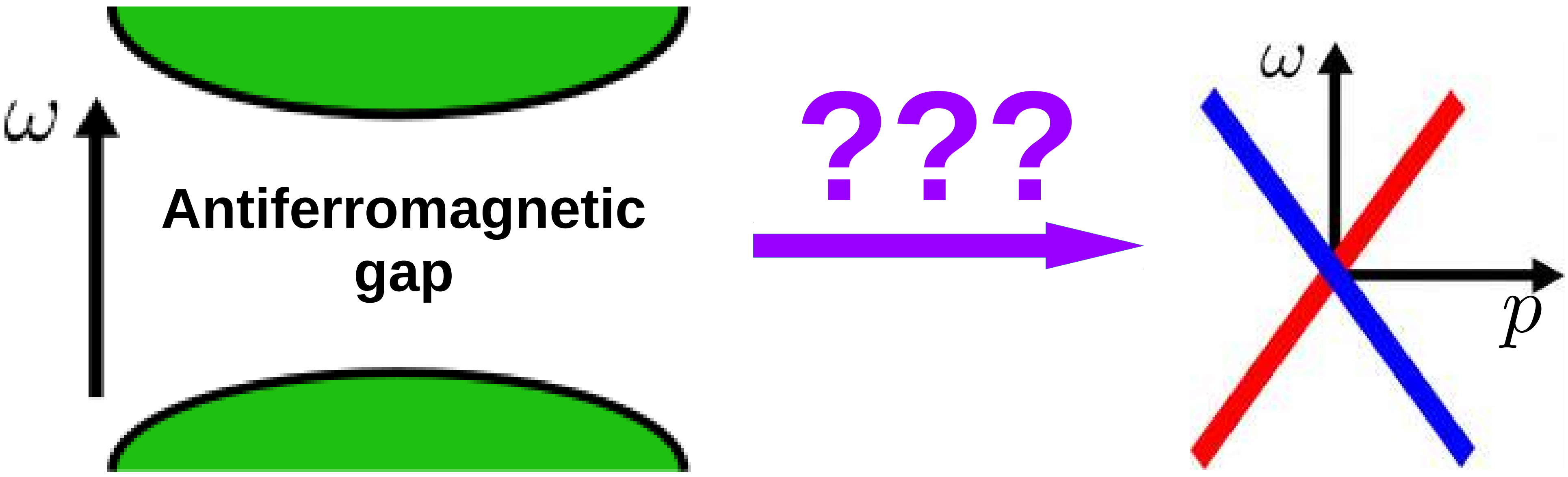
Bringing a new solid state platform to realize artificial topological superconductors

The system for TS in an AF insulator

2d topological superconductor at the interface

The initial problem

How can we get a topological phase starting from a trivial insulator?



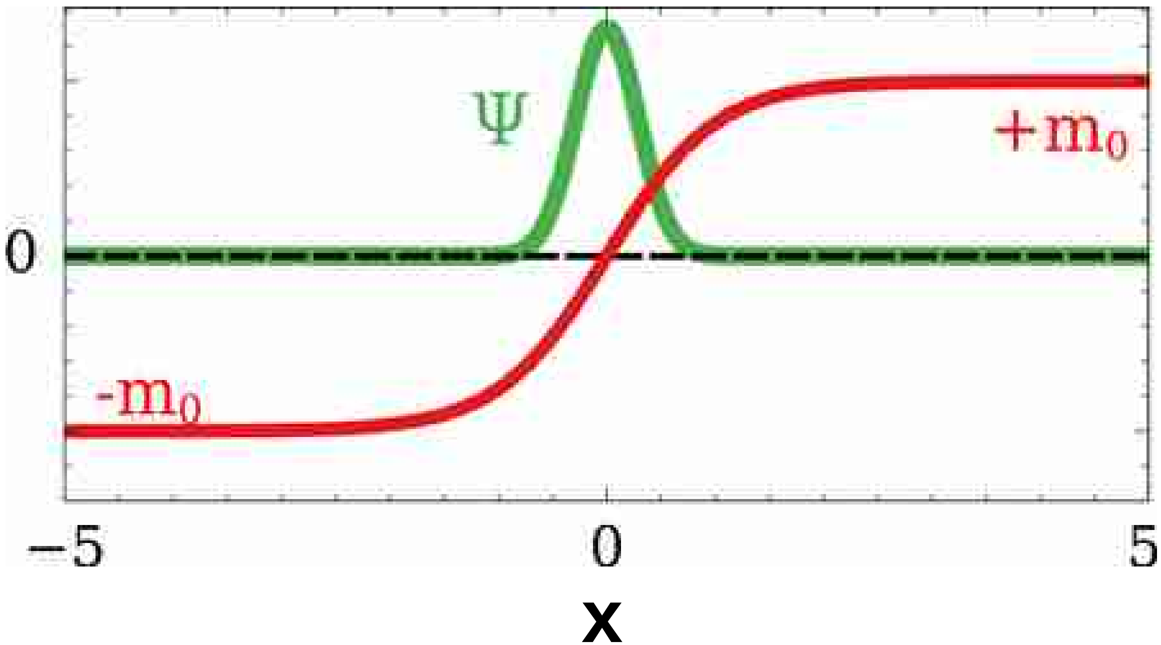
We need to create a "spinless" gapless state out of an insulator

Building an interface gas between two insulators

Lets take a 1d Dirac equation with spatially dependent mass

$$H = \begin{pmatrix} m(x) & p \\ p & -m(x) \end{pmatrix} = \sigma_x p + m(x)\sigma_z$$

$$m(\infty) = -m(-\infty) = m_0$$



Zero energy solution bound to the interface

$$\Psi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} e^{-\int_0^x m(\lambda) d\lambda}$$

$$H\Psi = 0$$

R. Jackiw and C. Rebbi
Phys. Rev. D 13, 3398 (1976)

Interface modes between a 1D AF and a 1D SC

Kinetic part

Antiferromagnetic order

Superconducting order

$$H = H_{kin} + H_{AF} + H_{SC}$$

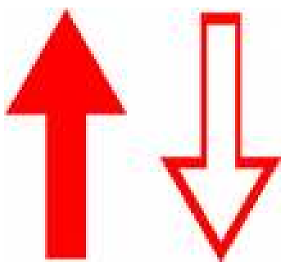
Interface modes between a 1D AF and a 1D SC

Total Hamiltonian

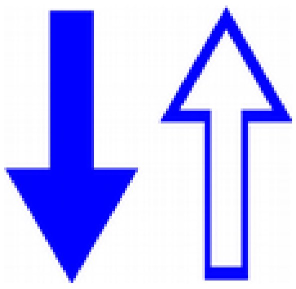
$$H = H_{kin} + H_{AF} + H_{SC}$$

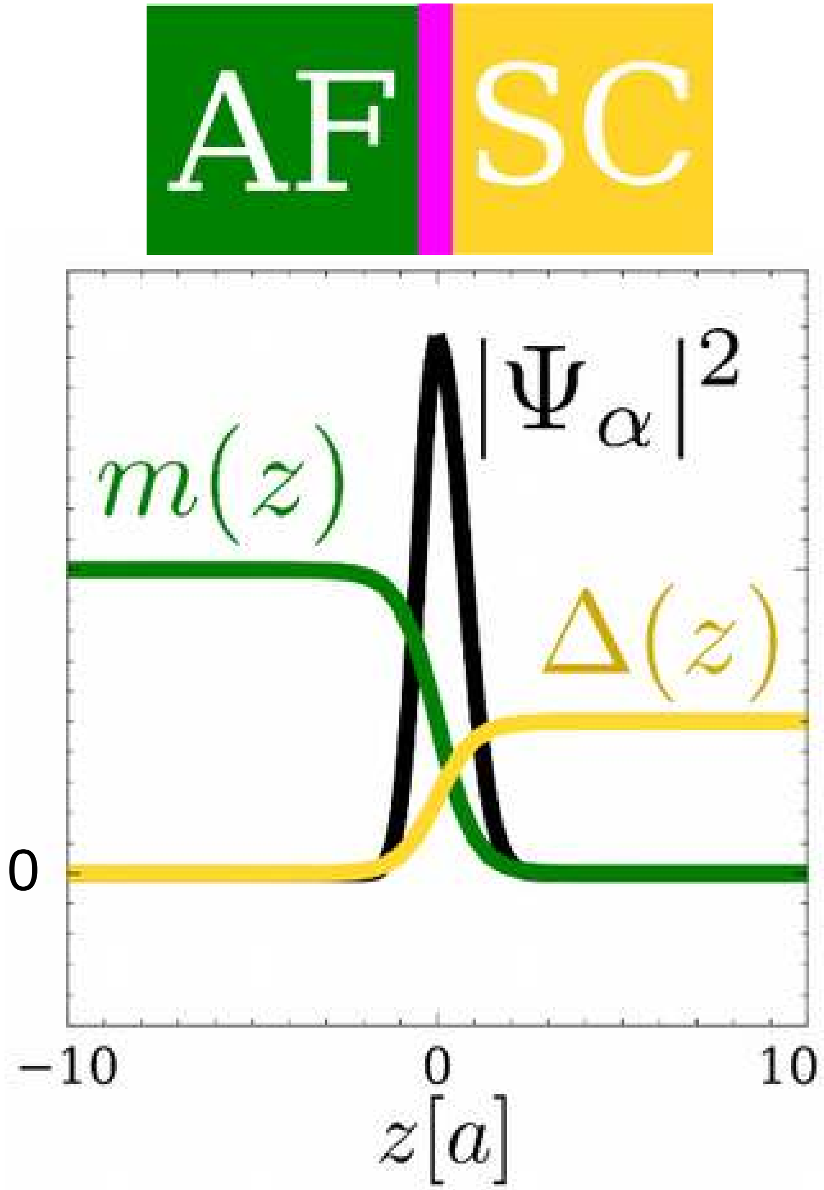
There will be two zero solutions

Sector #1
Up electron, down hole



Sector #2
Down electron, up hole





Phys. Rev. X 5, 041042 (2015)

Looking for a minimal lattice model hosting Dirac equations

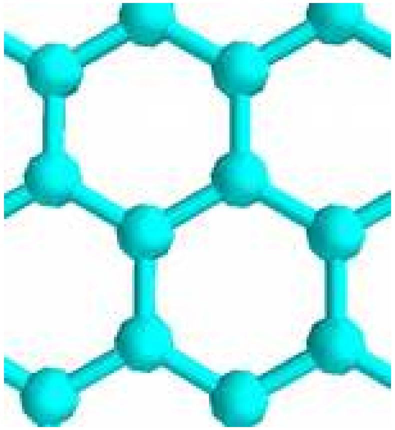
Bloch Hamiltonian

k.p Hamiltonian

$$H_k = \begin{pmatrix} 0 & f(\vec{k}) \\ f^*(\vec{k}) & 0 \end{pmatrix} = \begin{pmatrix} 0 & k_x + ik_y \\ k_x - ik_y & 0 \end{pmatrix}$$

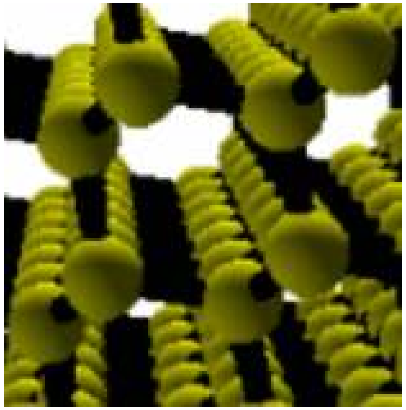
In 2D lattice

Honeycomb lattice (graphene)



In a 3D lattice

Diamond lattice



The full Hamiltonian

Tight binding model in a diamond lattice

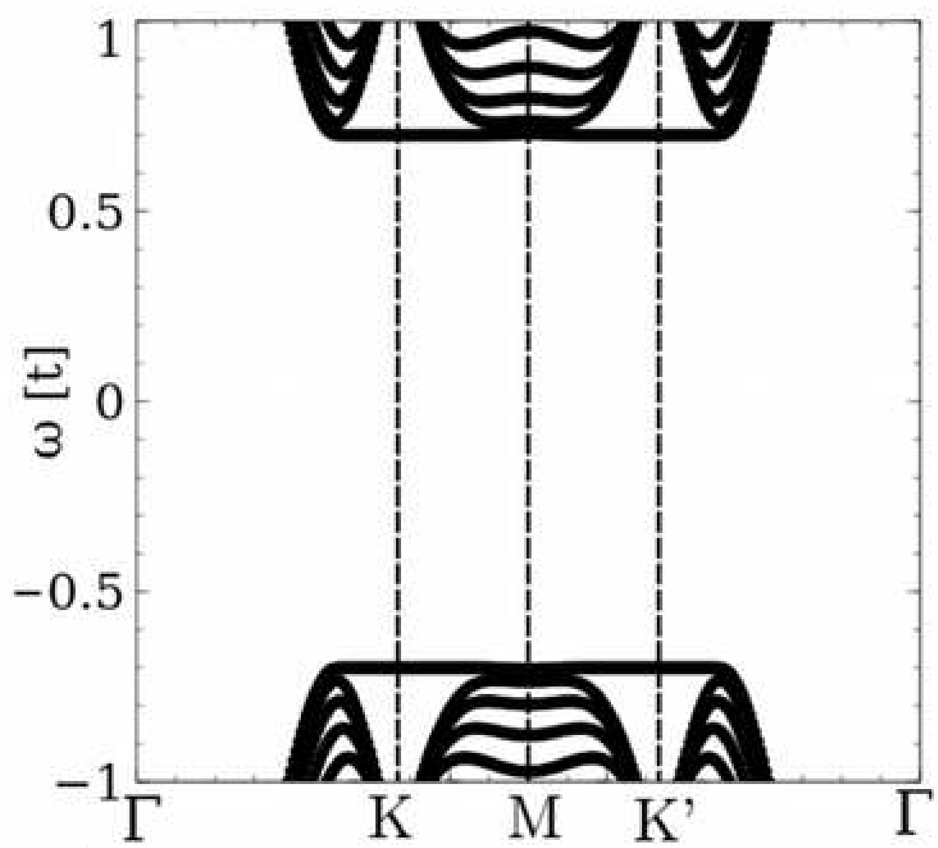
$$\hat{H} = \hat{H}_{kin} + \hat{H}_{AF} + \hat{H}_{SC} + \hat{H}_{SOC}$$

Kinetic energy	$\hat{H}_{kin} = \sum_{\langle ij\rangle,s} t_{ij} c^\dagger_{i,s} c_{j,s} - \sum_{i,s} \mu(z_i) c^\dagger_{i,s} c_{i,s}$
Antiferromagnetic order	$\hat{H}_{AF} = \sum_{i,s,s'} m(z_i) \tau_z^{i,i} \sigma_z^{s,s'} c^\dagger_{i,s} c_{i,s'}$
Superconducting order	$\hat{H}_{SC} = \sum_i \Delta(z_i)[c_{i,\downarrow} c_{i,\uparrow} + c^\dagger_{i,\uparrow} c^\dagger_{i,\downarrow}]$
Intrinsic spin-orbit coupling	$\hat{H}_{SOC} = \sum_{\langle\langle ij\rangle\rangle} i\Lambda \vec{\sigma}^{s,s'} \cdot (\vec{r}_{il} \times \vec{r}_{lj}) c^\dagger_{i,s} c_{j,s'}$

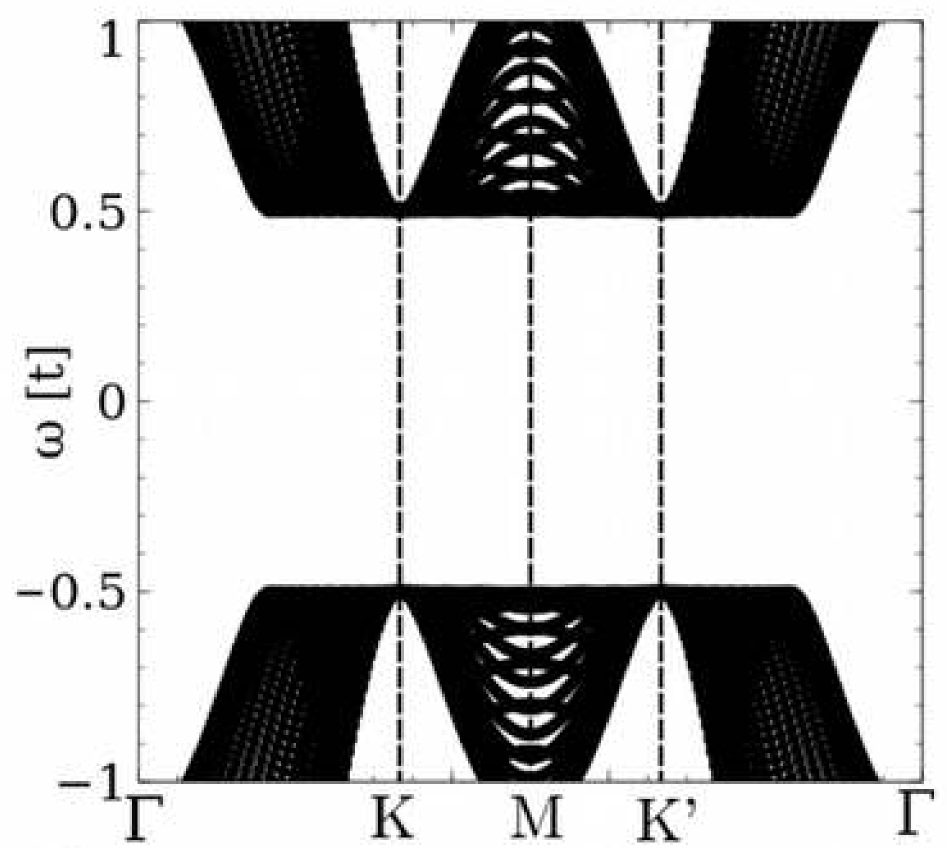
We will create an interface between a 3d antiferromagnet and a 3d superconductor

Spectra of isolated AF and SC

Antiferromagnet

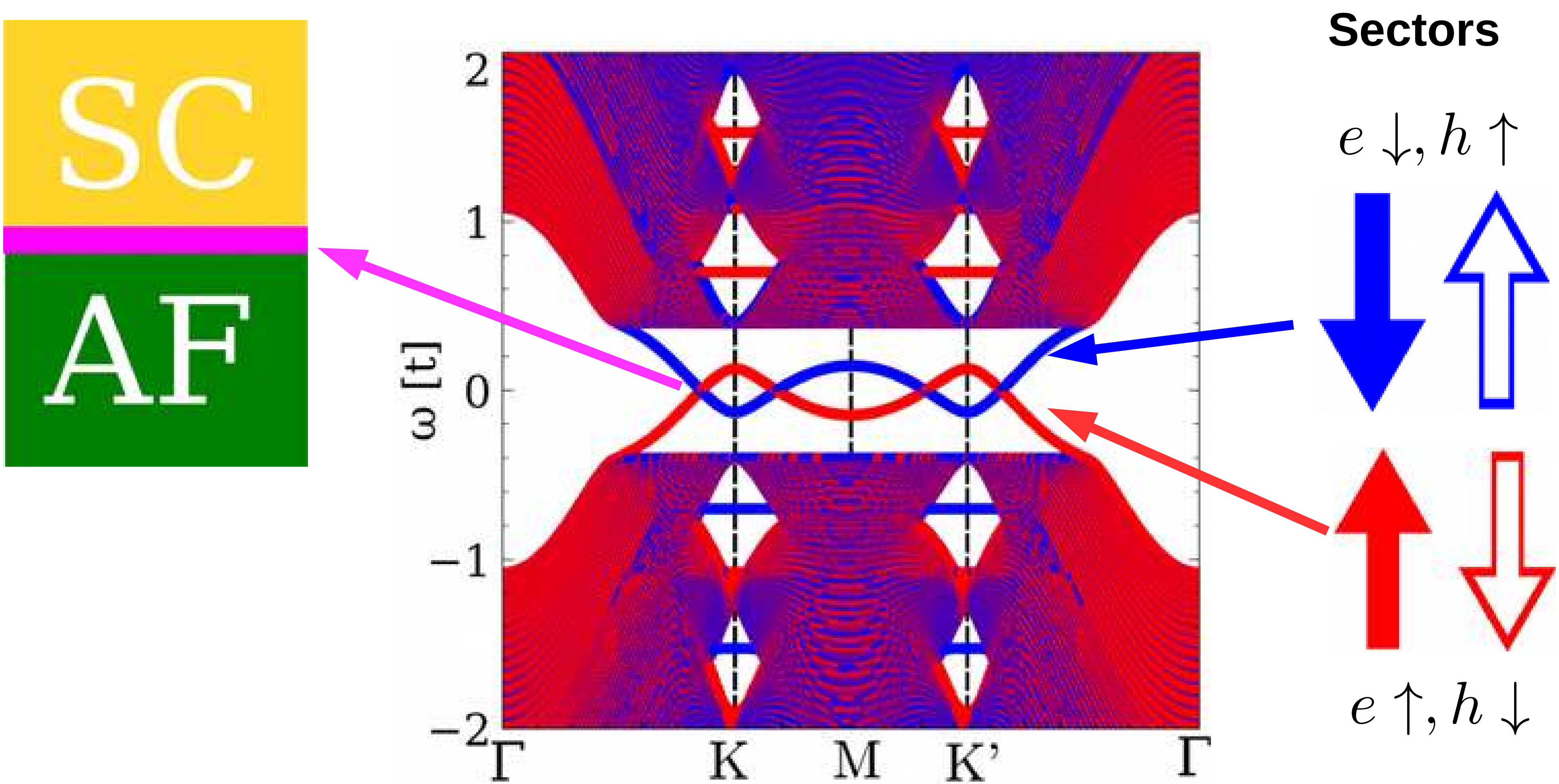


Superconductor



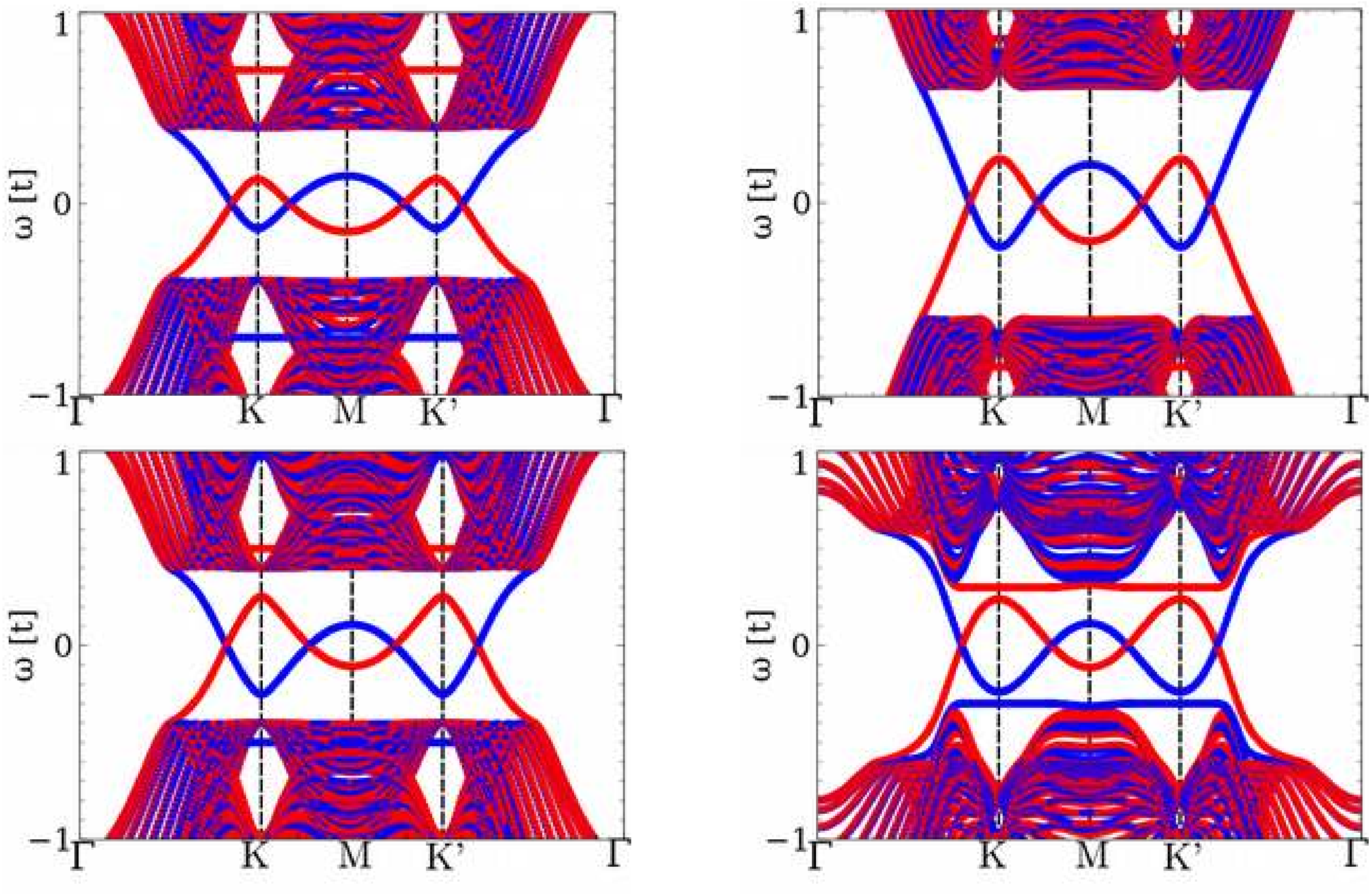
Both independent systems have a gap

Spectra of an antiferromagnet-superconductor heterostructure



The combined system AF-SC is gapless

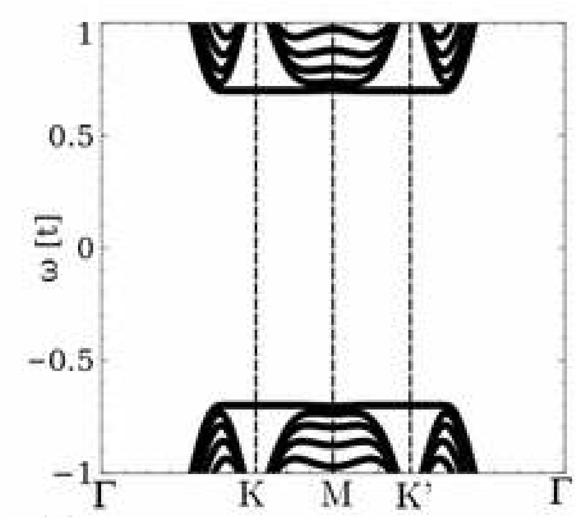
Spectra of AF-SC, different choices of parameters



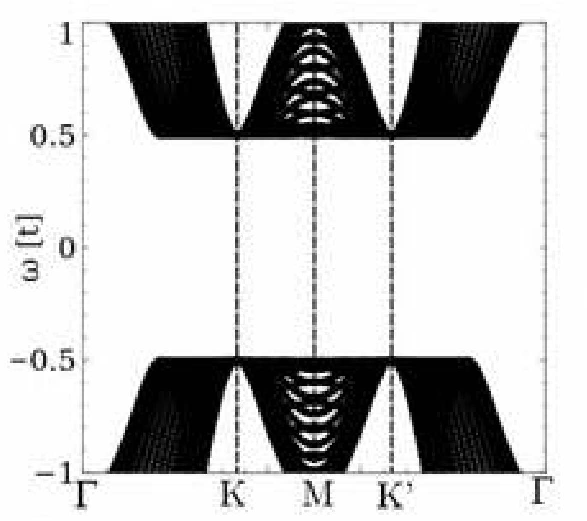
In-gap modes appear for generic antiferromagnetic and superconducting parameters

Emergence of interfacial modes

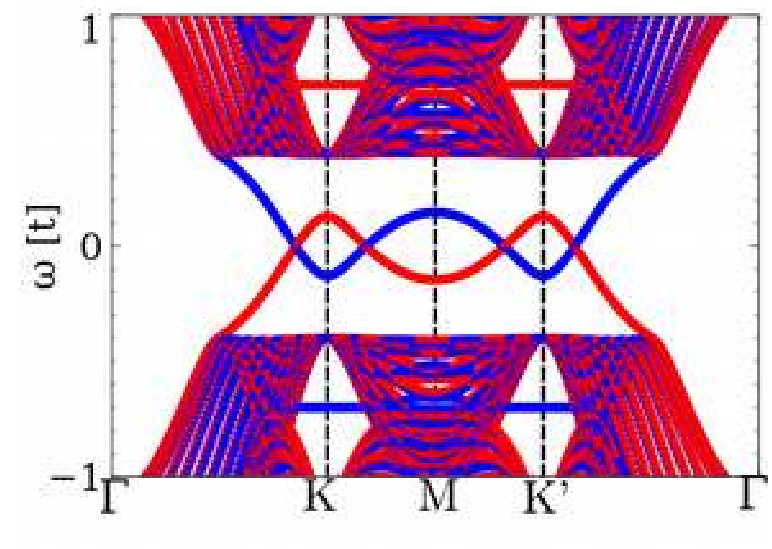
Antiferromagnet



Superconductor

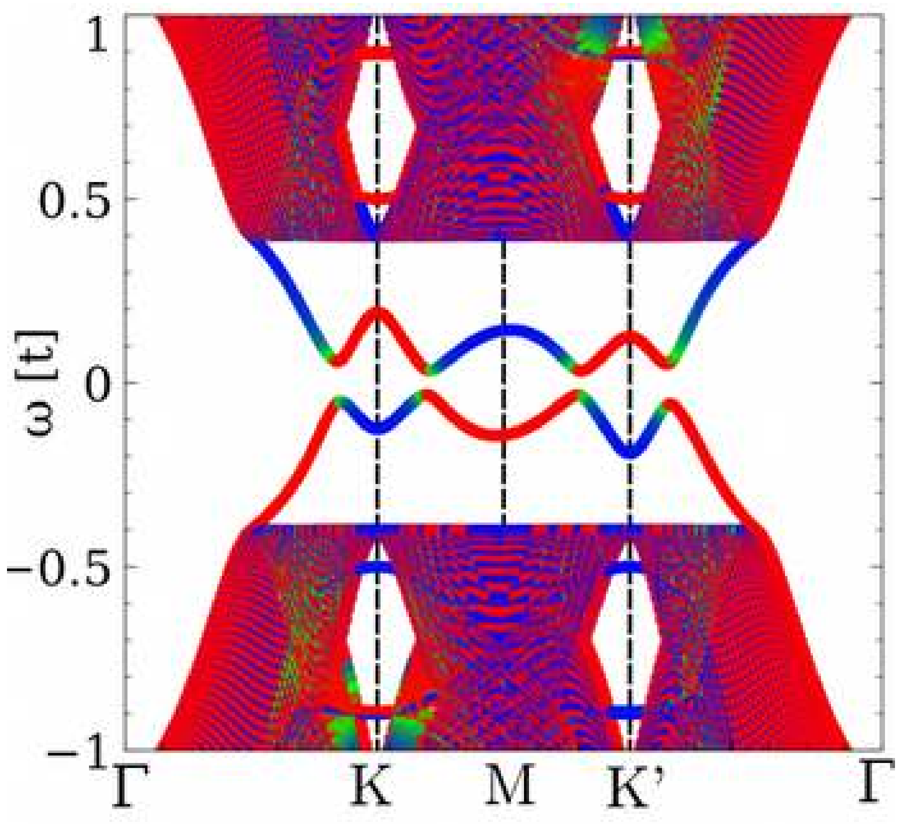


AF/SC heterostructure

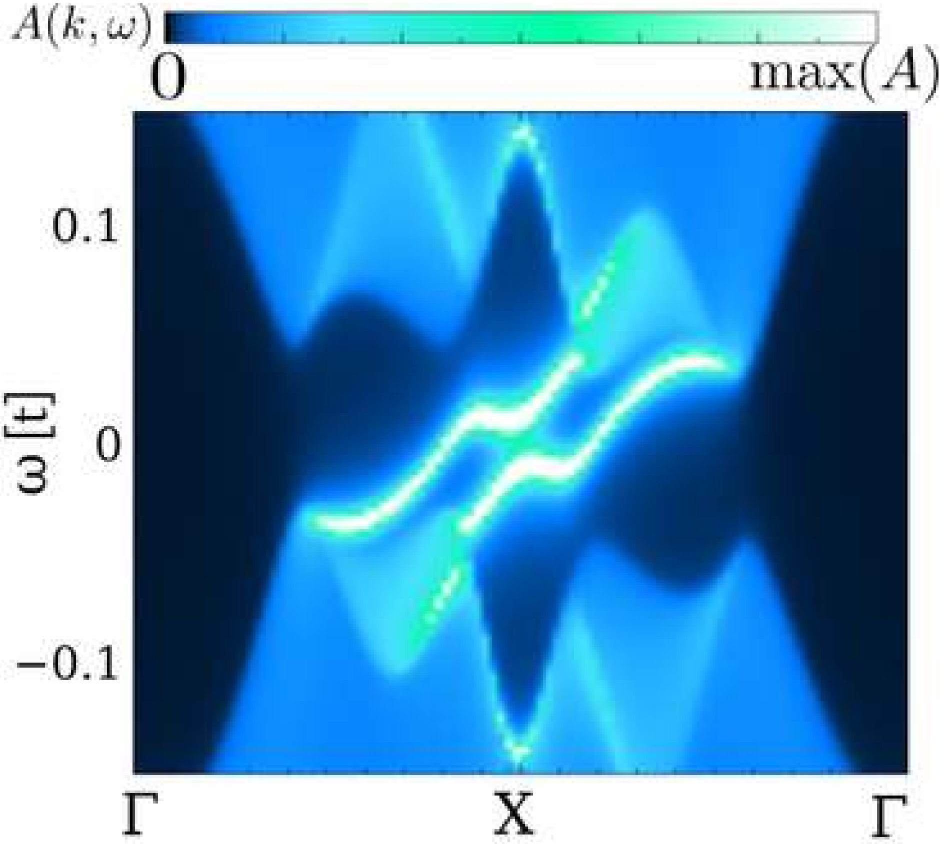


Adding spin-orbit coupling

Band structure

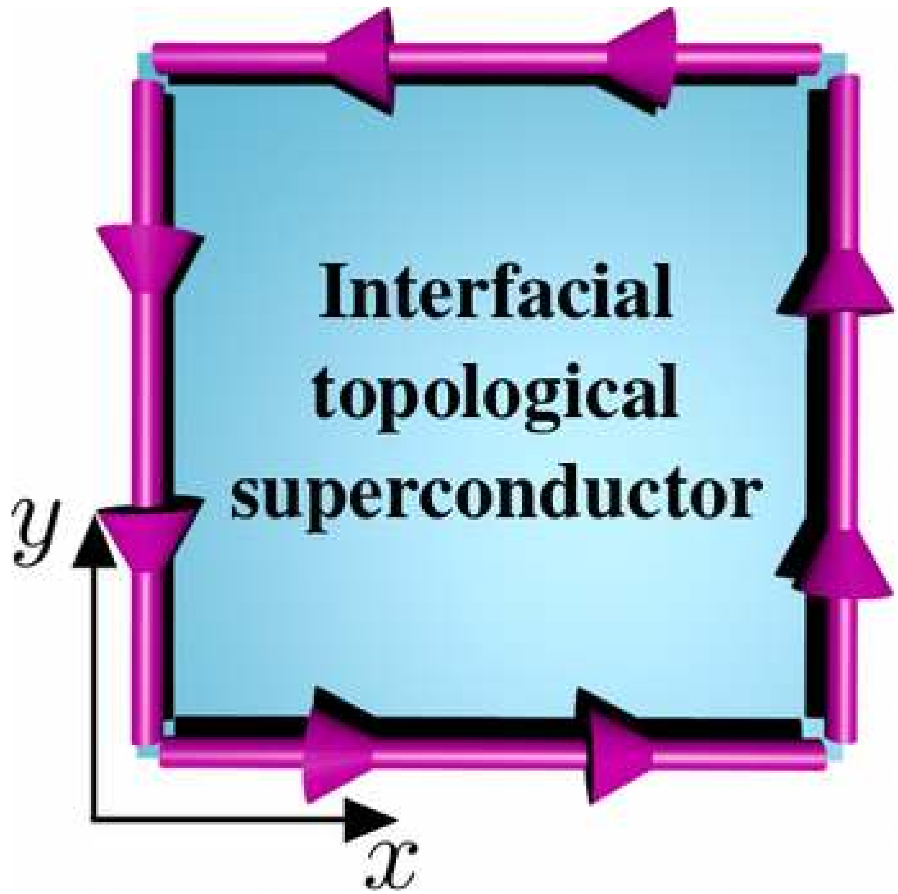


Edge spectral function

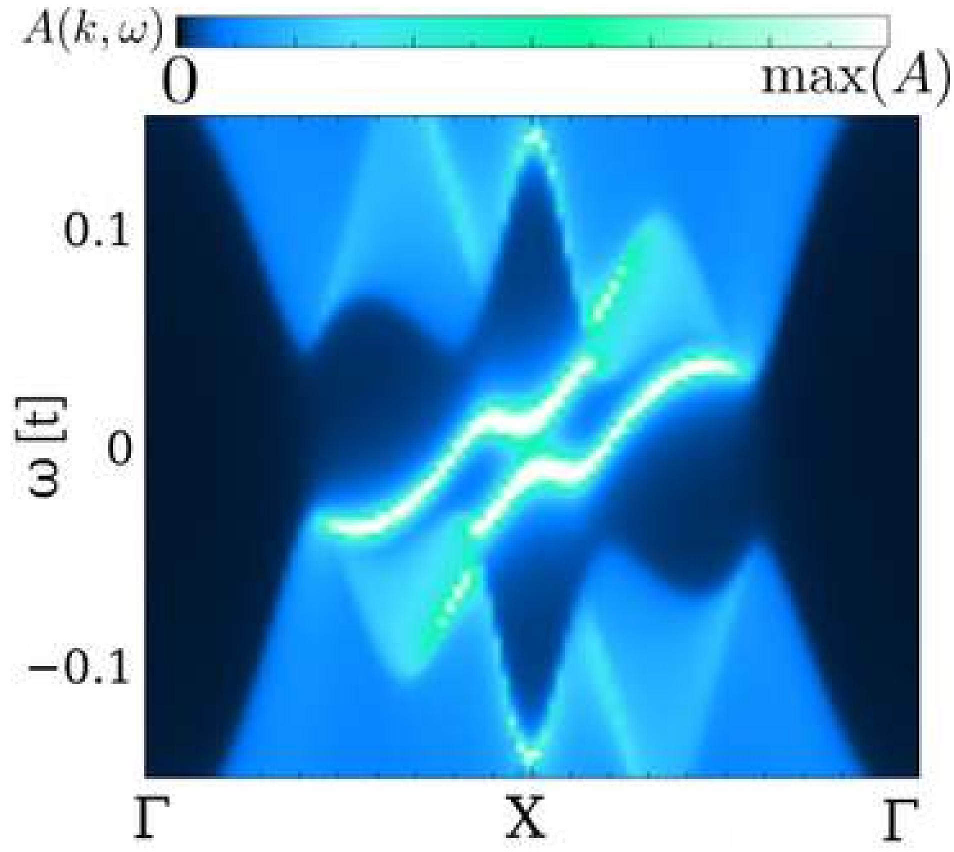


Two co-propagating edge modes

An interfacial topological superconductor



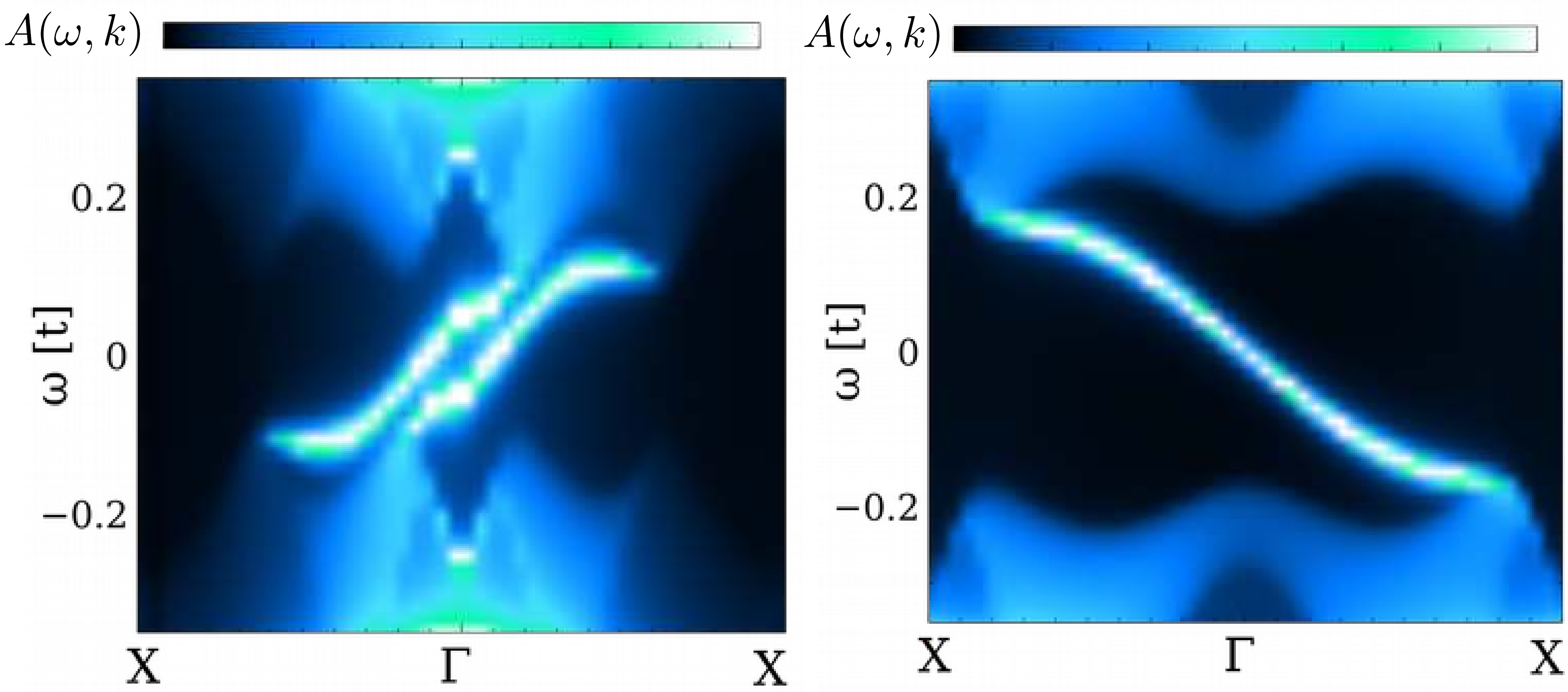
Edge spectral function



The interface of the heterostructure realized a topological superconductor

Topological state with strain

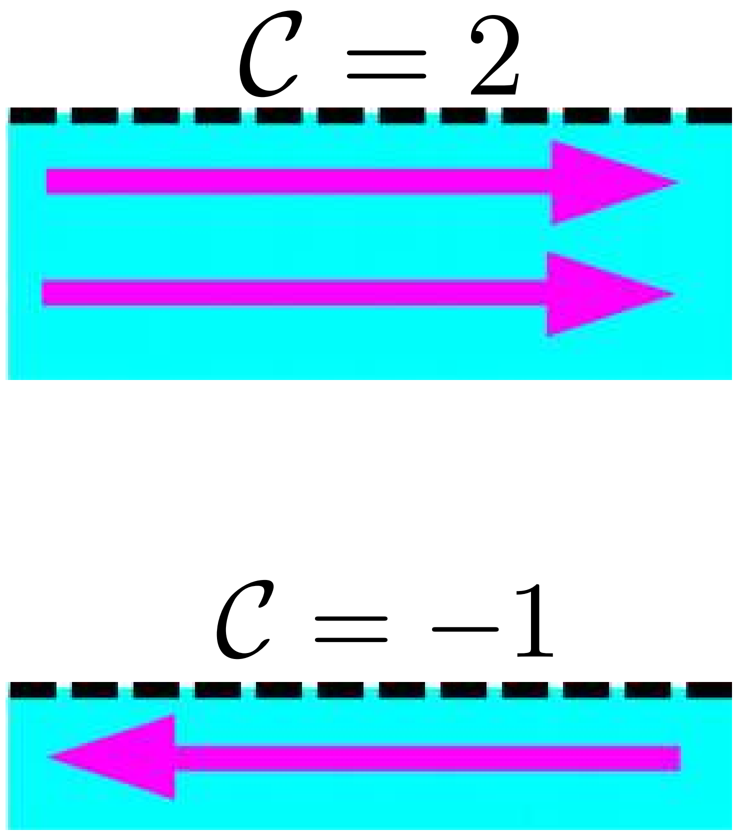
Tensile strain | **Compressive strain**

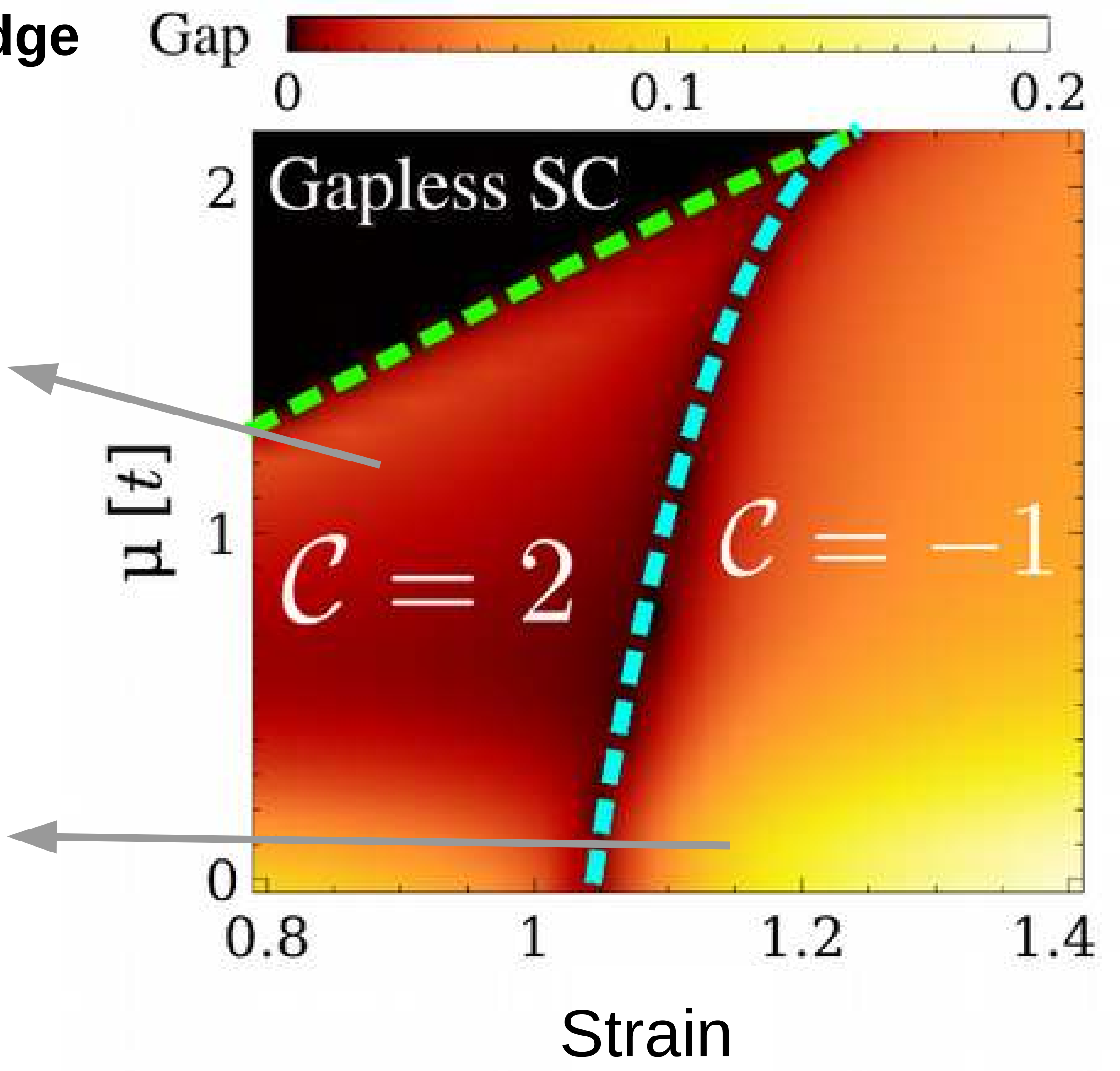


Two propagating modes | **One propagating mode**

Phase diagram with strain and doping

Propagating modes at the edge



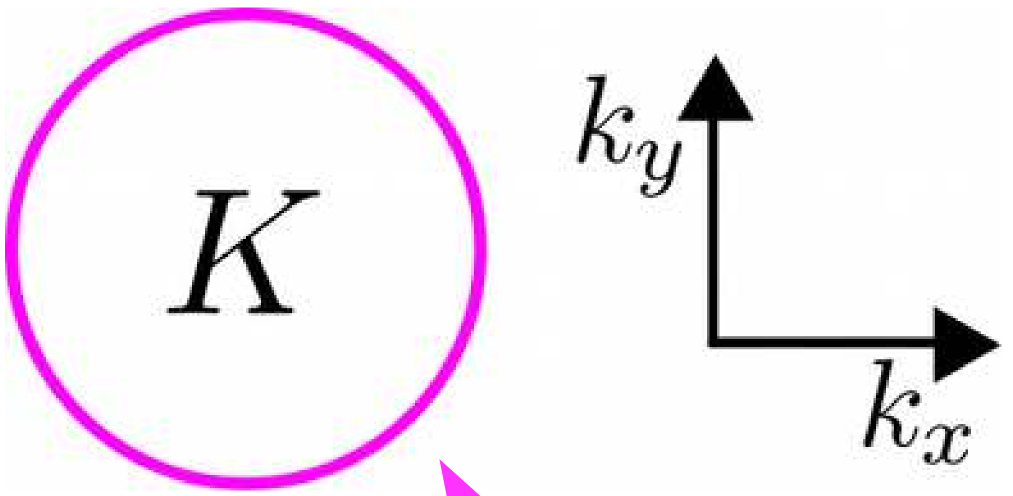


Open questions on the origin of the topological superconductor

- What is the ultimate origin of the interface modes?
- How is that spin-orbit coupling opens up a topological gap in the interface modes?
- Why is there a transition between topological phases with strain?

Understanding the origin of the interface modes

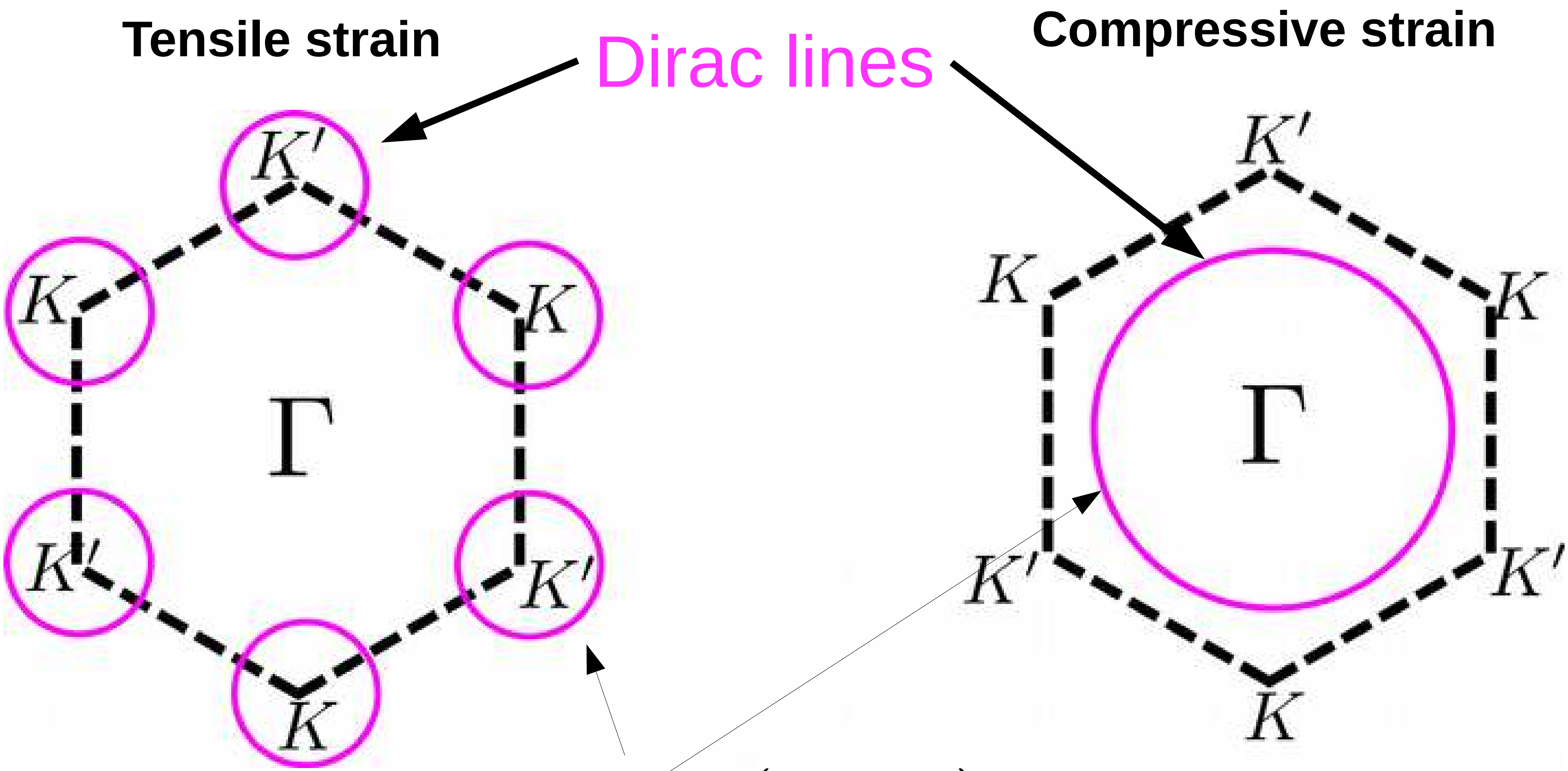
Lets take the Brillouin zone of the heterostructure



We define the **"Dirac line"** as the set of momenta where the Hamiltonian takes the form

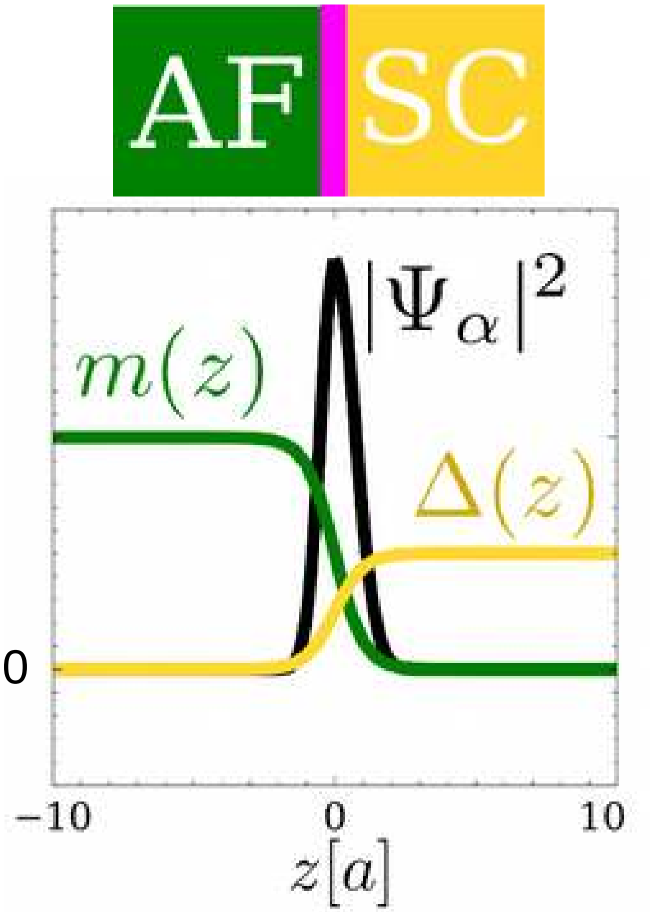
$$H = \begin{pmatrix} 0 & p_z \\ p_z & 0 \end{pmatrix}$$

Dirac lines in the diamond lattice

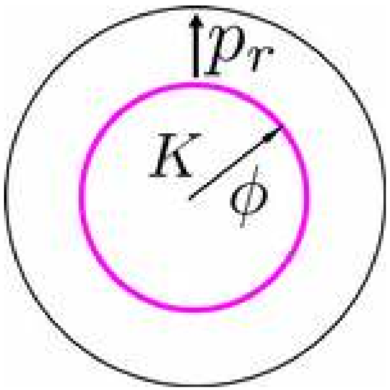


$$H = \begin{pmatrix} 0 & p_z \\ p_z & 0 \end{pmatrix}$$

Interface modes in an AF-SC heterostructure



Take local coordinates close to the Dirac line



$$H_{kin} = \begin{pmatrix} 0 & p_z + ip_r \\ p_z - ip_r & 0 \end{pmatrix}$$

Solve an inhomogeneous 1D problem

$$H = H_{kin} + H_{AF} + H_{SC}$$

There are two zero energy solutions $H\Psi = 0$

The Dirac line "imprints" zero energy modes in the interface

The mathematical structure of the interface modes

Nambu spinor $\varphi_1^\dagger = \begin{pmatrix} c^\dagger_{A,\vec{k}_\parallel,\uparrow} & c^\dagger_{B,\vec{k}_\parallel,\uparrow} & c_{A,-\vec{k}_\parallel,\downarrow} & c_{B,-\vec{k}_\parallel,\downarrow} \end{pmatrix}$

Hamiltonian

$$h_1 = \begin{pmatrix} m & p_z & \Delta & 0 \\ p_z & -m & 0 & \Delta \\ \Delta & 0 & m & -p_z \\ 0 & \Delta & -p_z & -m \end{pmatrix}$$

$$\mathcal{H}_1 = \varphi_1^\dagger h_1 \varphi_1$$

$$\Delta = \Delta(z)$$

$$m = m(z)$$

Zero energy solution

$$h_1 u_1 = 0$$

$$u_1(z) = \frac{1}{2} \begin{pmatrix} 1 \\ -i \\ -1 \\ -i \end{pmatrix} e^{\int_0^z [m(z') - \Delta(z')] dz'}$$

Asymptotic conditions

$$m(-\infty) = m_0 \qquad m(+\infty) = 0 \qquad \Delta(+\infty) = \Delta_0 \qquad \Delta(-\infty) = 0$$

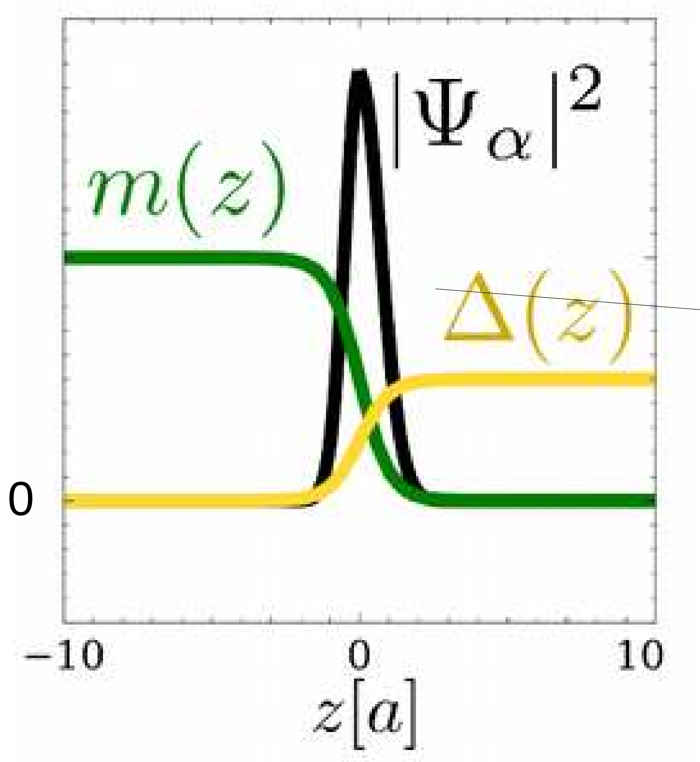
Assuming $m_0 > 0 \quad \Delta_0 > 0$

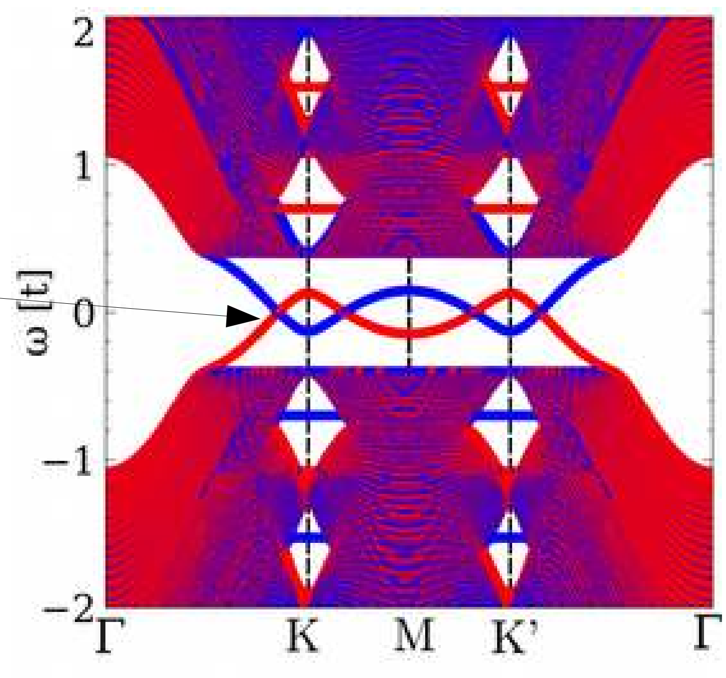
Interface states between AF and SC

Form of the solitonic excitations

$$\Psi_1(z) = g(z)[c_{A,\vec{k}_\parallel,\uparrow} - ic_{B,\vec{k}_\parallel,\uparrow} - c^\dagger_{A,-\vec{k}_\parallel,\downarrow} - ic^\dagger_{B,-\vec{k}_\parallel,\downarrow}]$$

$$\Psi_2(z) = g(z)[c_{A,\vec{k}_\parallel,\downarrow} + ic_{B,\vec{k}_\parallel,\downarrow} - c^\dagger_{A,-\vec{k}_\parallel,\uparrow} + ic^\dagger_{B,-\vec{k}_\parallel,\uparrow}]$$

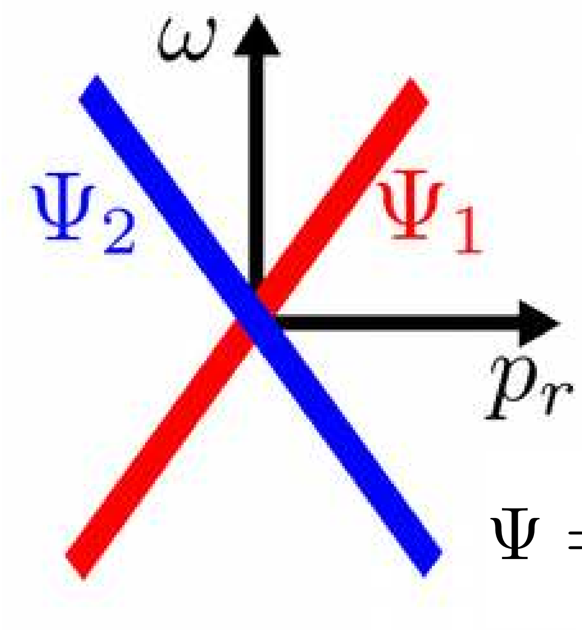




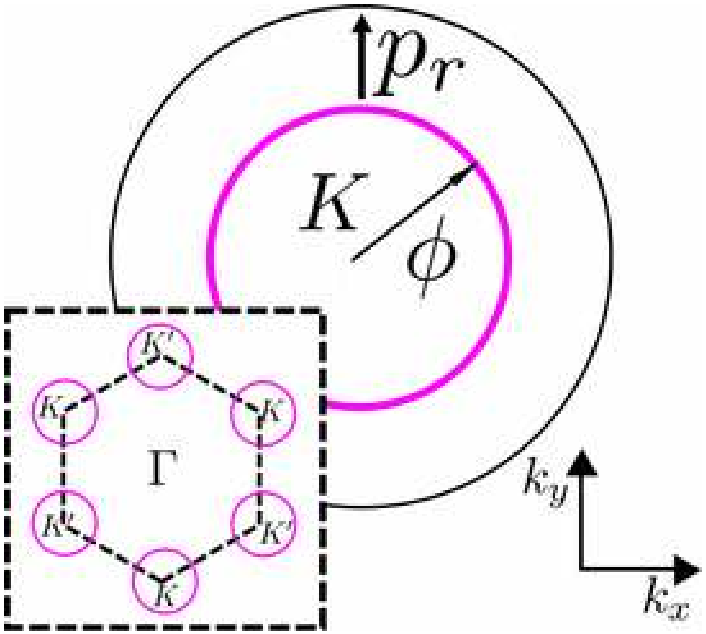
Effective model for the interface states

Effective Hamiltonian

$$\mathcal{H}(p_r, \phi) = \Psi^\dagger \begin{pmatrix} v_r p_r & 0 \\ 0 & -v_r p_r \end{pmatrix} \Psi$$



$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$



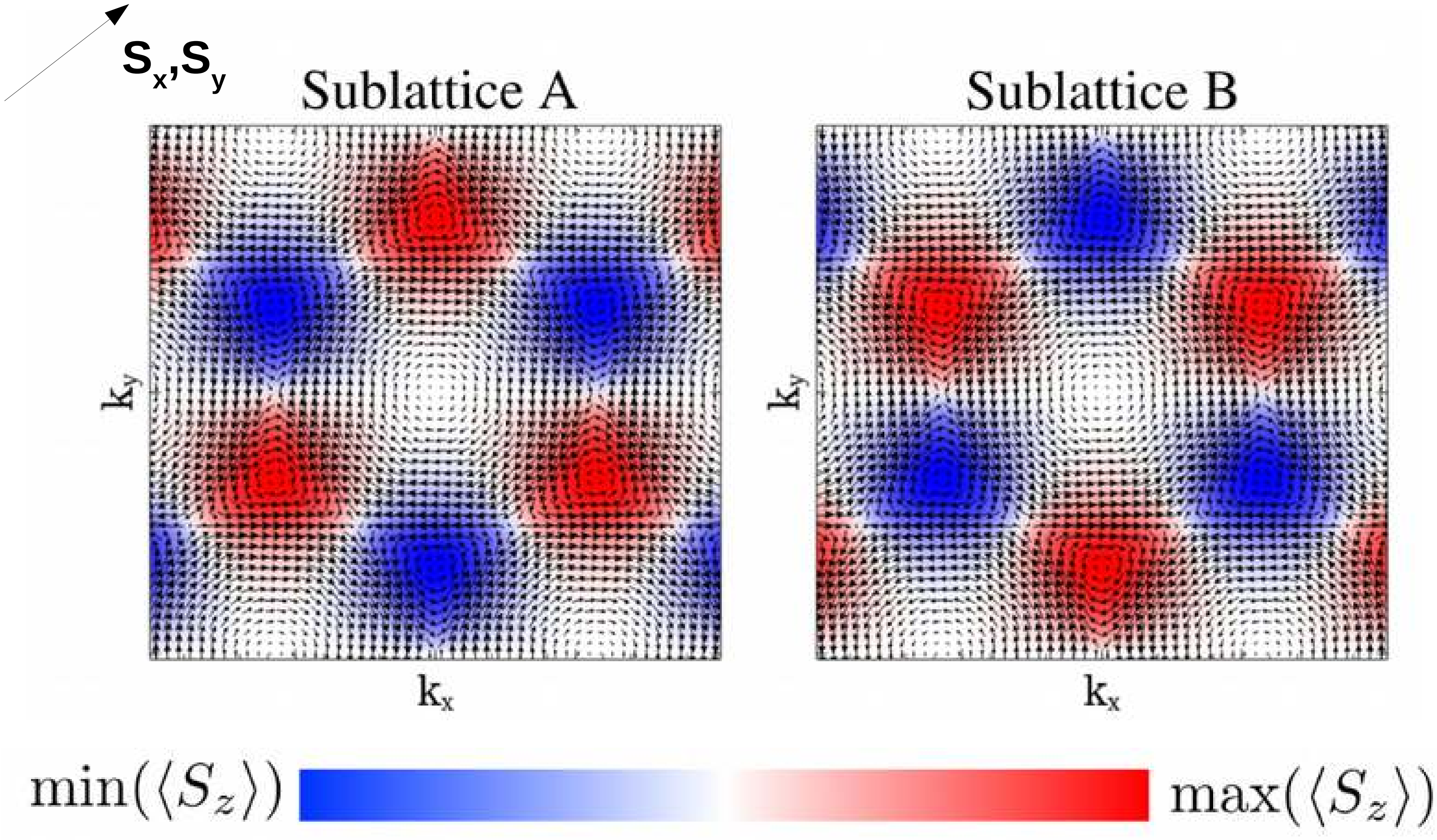
Effective model for the 2D topological superconductor

Can we see how a topological gap opens up when turning on spin-orbit coupling?

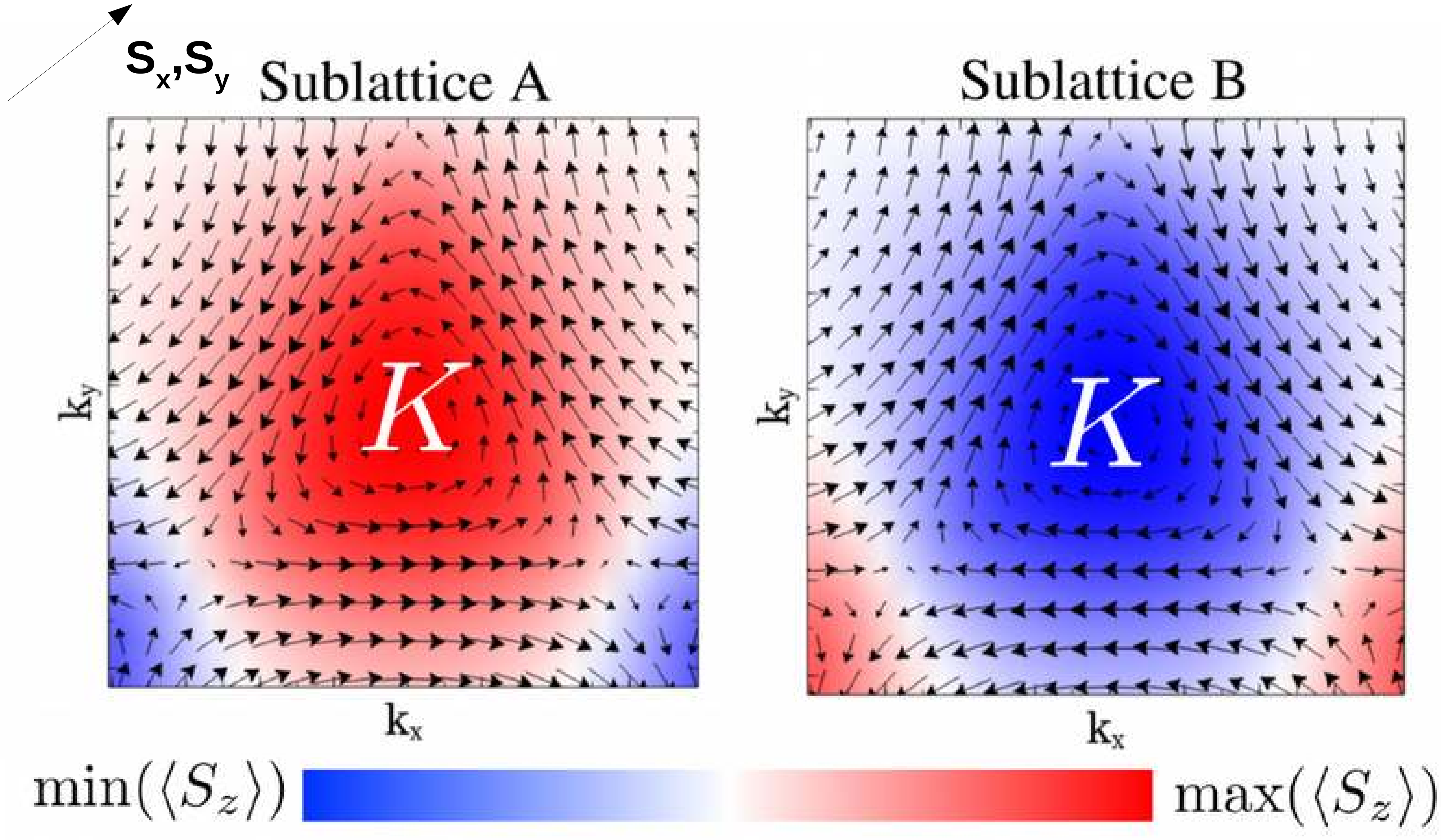
$$\mathcal{H}(p_r,\phi) = \Psi^\dagger \begin{pmatrix} v_r p_r & 0 \\ 0 & -v_r p_r \end{pmatrix} \Psi + \mathcal{H}_{SOC}$$

Hint: project the SOC on the solitonic manifold

How is the intrinsoc SOC in diamond?

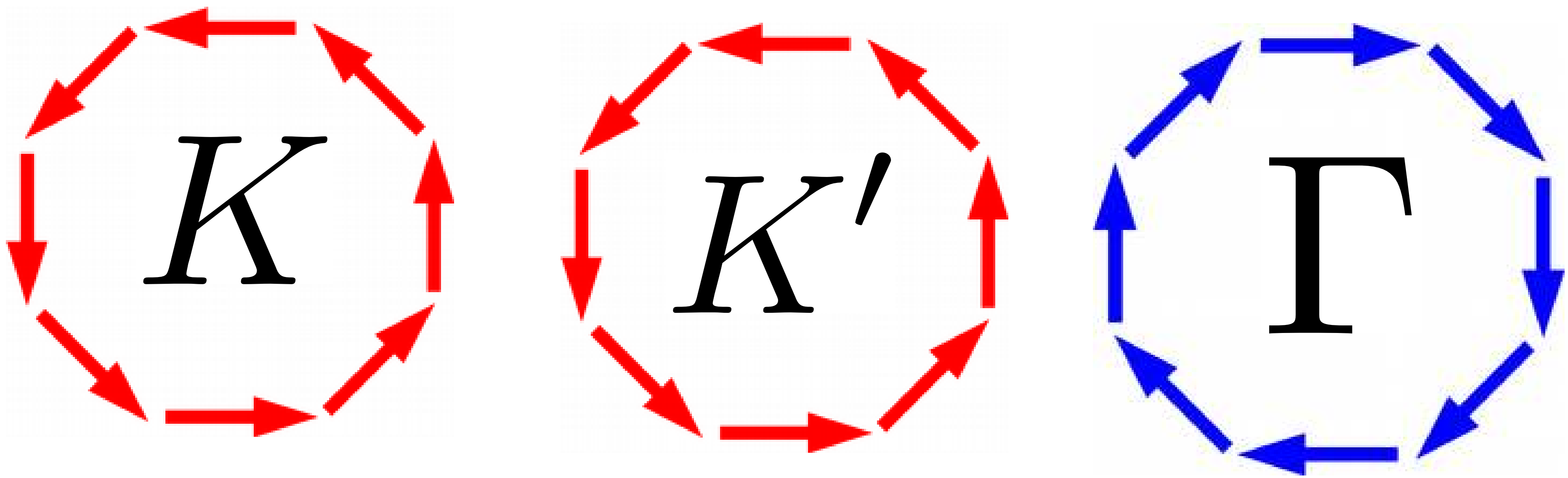


How is the intrinsoc SOC in diamond?



Vorticity induced by intrinsic spin-orbit coupling

SOC exchange induced in the A sublattice



For the B sublattice the exchange is inverted

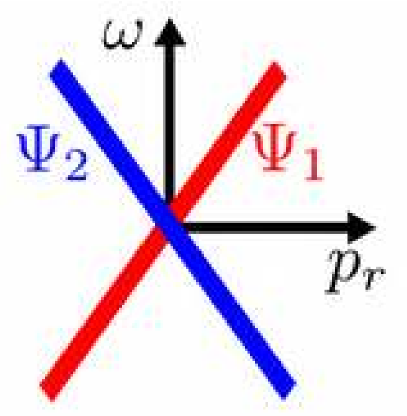
Effective model for the gapped interface modes

$$\hat{H}_{SO} = \lambda \tau_z^{i,i}[-\sin(\phi)\sigma_x^{s,s'} + \cos(\phi)\sigma_y^{s,s'}]c^\dagger_{i,\vec{k}_{||},s} c_{i,\vec{k}_{||},s'}$$

Projecting the Hamiltonian into the interface states we get

$$\mathcal{H}(p_r, \phi) = \Psi^\dagger \begin{pmatrix} v_r p_r & -i\lambda e^{i\phi} \\ i\lambda e^{-i\phi} & -v_r p_r \end{pmatrix} \Psi$$

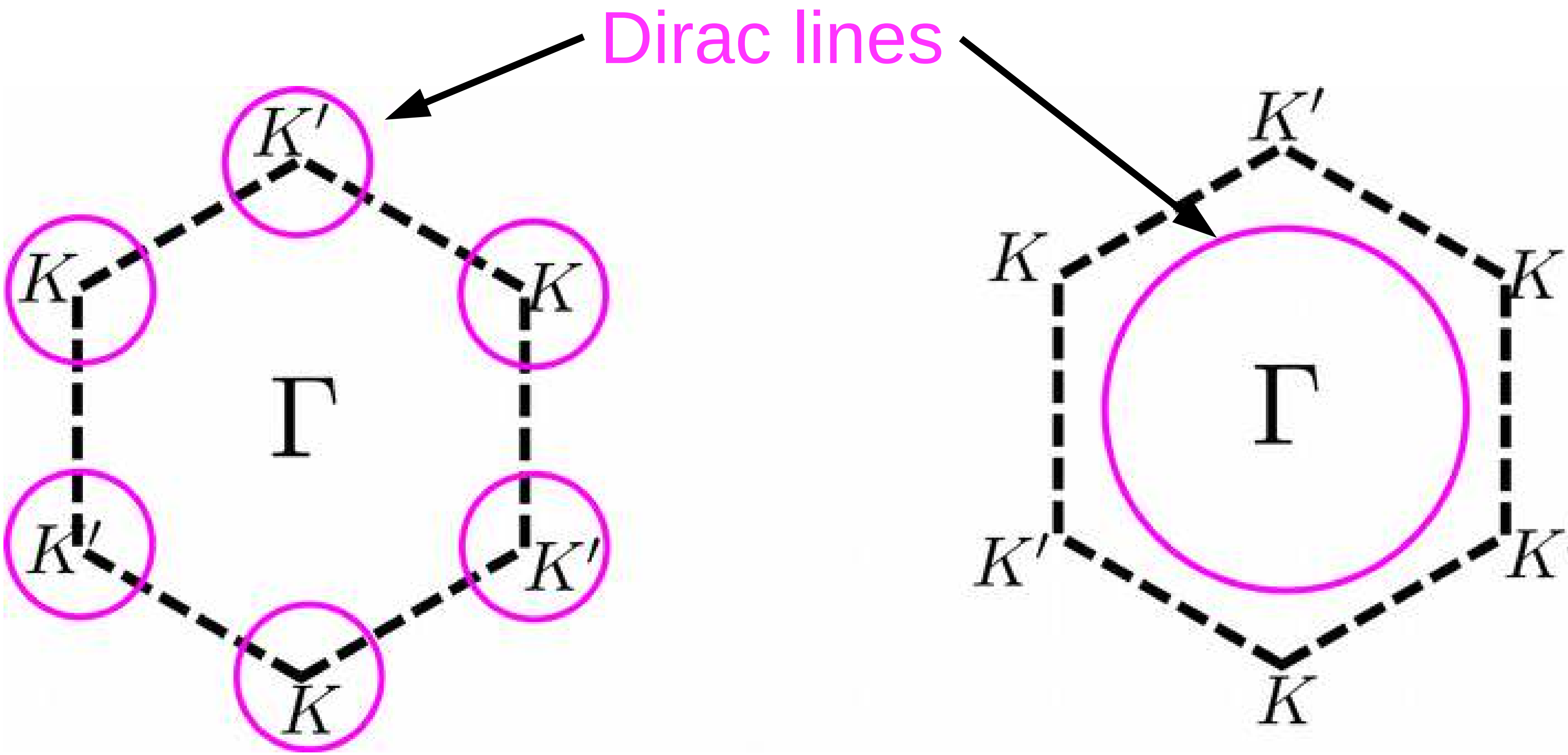
$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$



Spin-orbit coupling takes the form of a chiral pairing

Dirac lines, zero modes and Chern numbers

Tensile strain **Compressive strain**



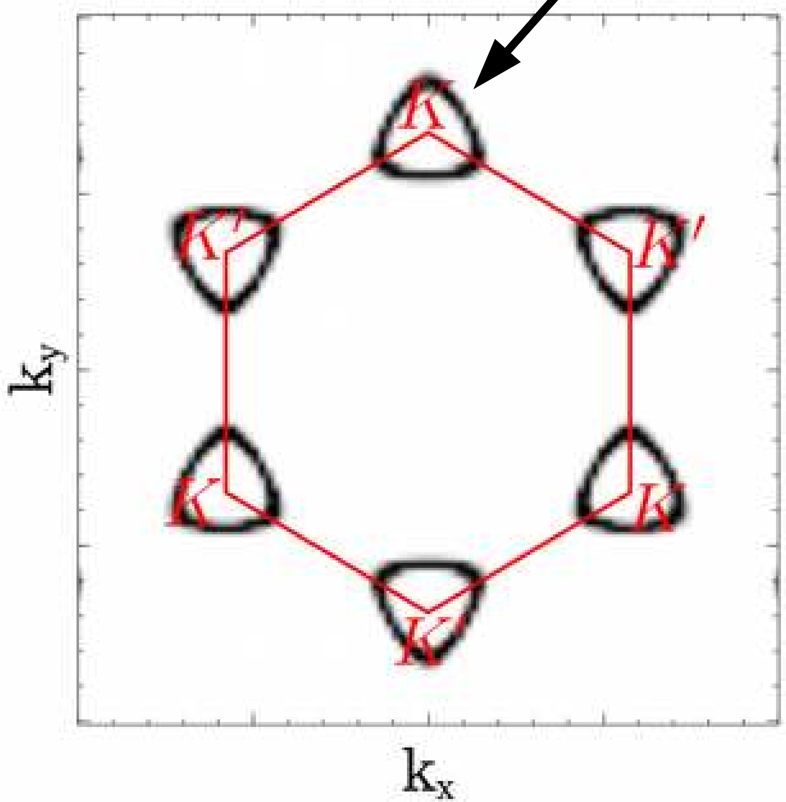
Dirac lines, zero modes and Chern numbers

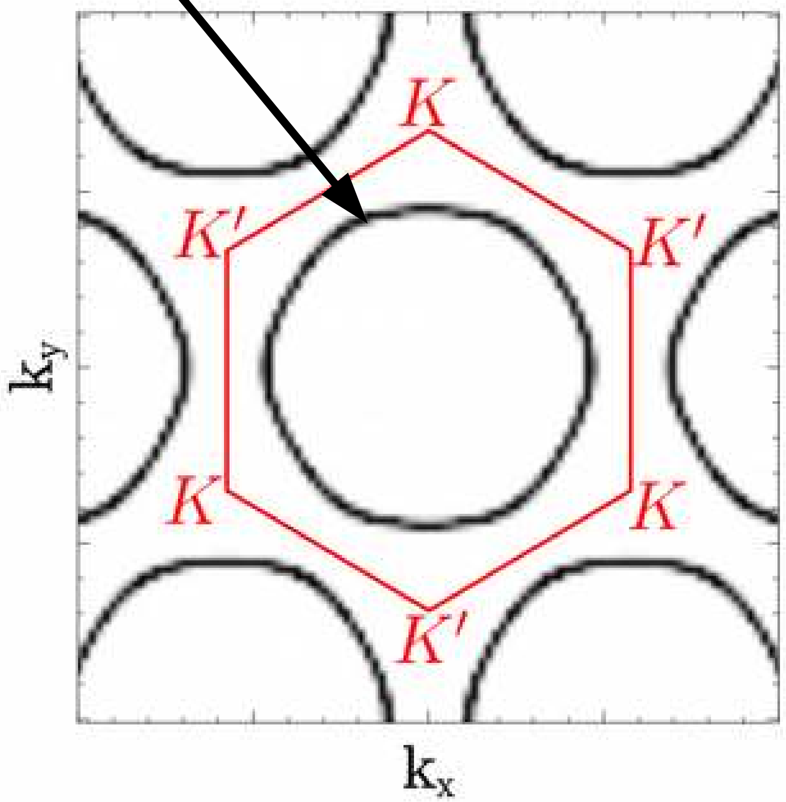
Tensile strain

Zero modes

$$\omega(k_{\parallel}) = 0$$

Compressive strain



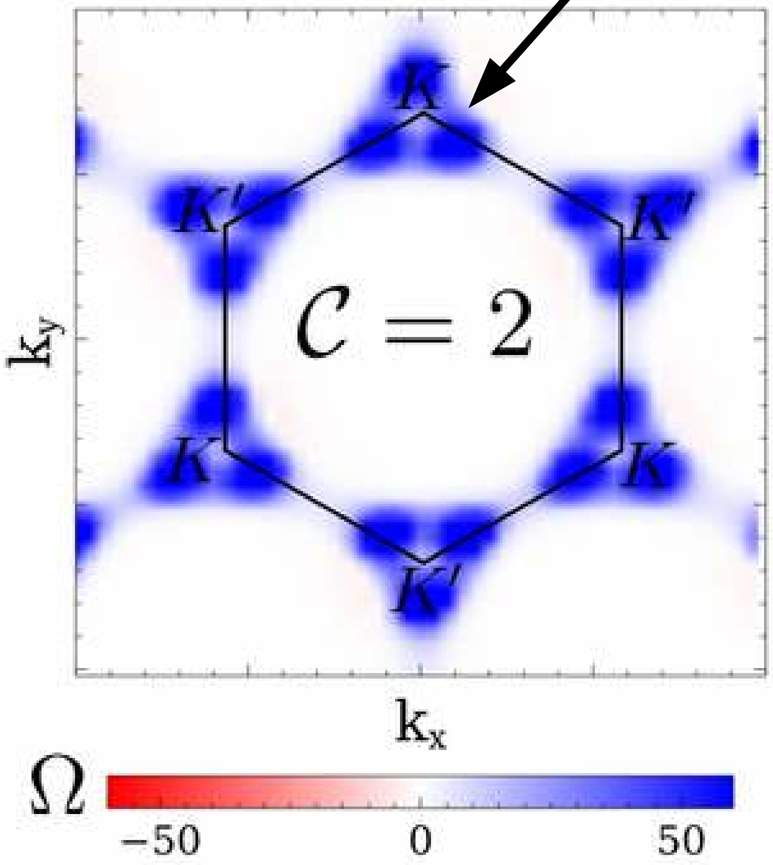


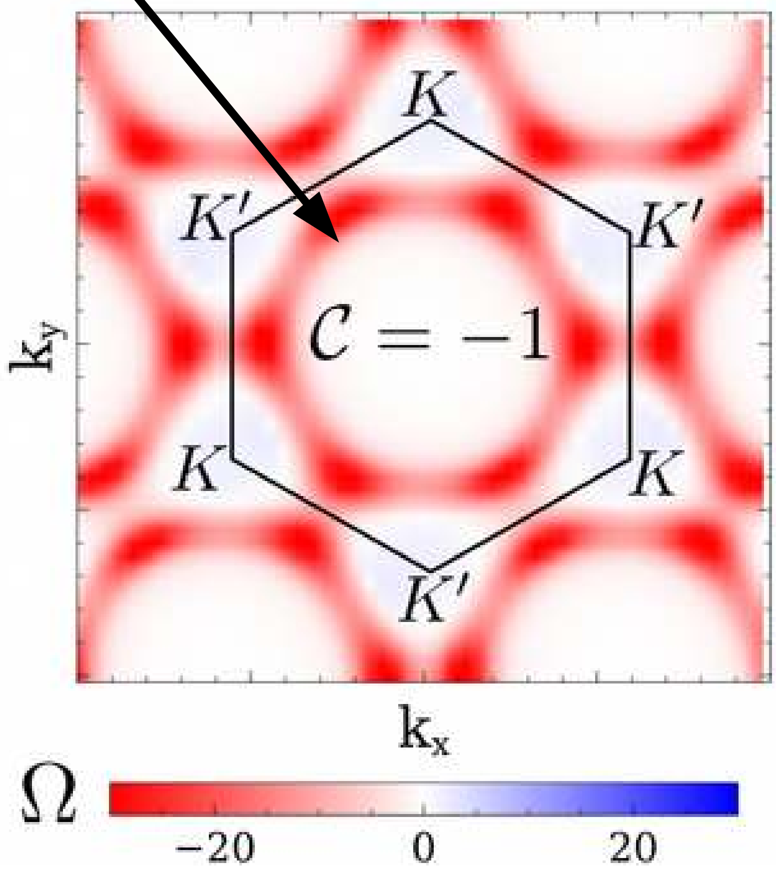
Dirac lines, zero modes and Chern numbers

Tensile strain

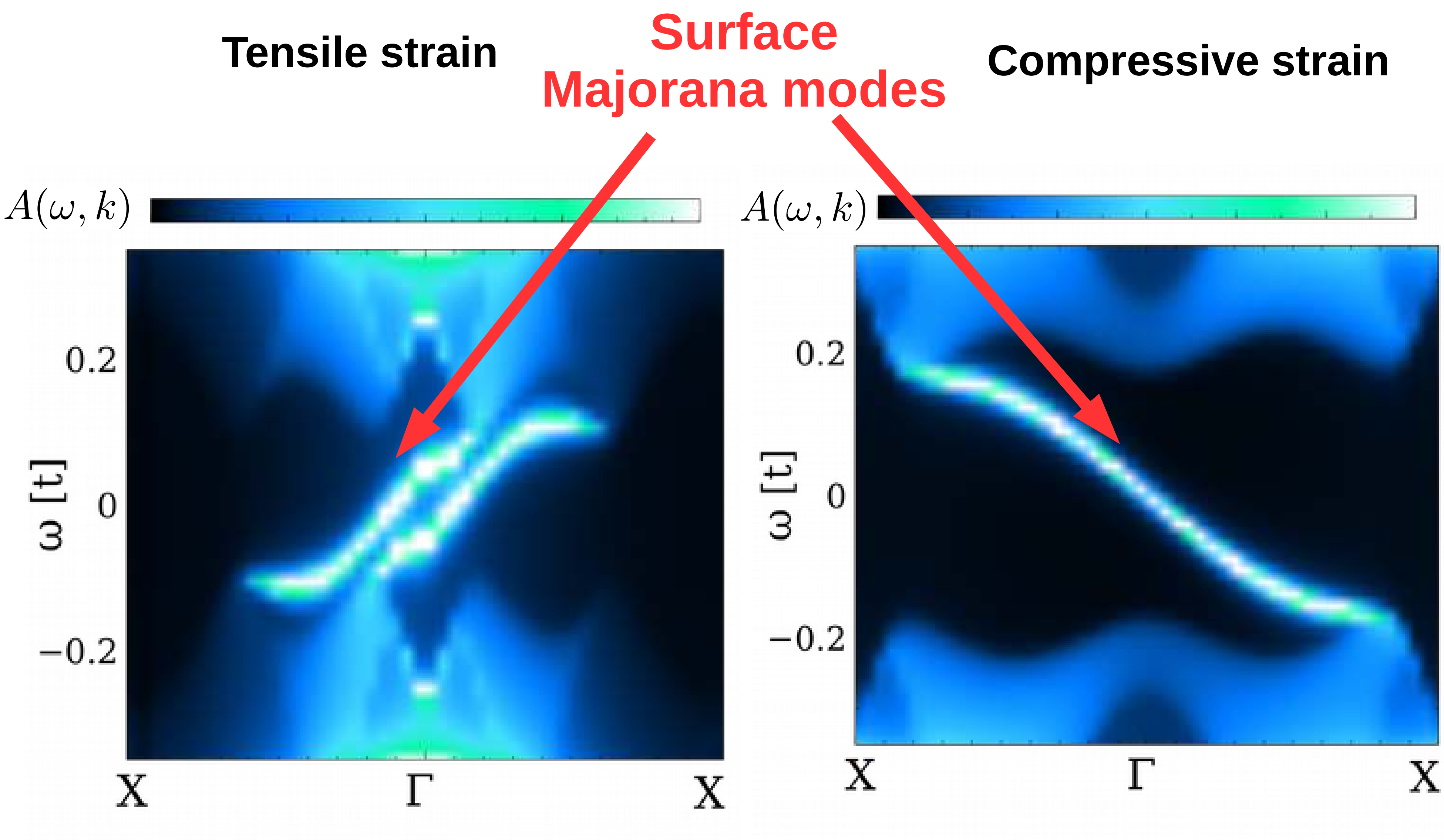
Berry curvature

Compressive strain



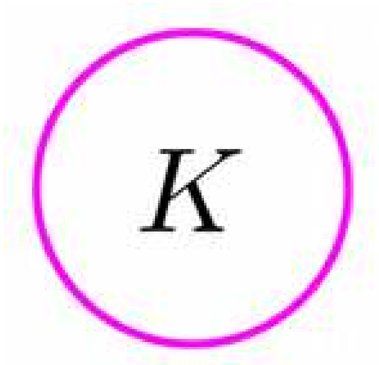


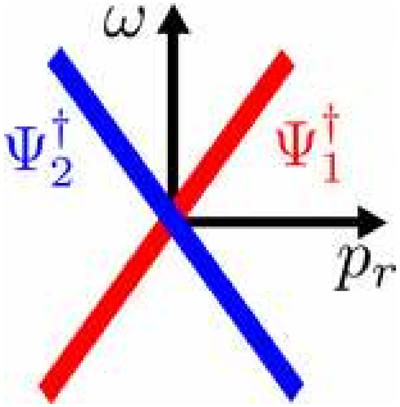
Dirac lines, zero modes and Chern numbers

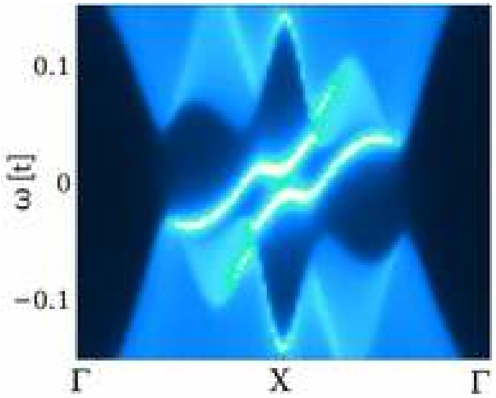


Ingredients to engineer a TS with an AF insulator

- Dirac lines in the nonmagnetic state
- Antiferromagnetic ordering
- Intrinsic spin-orbit coupling
- Conventional superconductor





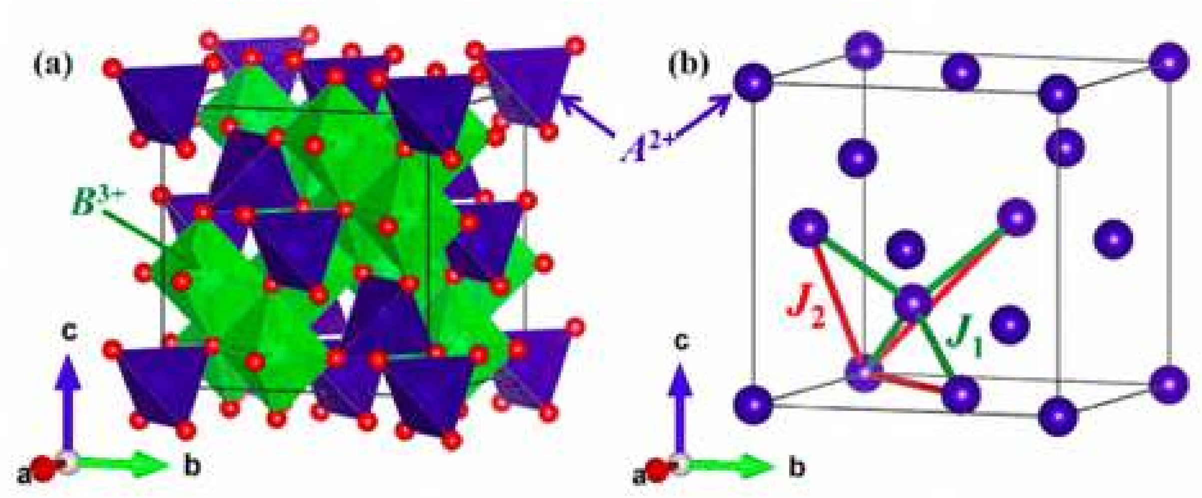


Possible materials, spinels

- Antiferromagnet forming a diamond lattice

Antiferromagnetic spinels

$CoAl_2O_4$



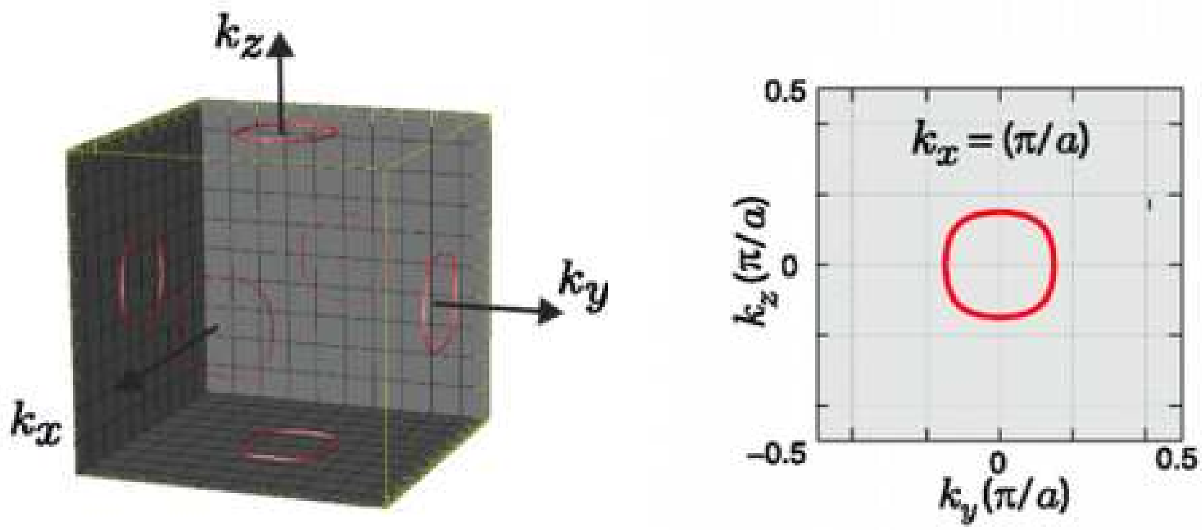
Co atoms form a diamond lattice

Phys. Rev. B 95, 094404 (2017)

Possible materials, Dirac materials in the AF phase

Dirac lines in the absence of spin-orbit coupling and magnetism

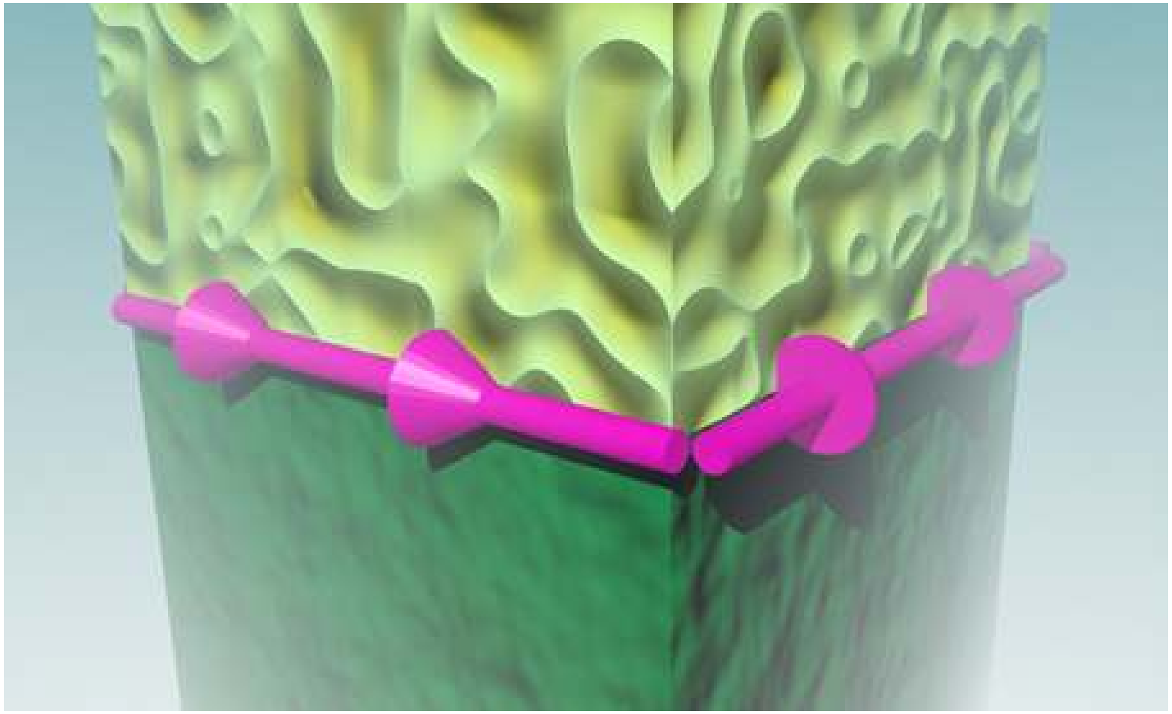
Phys. Rev. Lett. 115, 036806 (2015)



Compounds that in the non-magnetic state would have Dirac lines but that they are experimentally antiferromagnetic

Take home

Antiferromagnetic insulators can yield a robust platform to engineer two dimensional topological superconductors



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