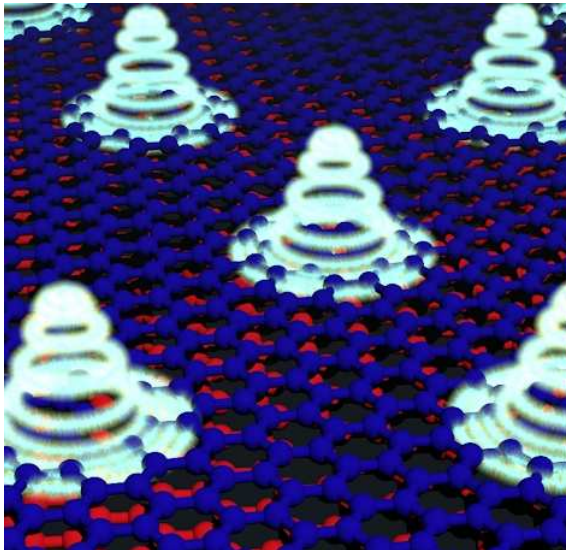


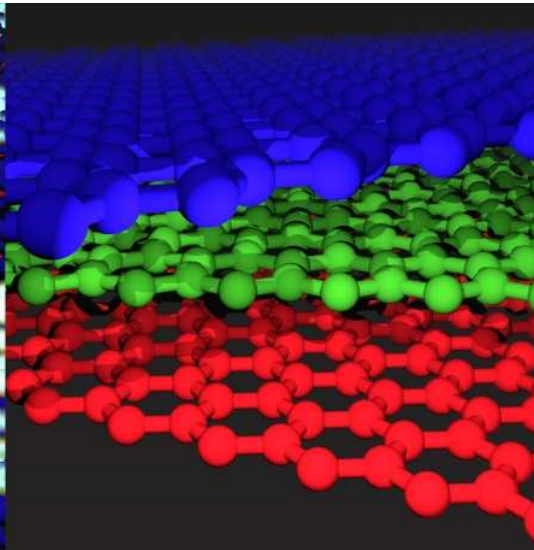
Atomic defects in topological and correlated twisted van der Waals heterostructures

Jose Lado

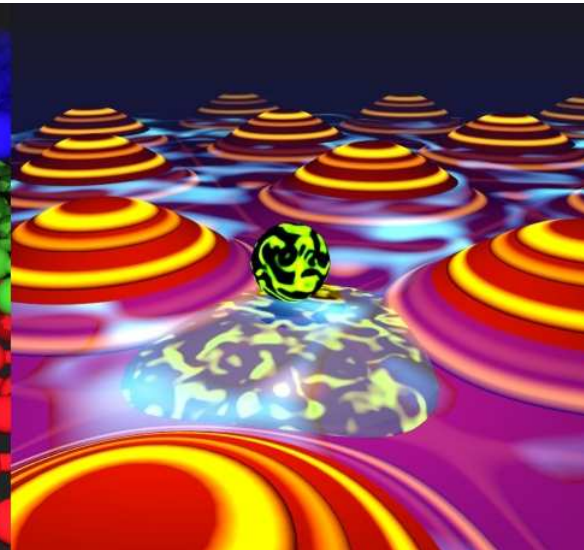
Department of Applied Physics, Aalto University, Finland



Phys. Rev. B 99, 245118 (2019)
Phys. Rev. Materials 3, 084003 (2019)



Phys. Rev. Research 2, 033357 (2020)



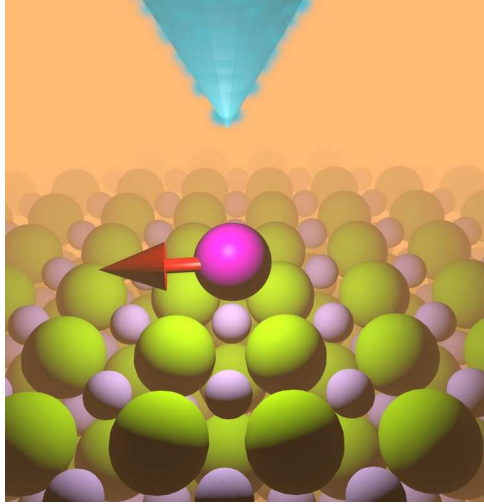
arXiv:2202.11003 (2022)

Condensed matter theory of 2D materials seminar, University of Manchester, UK

June 8th 2022

Building quantum matter with artificial lattices

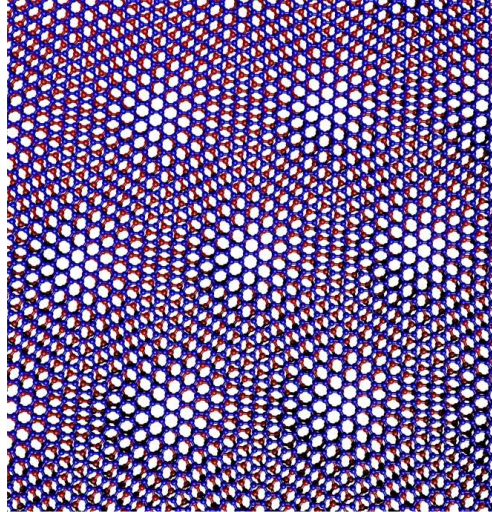
Atomic lattices



$a = 0.5 \text{ nm}$

Nature Physics 12 (7), 656 (2016)
Nature Physics 13 (7) 668 (2017)
Science 335 (6065), 196-199 (2012)

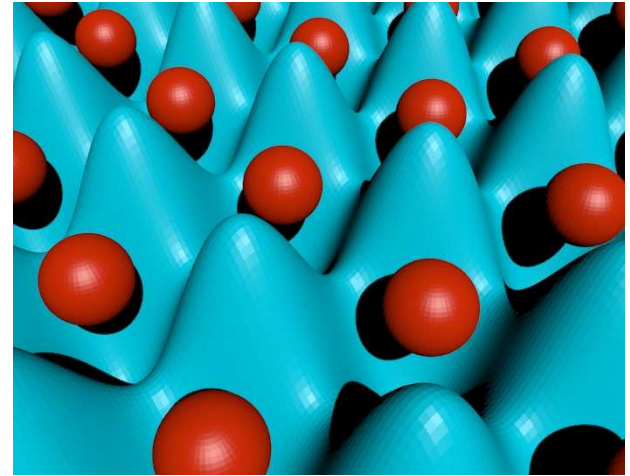
Twisted 2D moire materials



$a = 30 \text{ nm}$

Phys. Rev. Lett. 99, 256802 (2007)
Nature Physics 6, 109–113 (2010)
PNAS 108 (30) 12233-12237 (2011)
Nature 556, 80–84 (2018)

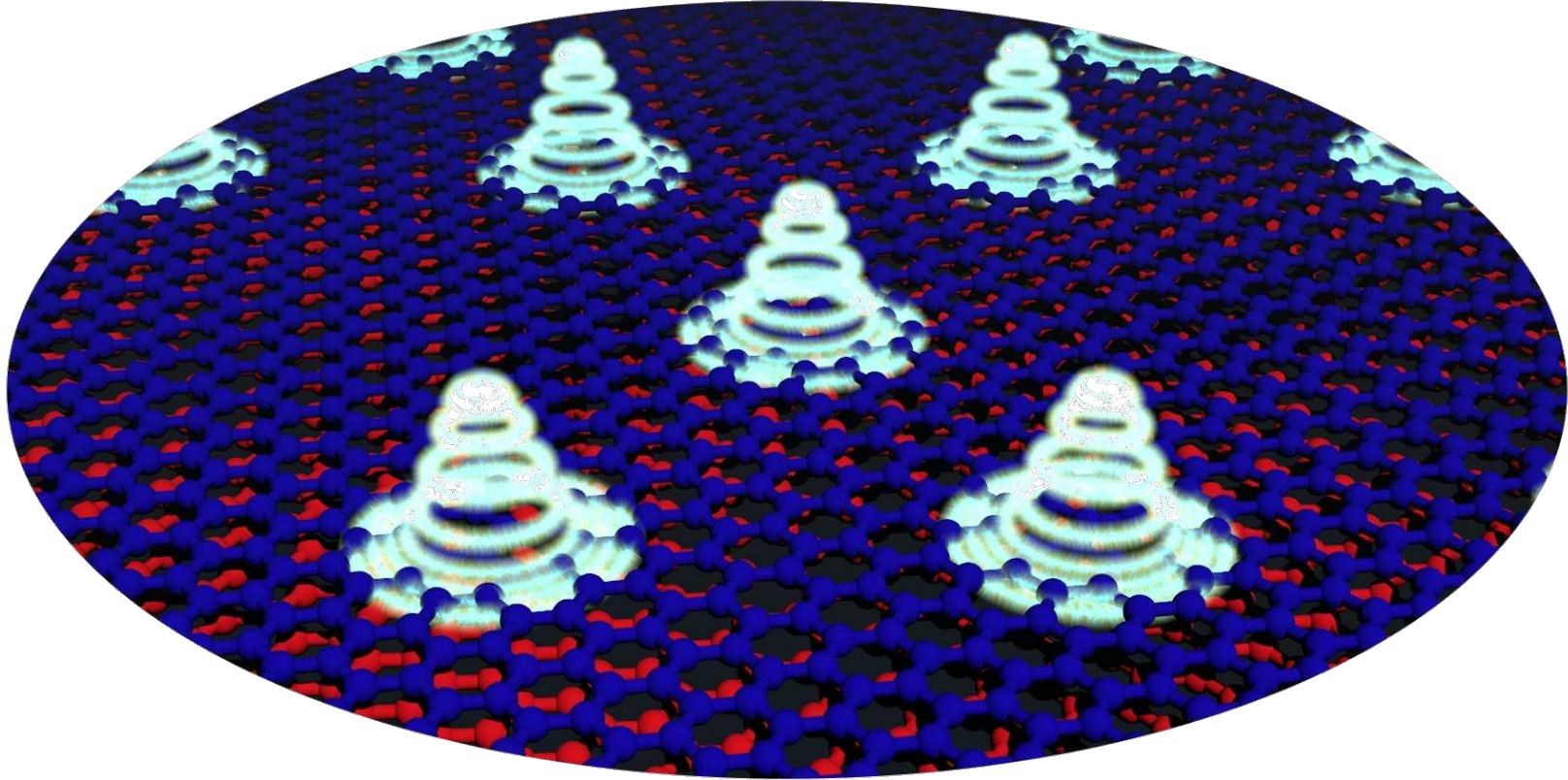
Cold atoms



$a = 1000 \text{ nm}$

Nature 545, 462–466 (2017)
Nature 455, 204–207 (2008)
Phys. Rev. Lett. 111, 185307 (2013)

A new universe in each new twisted material

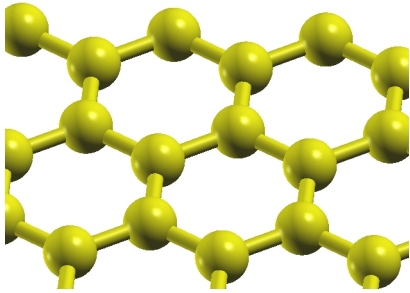


Each twisted material is a new universe for electrons, with laws that change from twist to twist

The two-dimensional materials world

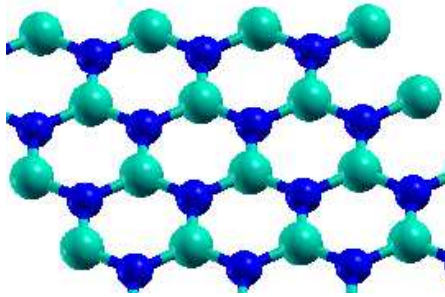
Semimetal

Graphene



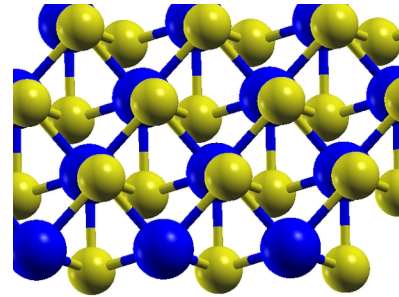
Insulator

BN



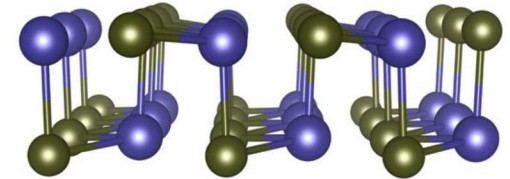
Superconductor

NbSe₂



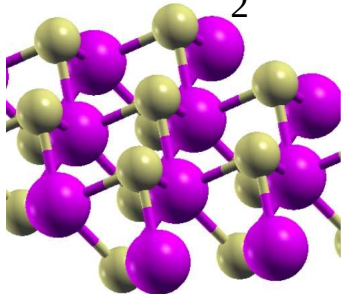
Ferroelectric

SnTe



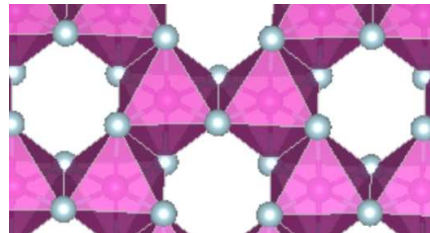
Semiconductor

WSe₂



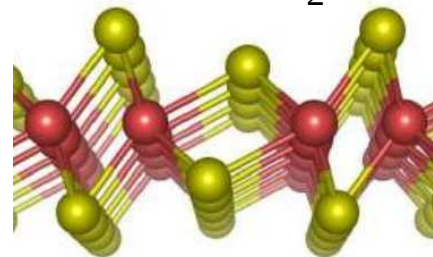
Ferromagnet

CrI₃



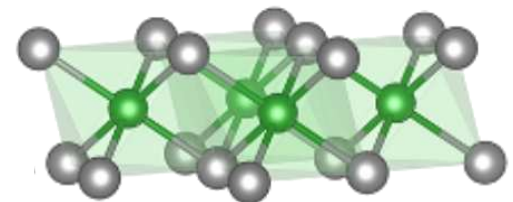
**Quantum spin
Hall insulator**

WTe₂



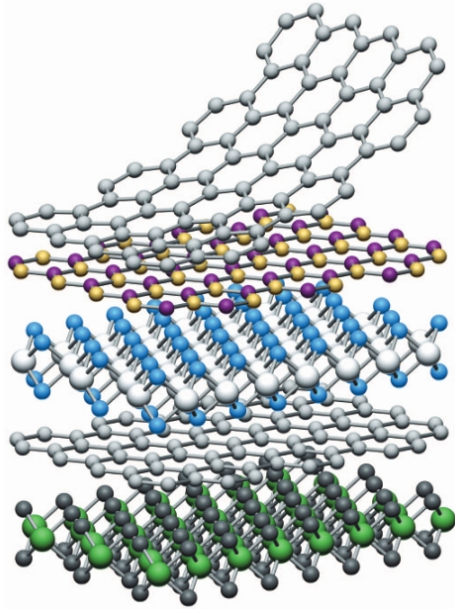
Multiferroic

NiI₂



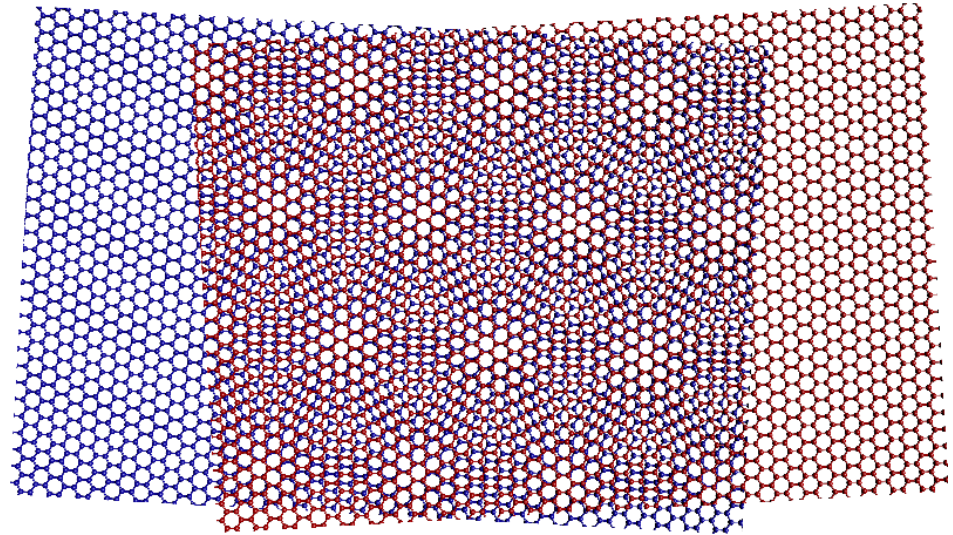
The flexibility of two-dimensional materials

They can be stacked



Nature 499.7459 419. (2013)

They can be rotated

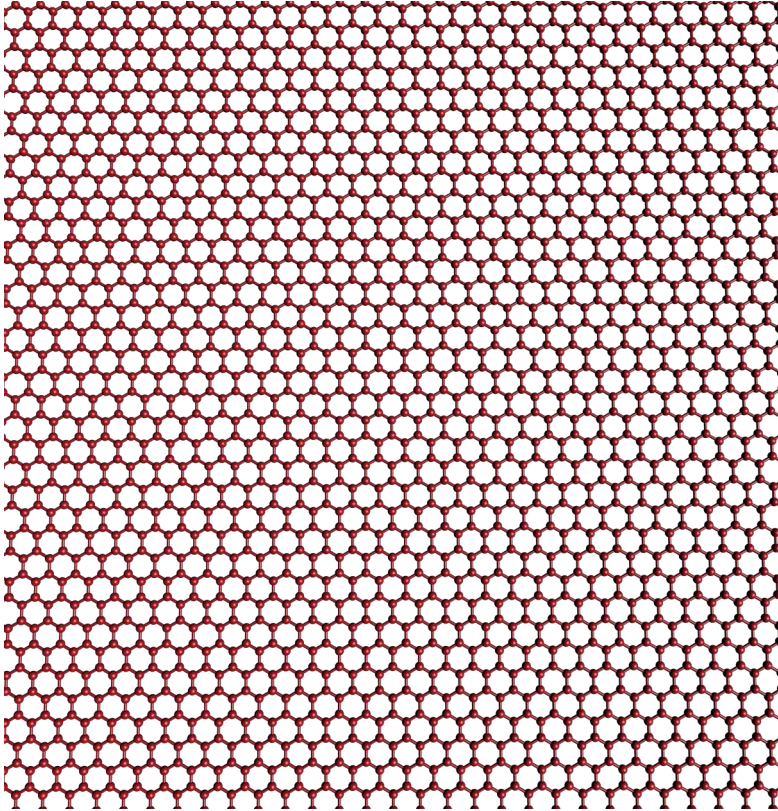


Science 361.6403 690-693. (2018)

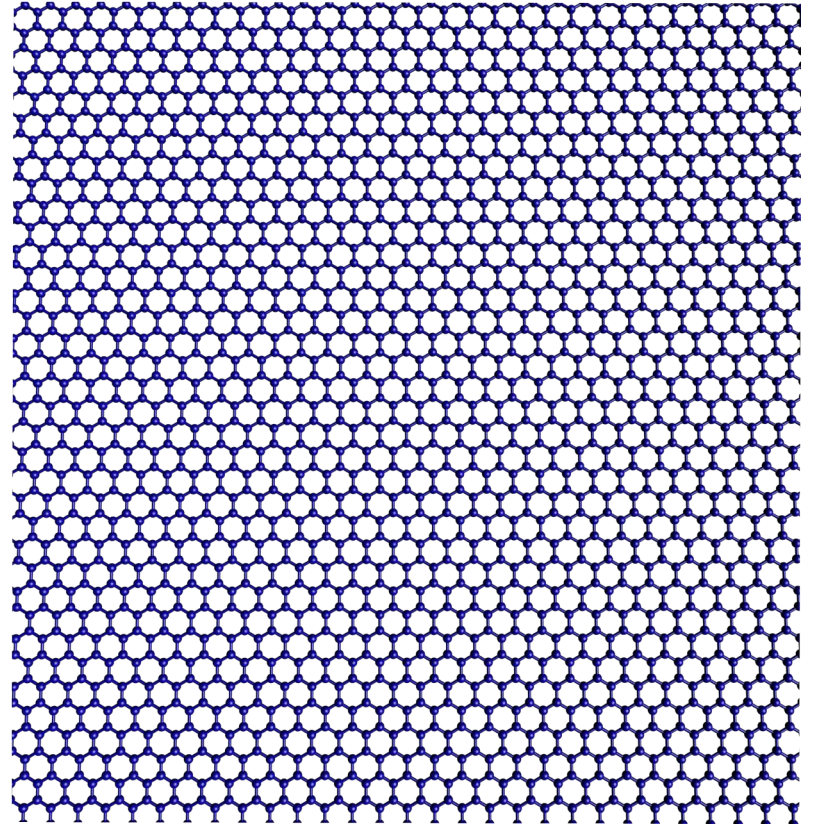
These are unique features of two-dimensional materials

Moire in twisted bilayer graphene

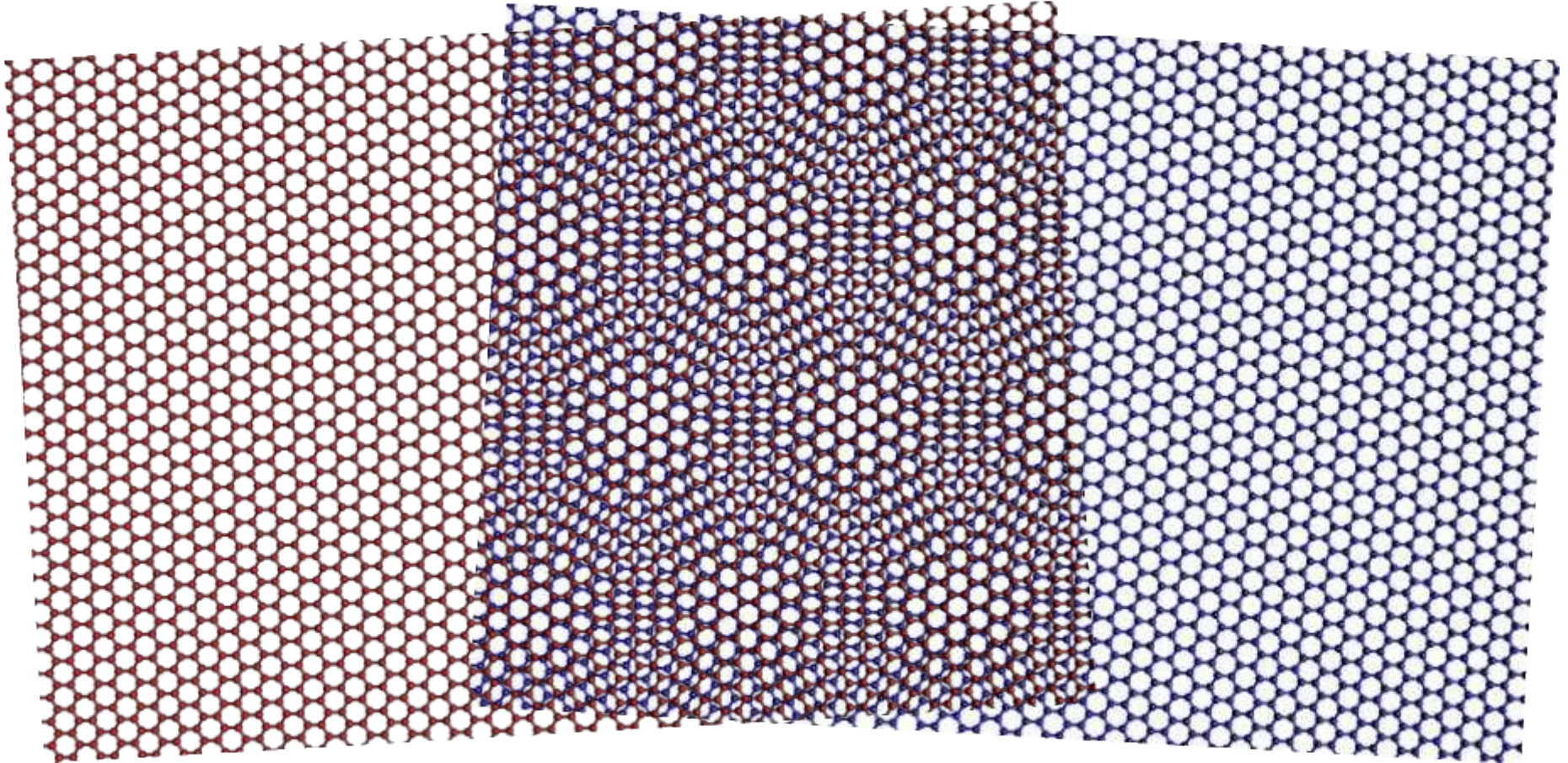
Upper graphene layer



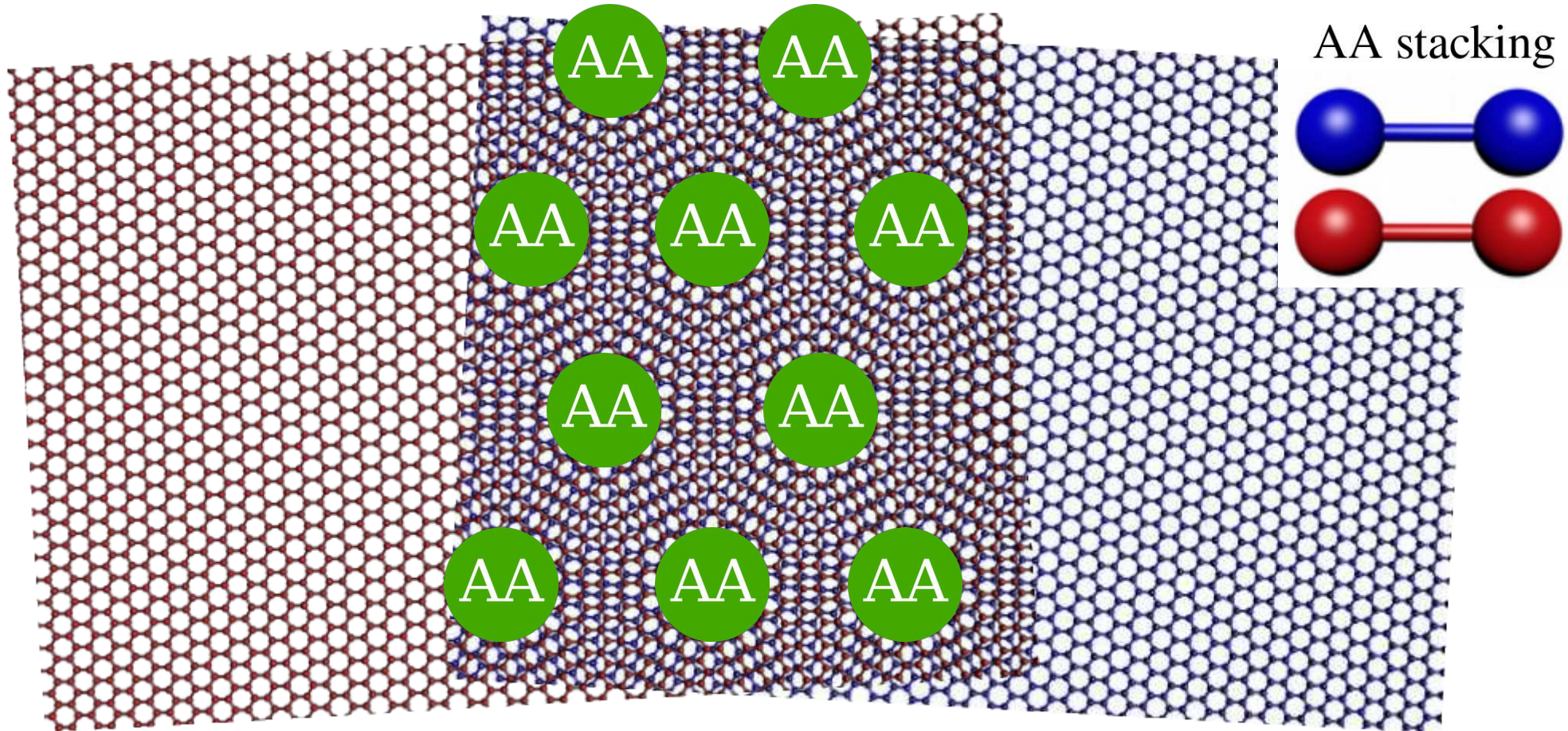
Lower graphene layer



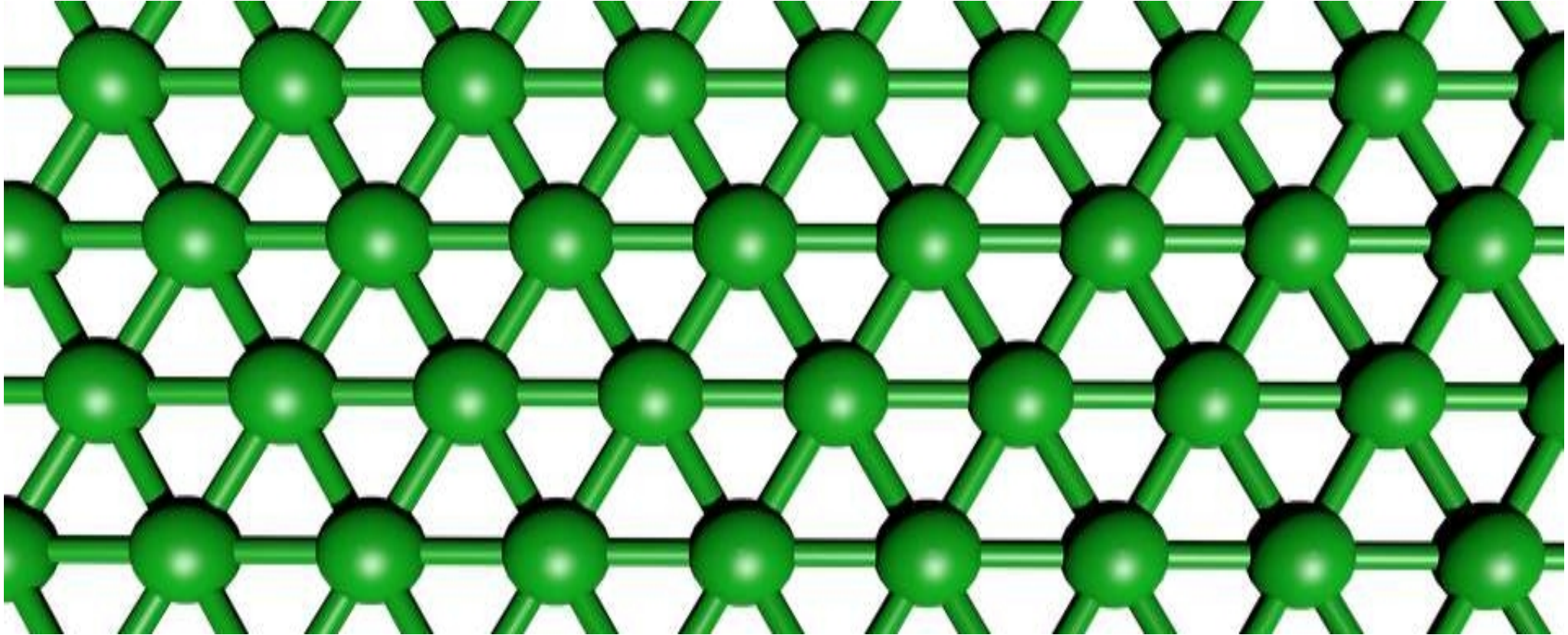
Moire in twisted bilayer graphene



Moire in twisted bilayer graphene

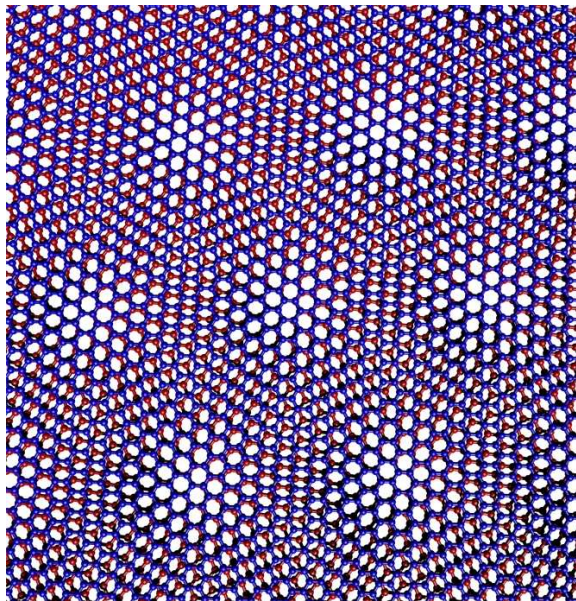


Moire in twisted bilayer graphene

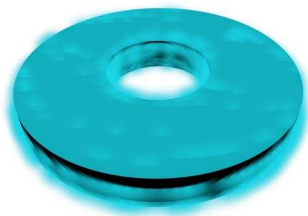


One material, a zoo of electronic phases

Twisted bilayer graphene

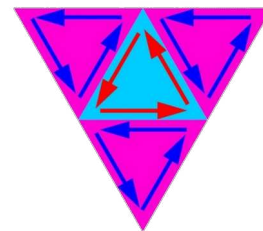


Superconductivity



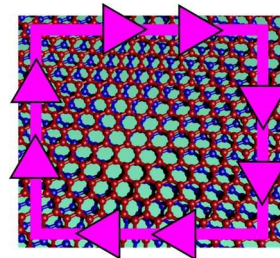
Nature 556, 43–50 (2018)

Topological networks



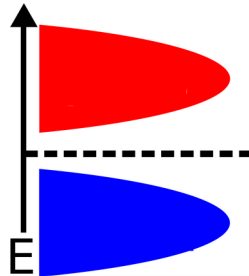
Nano Lett. 18, 11,
6725-6730 (2018)

Chern insulators



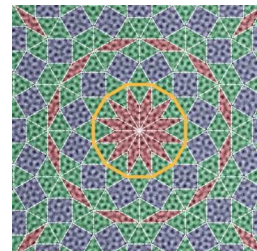
Science 365, 605-608 (2019)

Correlated insulators



Nature 556, 80–84 (2018)

Quasicrystalline physics



Science 361, 782-786 (2018)

Fractional Chern insulators



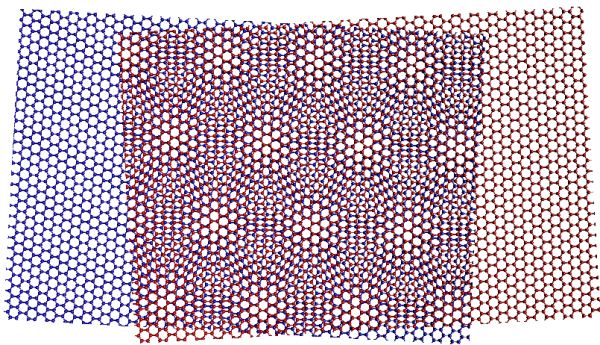
Nature 600, 439–443 (2021)

Twisted graphene multilayers provide a powerful platform for emergent phenomena

Unique properties of moire systems

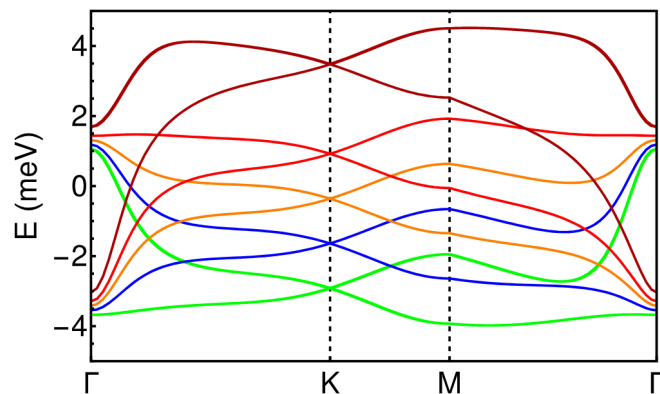
In addition to all the features of conventional materials, twisted multilayers feature (among others)

Control via twist engineering



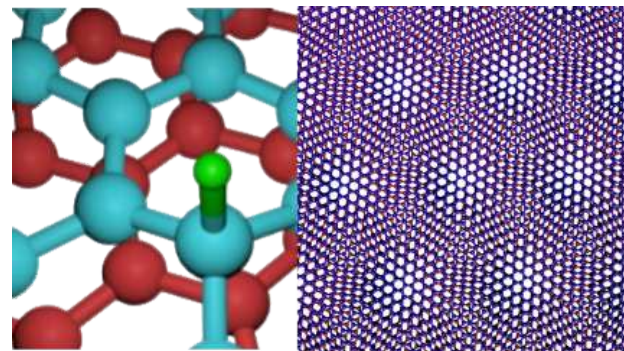
Phys. Rev. B 82, 121407(R) (2010)
PNAS 108 (30) 12233-12237 (2011)
Nature Reviews Materials 6, 201–206 (2021)

Strong electrostatic band distortions



PNAS 115 (52), 13174-13179 (2018)
Physical Review B 99 (20), 205134 (2019)

Defects in two different length scales

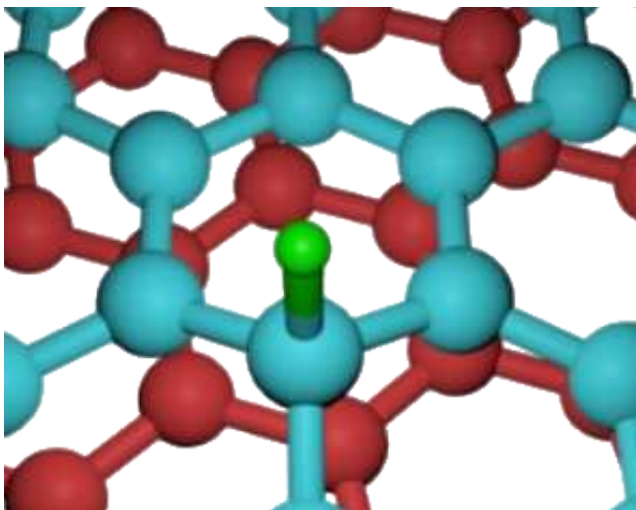


Nature 581, 47–52 (2020)
Today's plan

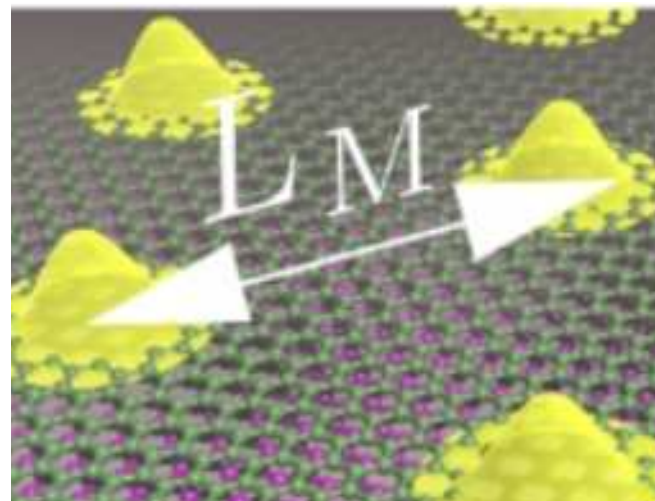
Moire and impurities

Two length scales in moire systems

The original atomic scale



The emergent moire scale

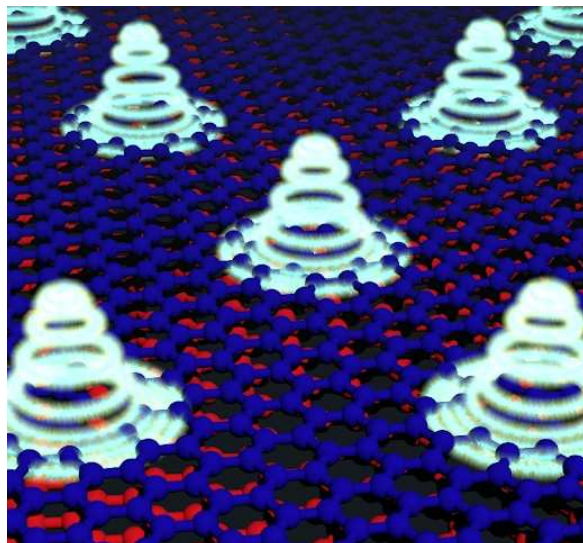


The coexistence of these two different scales is unique to moire systems

How do atomic impurities impact the physics emerging from much bigger length scales?

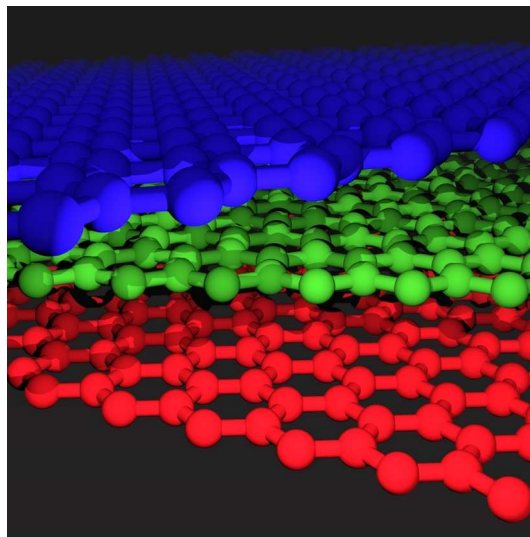
Today's plan

Impurities in twisted graphene bilayers



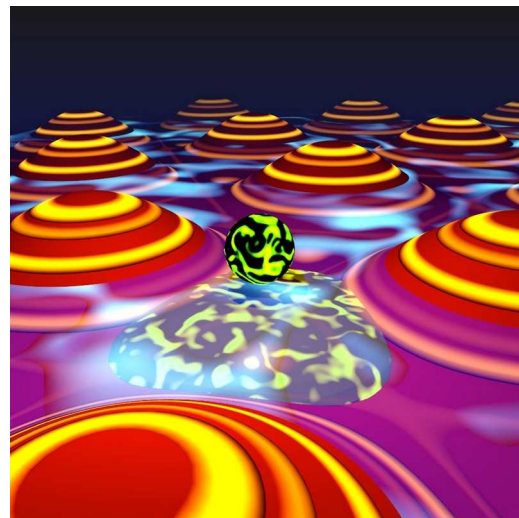
Phys. Rev. B 99, 245118 (2019)
Phys. Rev. Materials 3, 084003 (2019)

Impurities in twisted graphene trilayers



Phys. Rev. Research 2, 033357 (2020)

Impurities in twisted moire topological superconductors



arXiv:2202.11003 (2022)

Behind the scenes

*Impurities in twisted
graphene bilayers*



Aline Ramires

Phys. Rev. B 99, 245118 (2019)

*Impurities in twisted
graphene trilayers*



Alejandro Lopez Bezanilla

Phys. Rev. Research 2, 033357 (2020)
Phys. Rev. Materials 3, 084003 (2019)

*Impurities in twisted
moire topological superconductors*

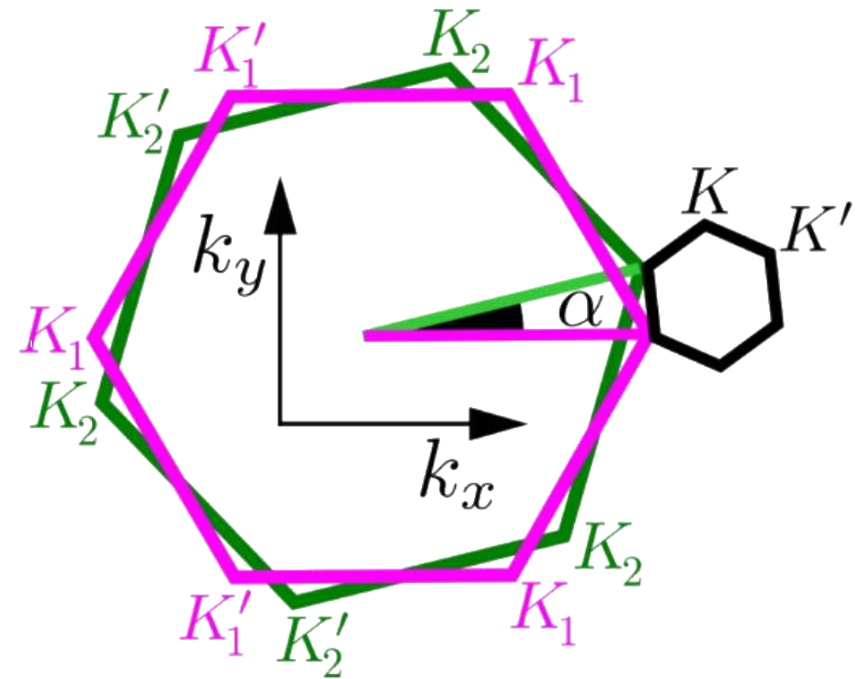
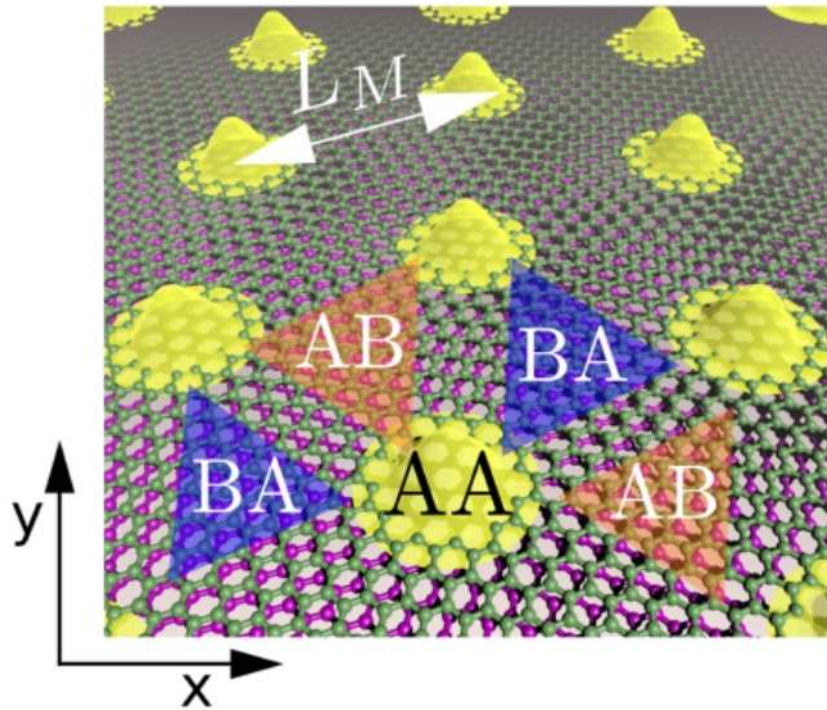


Maryam Khosravian

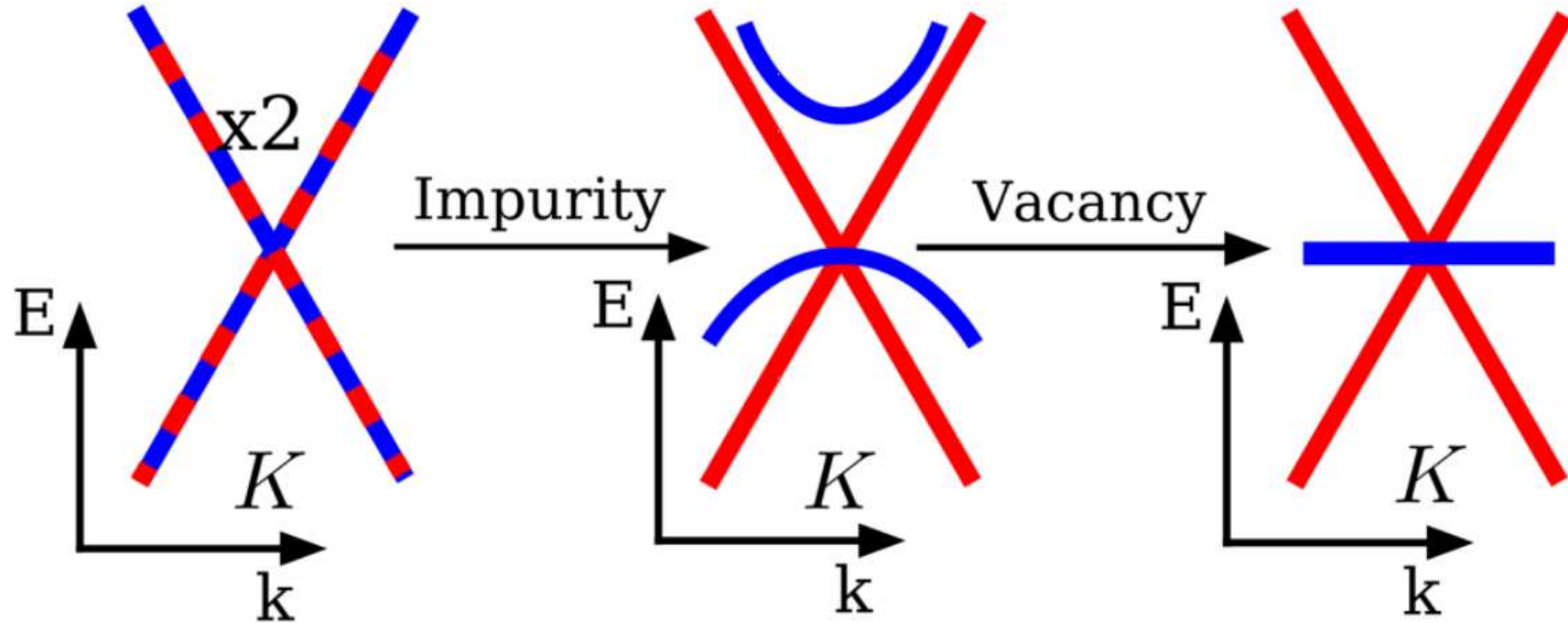
arXiv:2202.11003 (2022)

Impurities in twisted bilayer graphene

Twisted graphene bilayers



Impact of single impurities in the twisted bilayer electronic structure



Single impurities give rise to a triple point crossing in the electronic structure

Model for impurities in twisted multilayers

Full Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{W}$$

Pristine twisted bilayer graphene

$$\mathcal{H}_0 = t \sum_{\langle i,j \rangle} c_i^\dagger c_j + \sum_{i,j} \bar{t}_\perp(\mathbf{r}_i, \mathbf{r}_j) c_i^\dagger c_j$$

Intra-layer
hopping

Inter-layer
hopping

Local impurity

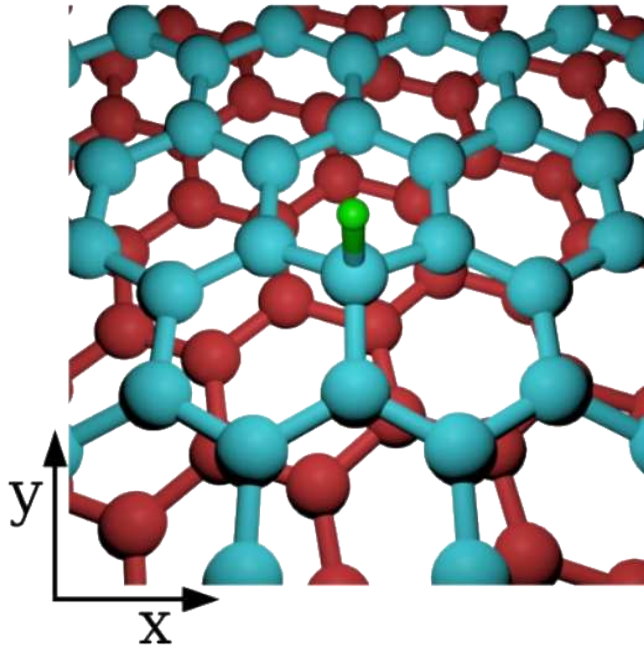
$$\mathcal{W} = w c_n^\dagger c_n$$

The limit of infinite potential corresponds to removing a site

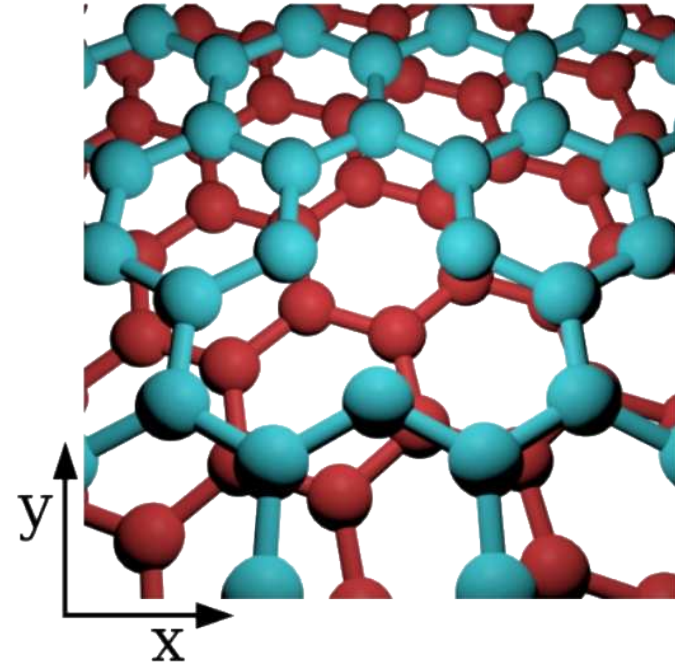
$$w \rightarrow \infty$$

The strong impurity limit

Adsorbed hydrogen in TBG



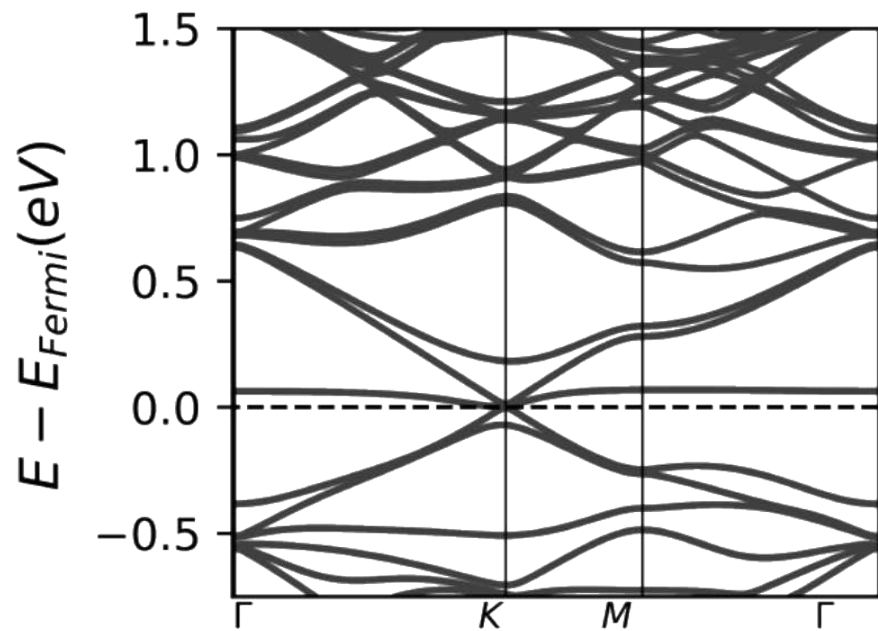
Effective tight binding model



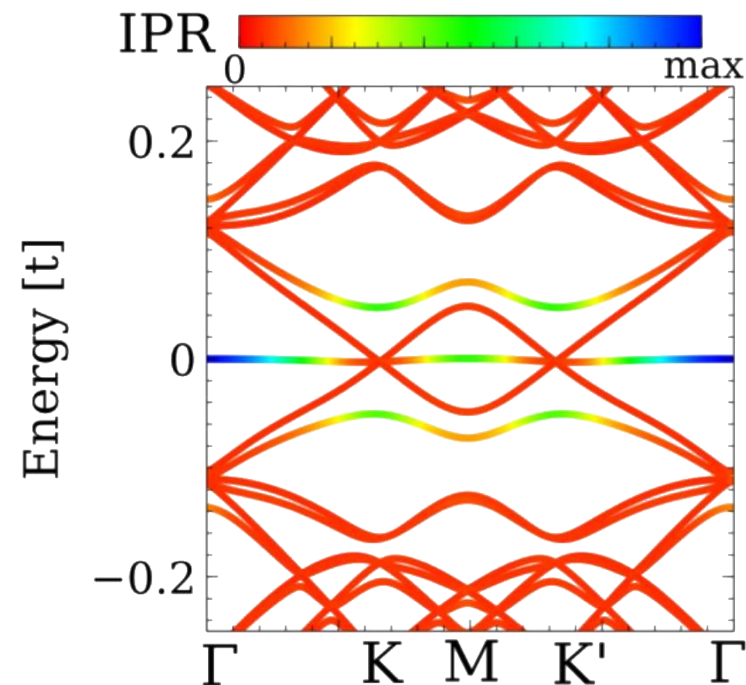
Hydrogen adsorption effectively gives rise to a vacancy in the low energy model

DFT versus tight binding

Density functional theory



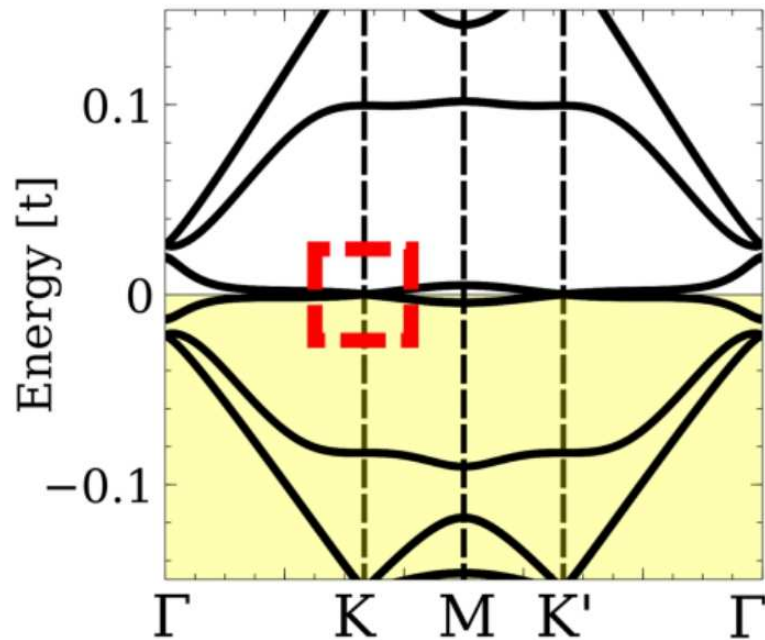
Tight binding



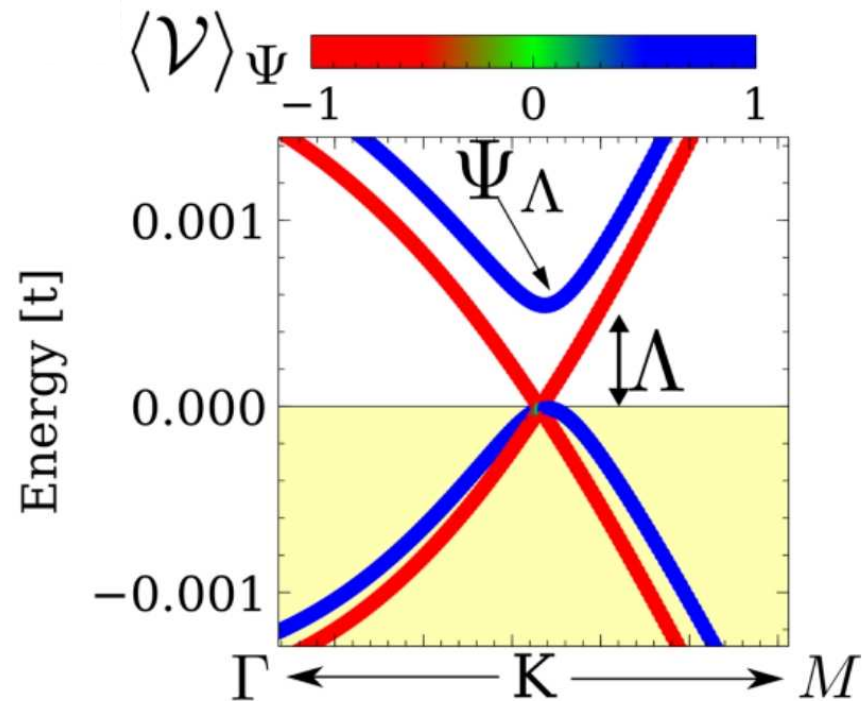
DFT and tight binding give rise to comparable electronic structures

Impact of a single weak defect

Band structure



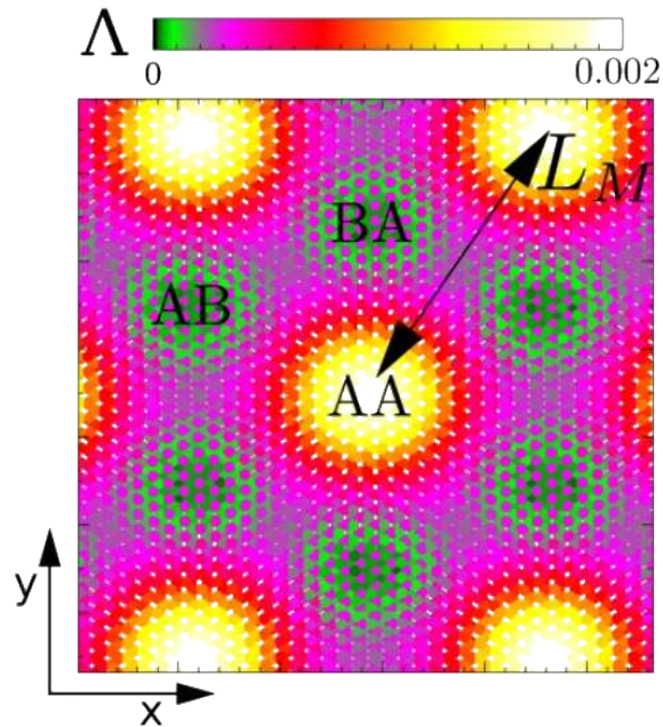
Valley expectation value



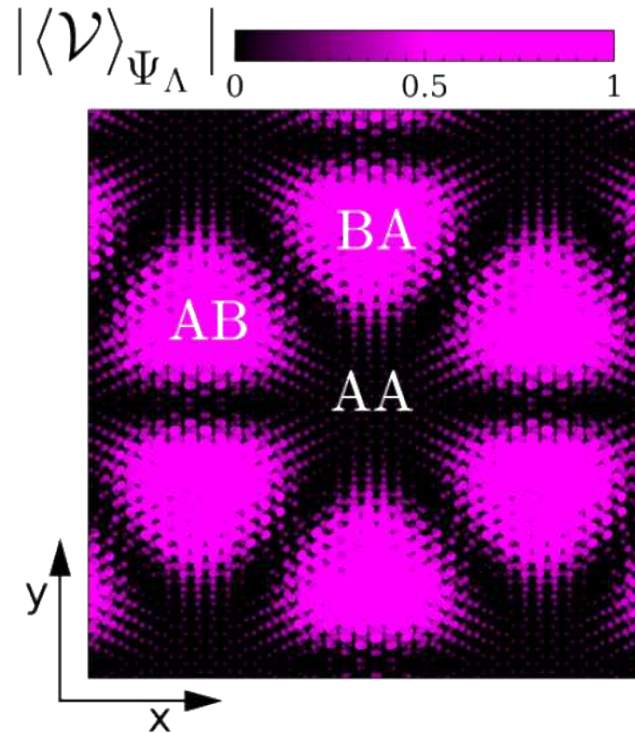
A weak impurity generates a triple band crossing at the Dirac point

Position-dependent valley mixing

Gap opening depending on the location



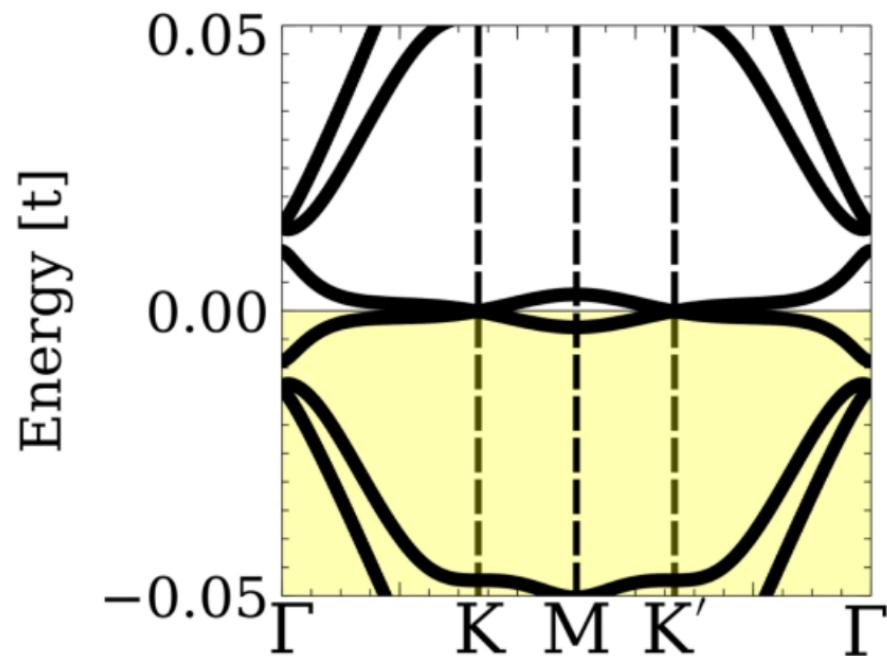
Valley polarization depending on the location



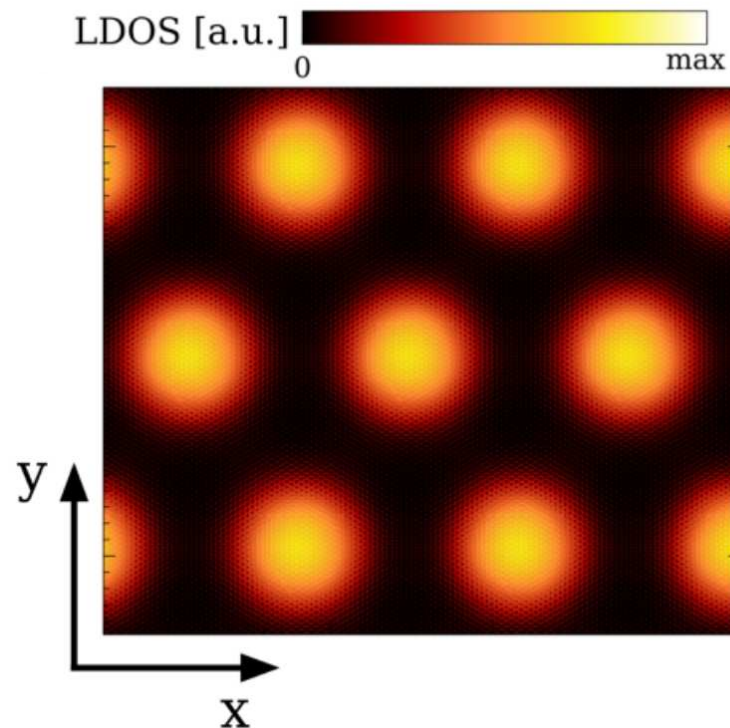
The location of the impurities determines the strength of the valley mixing

Low energy states, pristine bilayer

Band structure



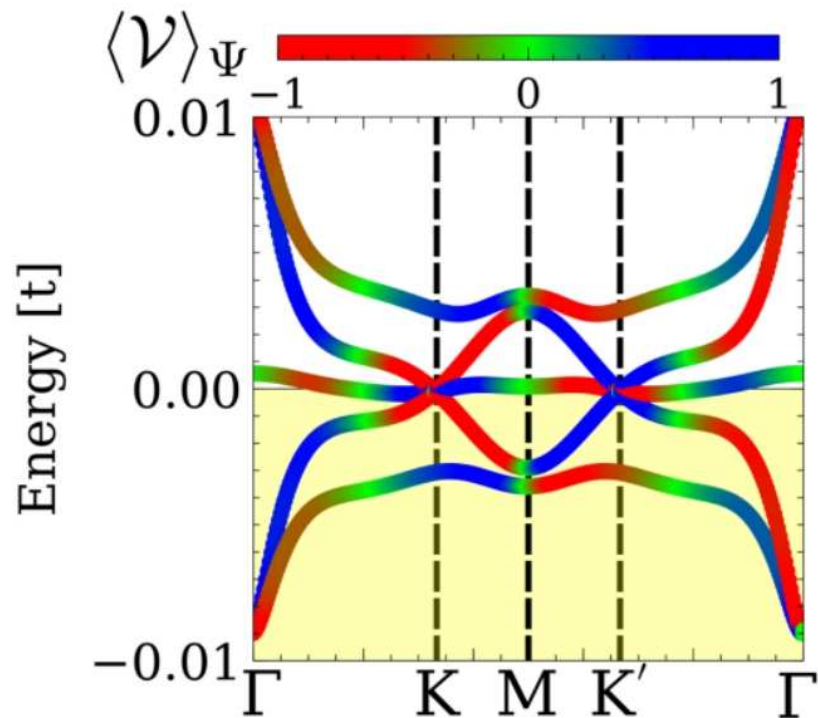
Spatial distribution of the low energy states



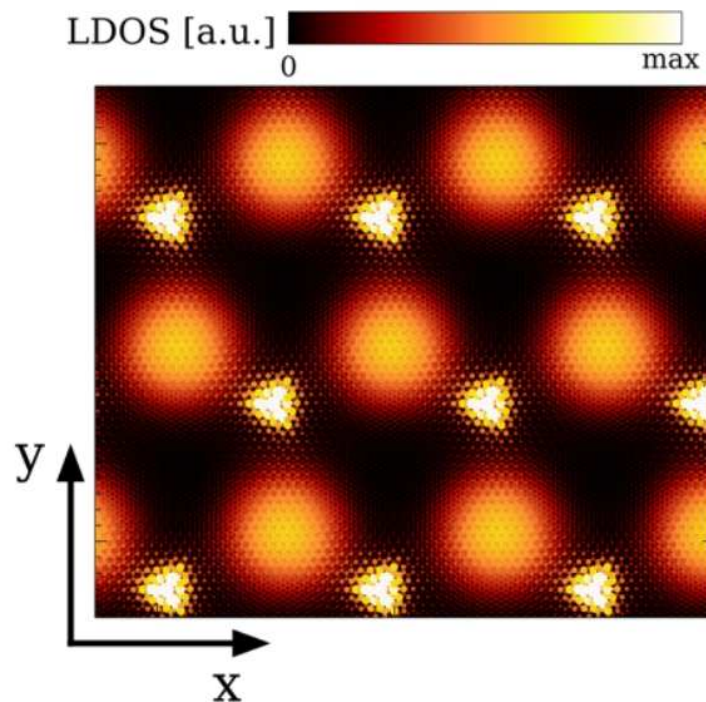
The nearly flat bands create an emergent triangular superlattice

Low energy states, strong single defect

Band structure



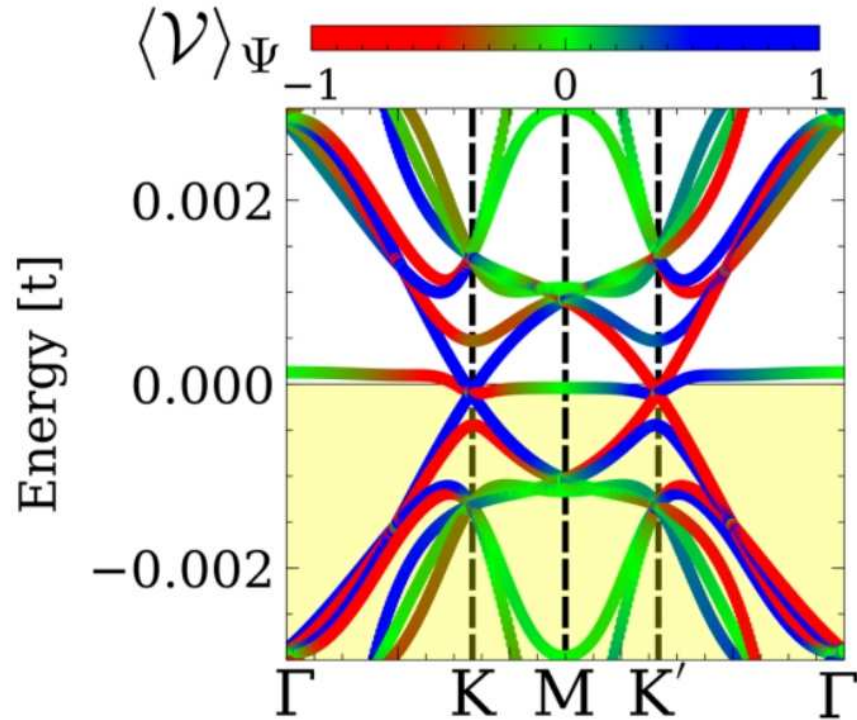
Spatial distribution of the low energy states



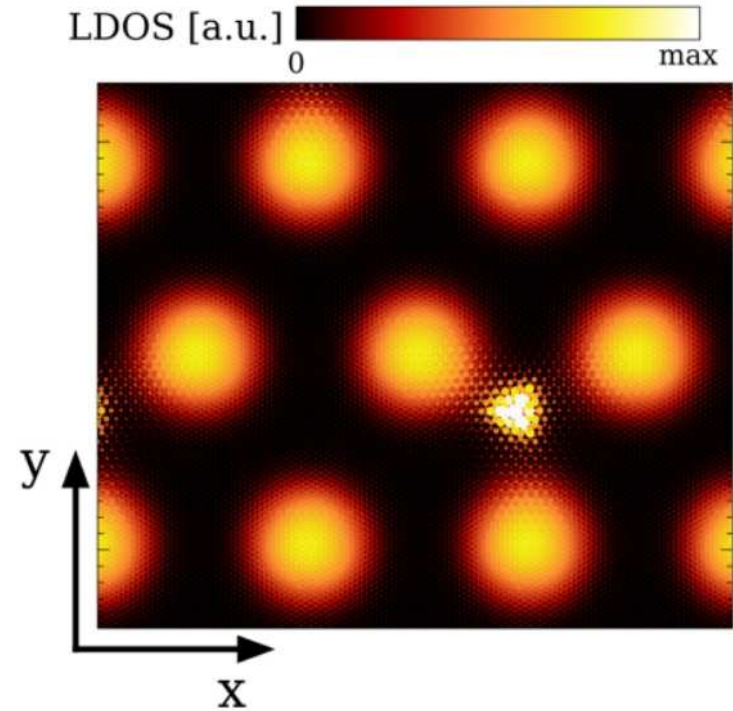
The nearly flat bands coexist with a localized vacancy state

Low energy states, strong single defect in a 4x4 supercell

Band structure



Spatial distribution of the low energy states



The nearly flat bands coexist with a localized vacancy state

Including interactions in a minimal way

We will include interactions with a minimal Hubbard repulsion

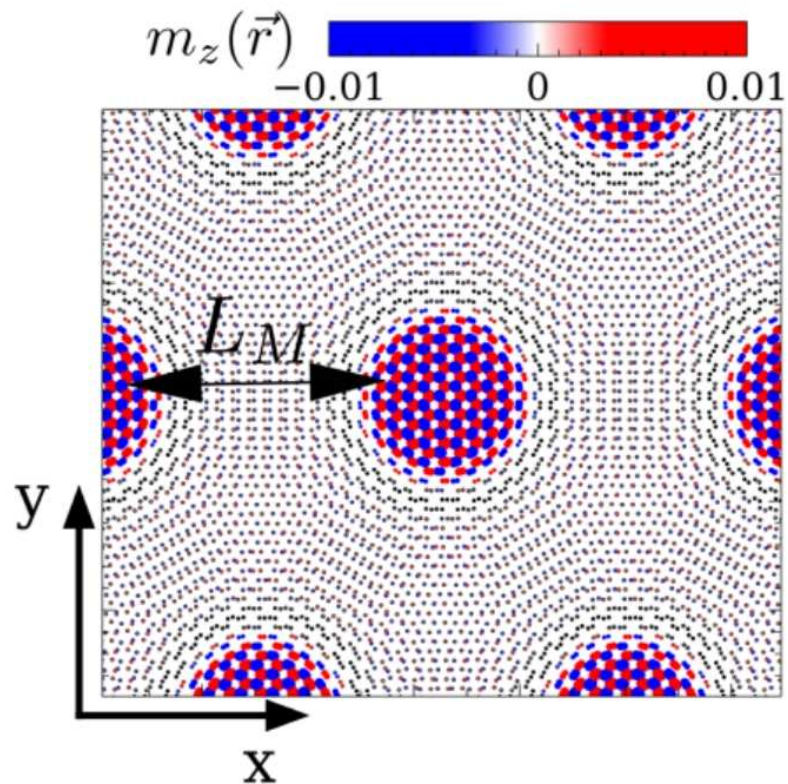
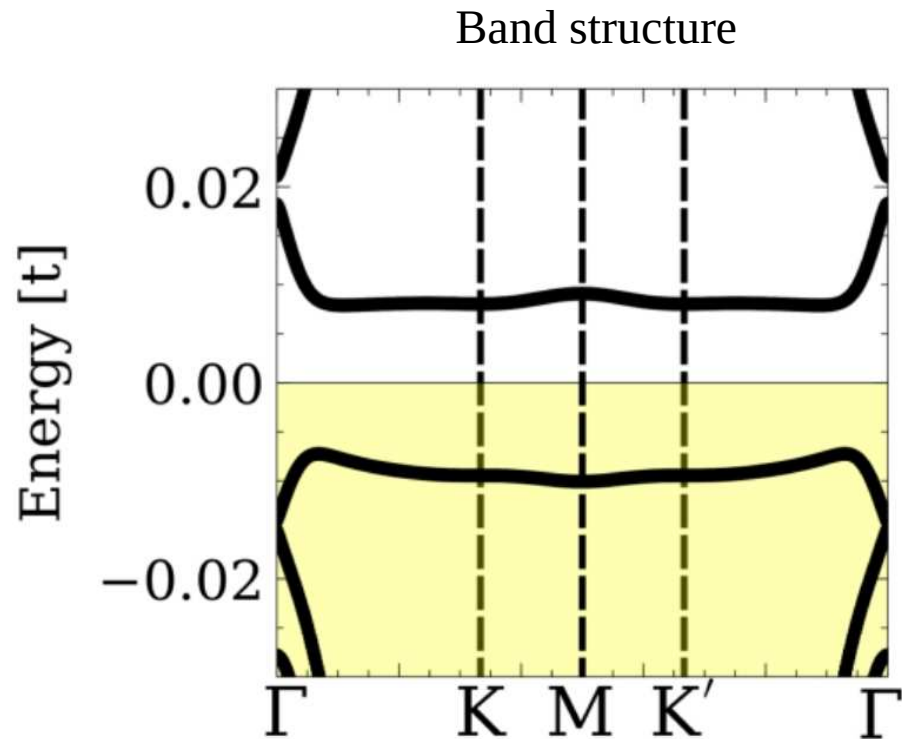
$$H_U = \sum_i U c_{i,\uparrow}^\dagger c_{i,\uparrow} c_{i,\downarrow}^\dagger c_{i,\downarrow} = \sum_i U n_{i,\uparrow} n_{i,\downarrow}$$

Solved at the mean-field level

$$H_U^{\text{MF}} = \sum_i U [\langle n_{i,\uparrow} \rangle n_{i,\downarrow} + \langle n_{i,\downarrow} \rangle n_{i,\uparrow} - \langle n_{i,\uparrow} \rangle \langle n_{i,\downarrow} \rangle]$$

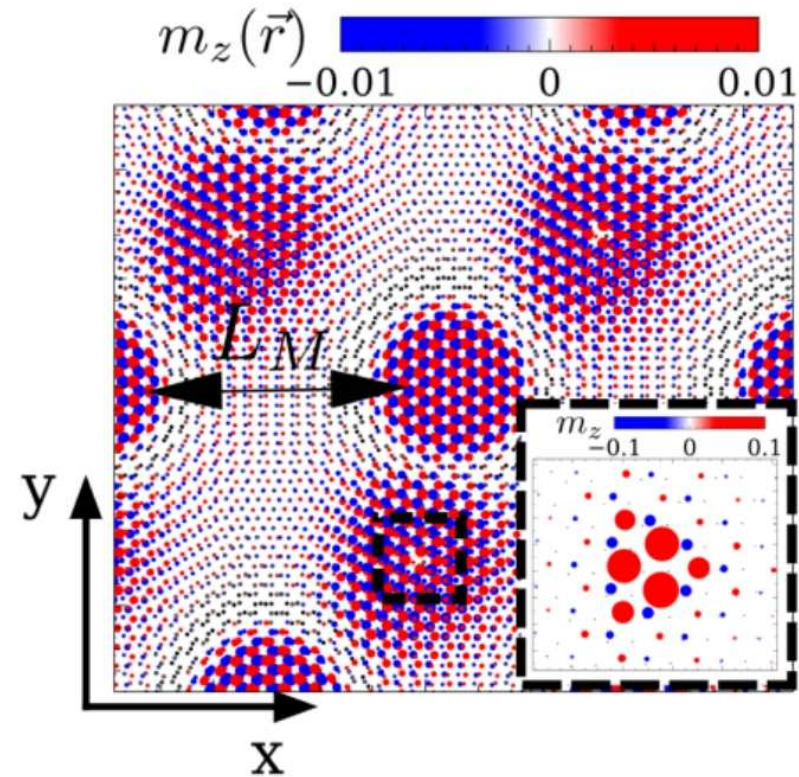
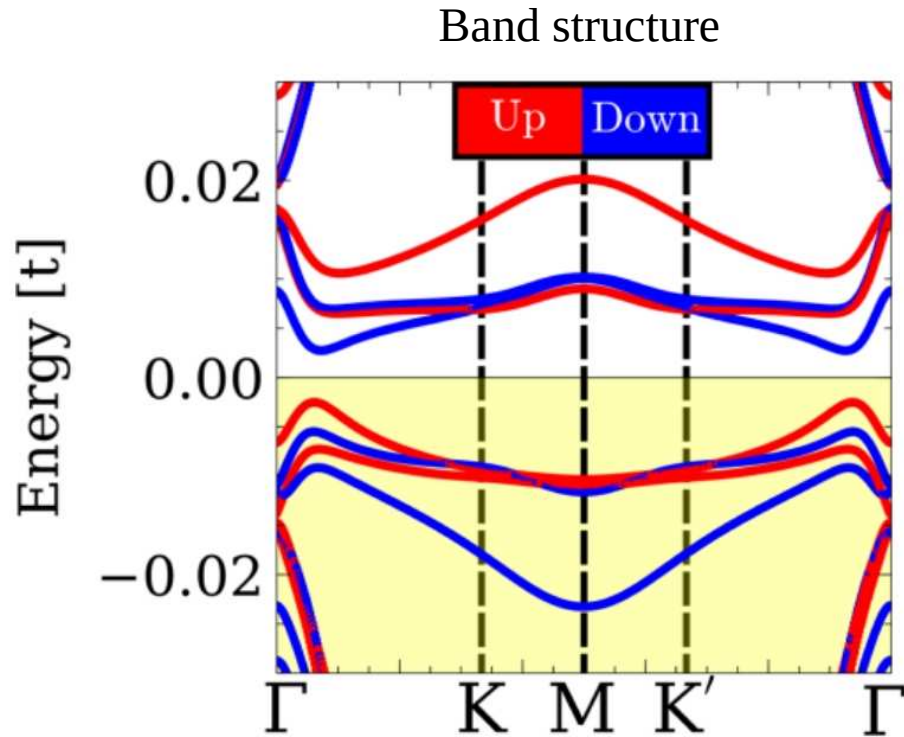
Let us see the impact on the low energy states

Impact of interactions, pristine system



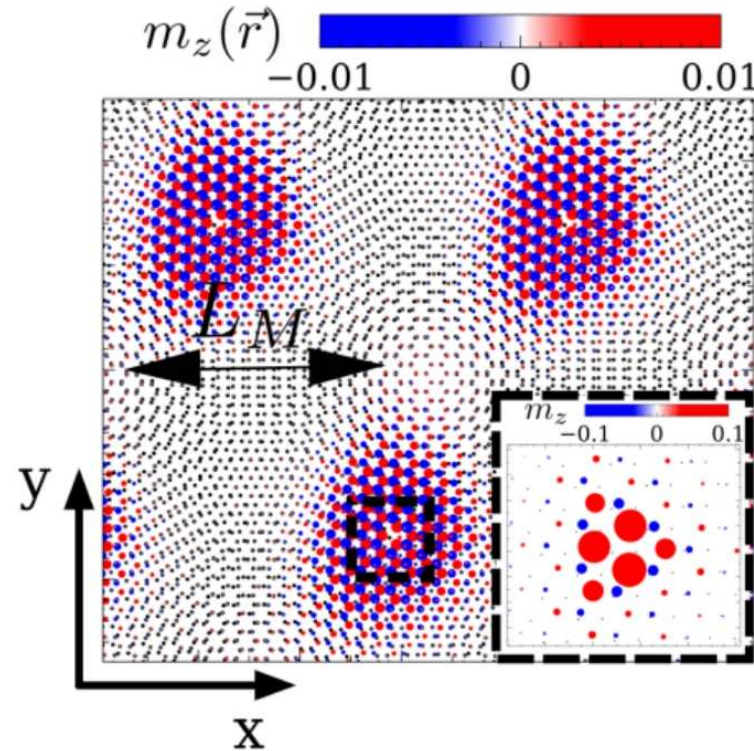
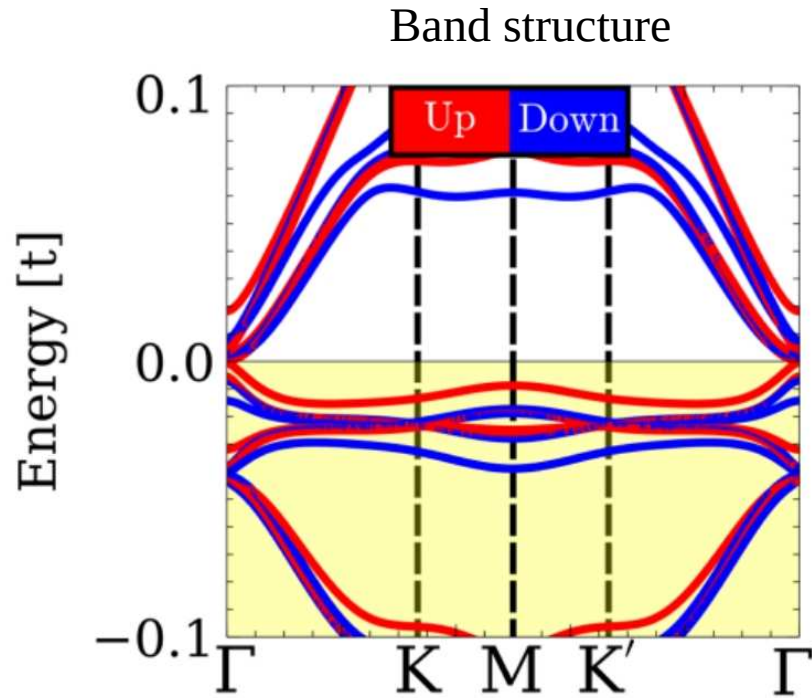
Gap opening at charge neutrality

Impact of interactions, single vacancy at charge neutrality



Interaction-driven symmetry breaking in the impurity and flat bands coexists

Impact of interactions, single vacancy at 4e per moire unit cell

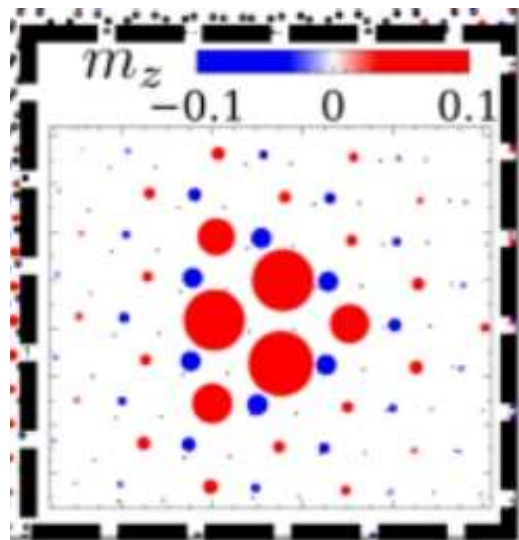


The vacancy magnetism survives full filling of the original twisted bilayer flat bands

Interplay between impurity magnetism and twisted bilayer superconductivity

The impurity induced moment survives doping of the flat bands

What happens when its magnetism coexists with the twisted bilayer superconducting state?

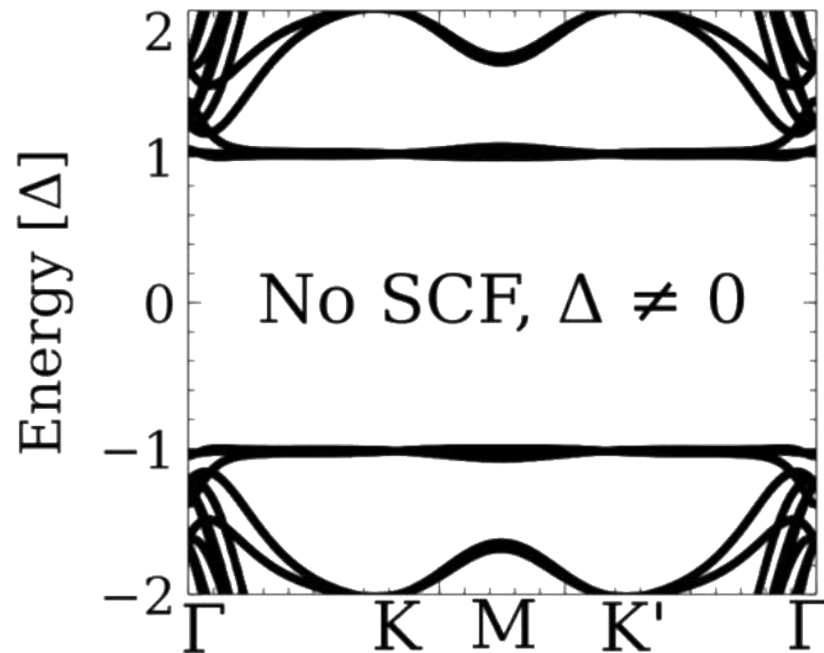
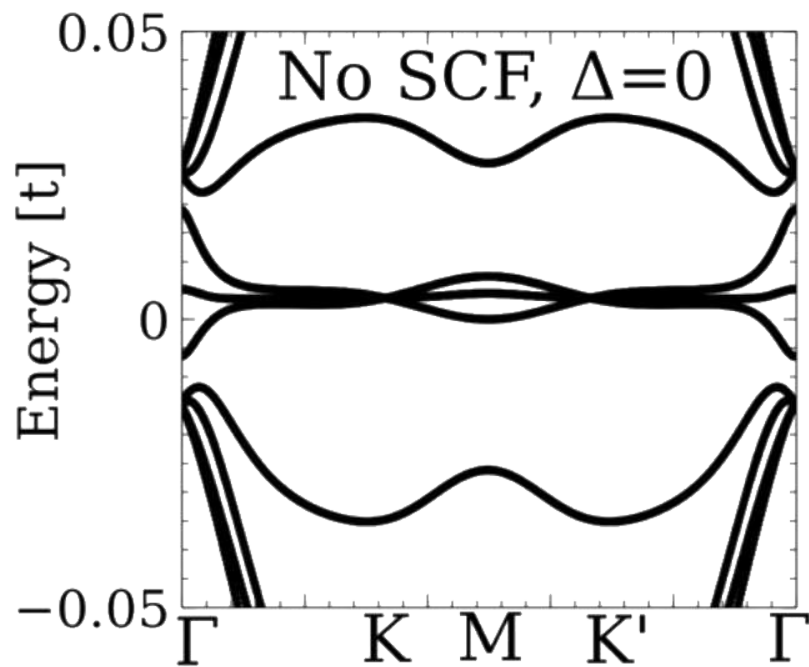


We will now include a superconducting state

$$H_{\text{SC}} = \sum_i \Delta_i [c_{i,\uparrow} c_{i,\downarrow} + c_{i,\downarrow}^\dagger c_{i,\uparrow}^\dagger]$$

Working in the regime of (close to) 2 holes per moire unit cell

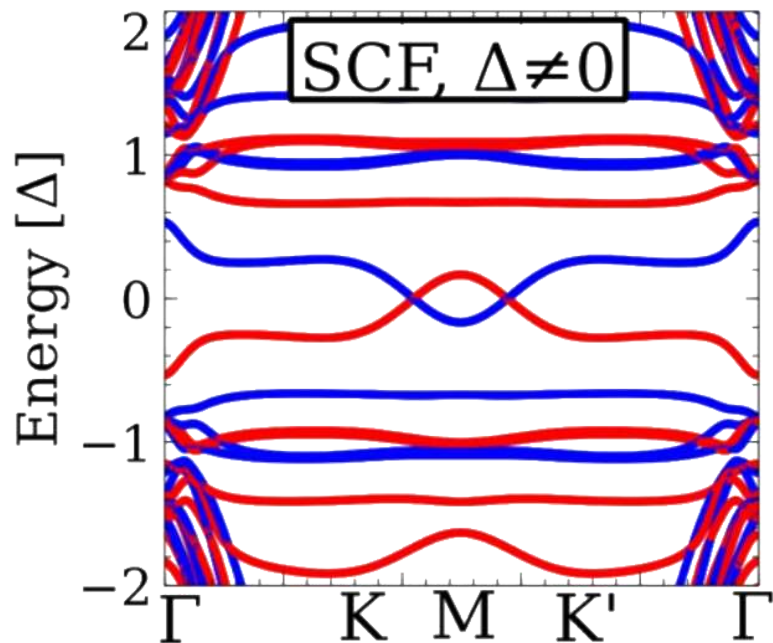
Electronic structure without including interactions



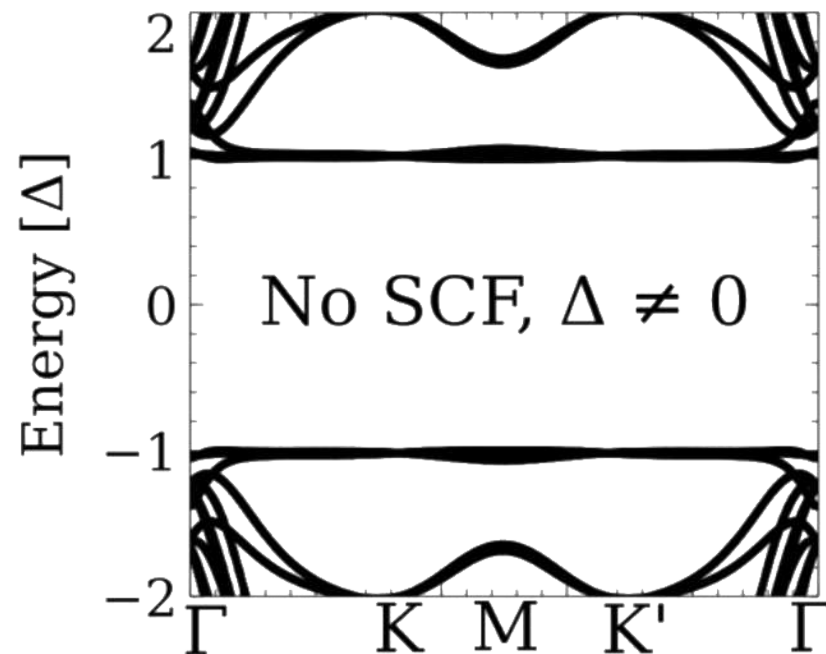
s-wave superconductivity opens up a gap in the electronic spectra

Electronic structure including repulsive interactions

With interaction-induced
magnetism



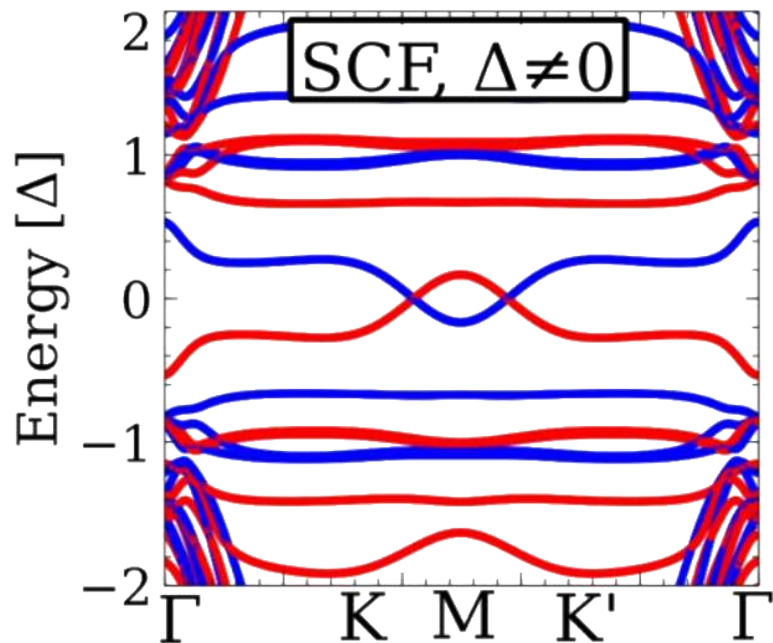
Without interaction-induced
magnetism



Impurity-induced magnetism and s-wave superconductivity gives rise to in-gap Yu-Shiba-Rusinov states

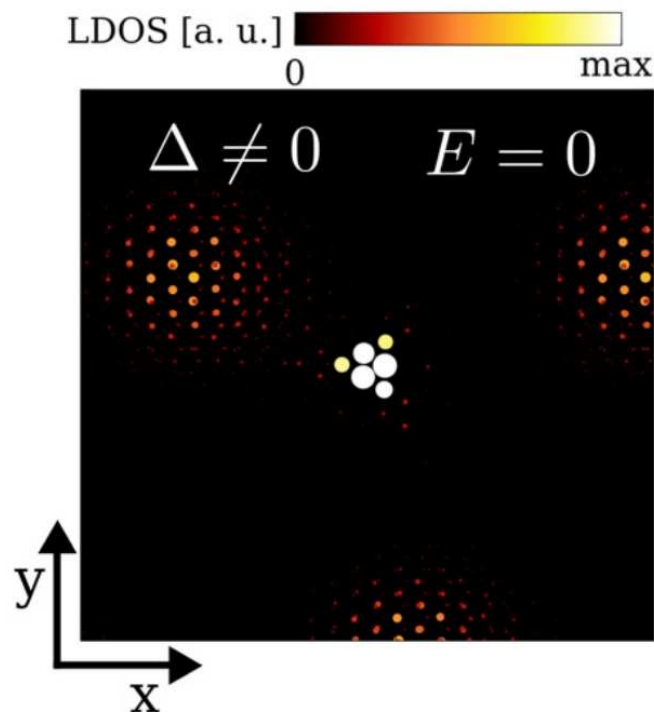
Electronic structure including repulsive interactions

With interaction-induced magnetism



In-gap modes appear around the interaction-induced impurity magnetic moment

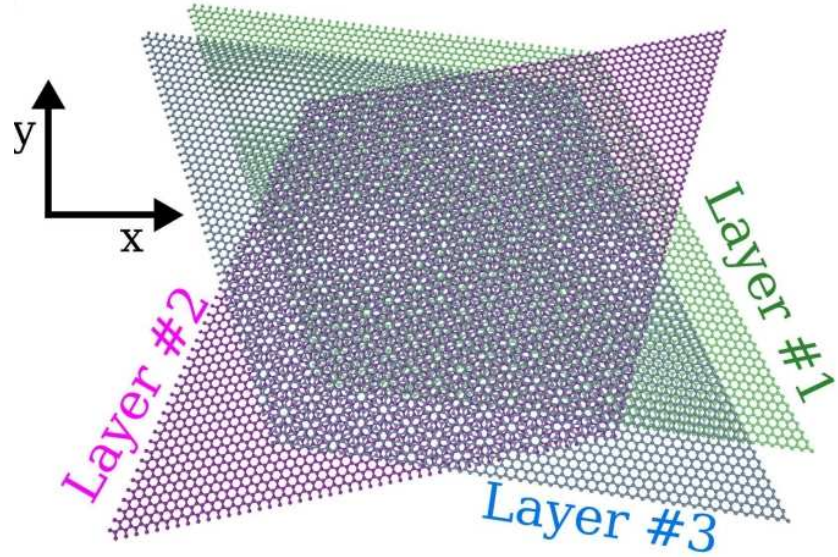
Spatial distribution of the in-gap YSR states



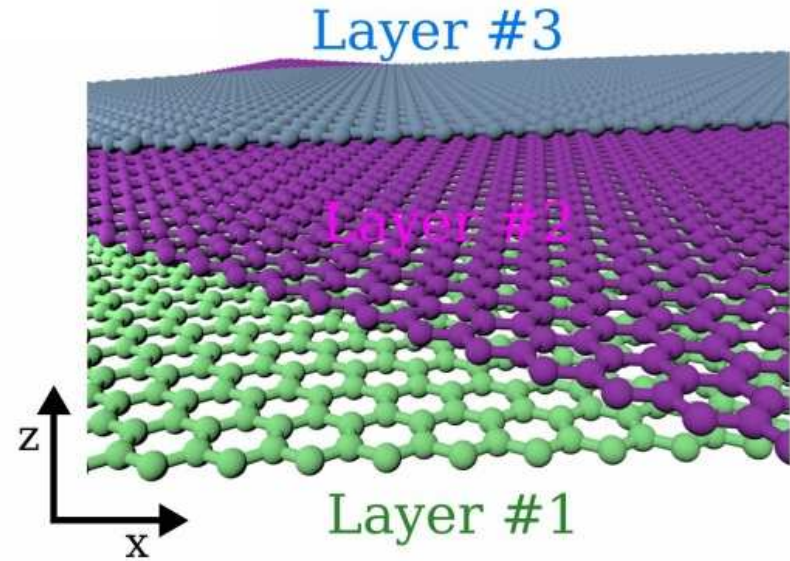
Impurities in twisted trilayer graphene

Structure of mirror-symmetric twisted graphene trilayer

Top view

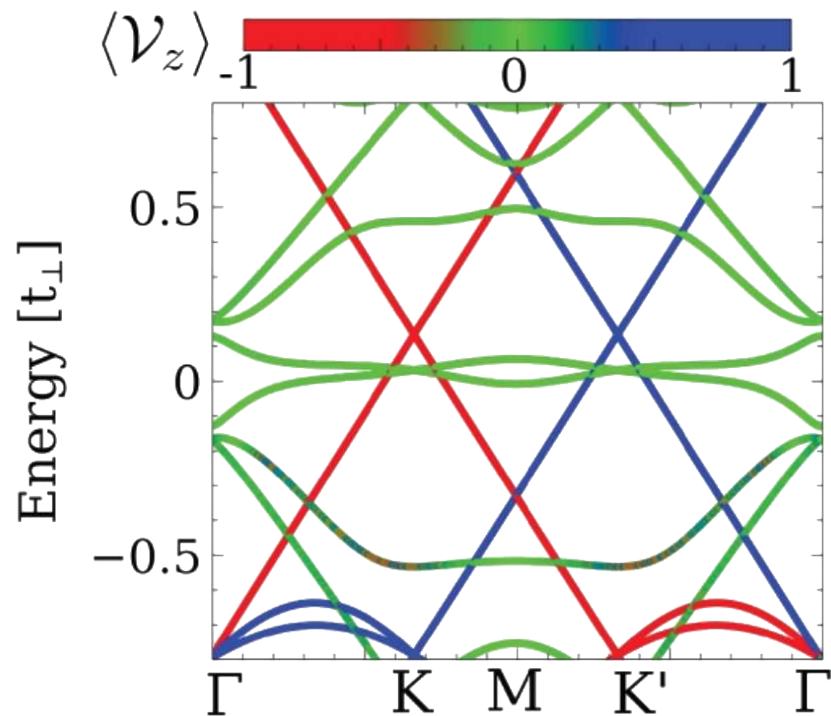


Lateral view

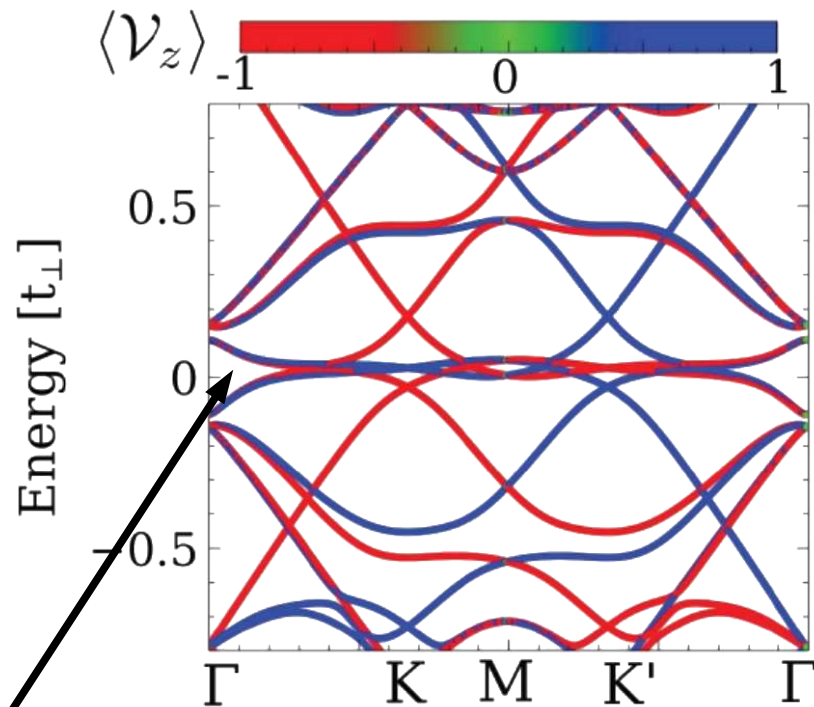


Electronic structure of twisted trilayer

Without an interlayer bias



With an interlayer bias



An interlayer bias creates internal topological valley currents

Internal topological valley currents in a twisted trilayer

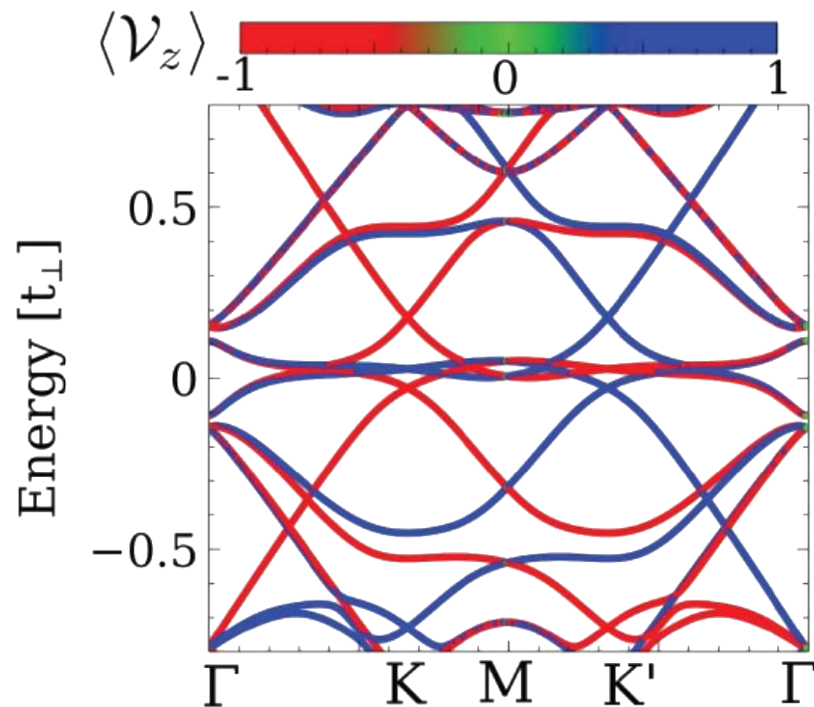
The spatially resolved valley current is defined from the valley Chern number

$$C_V = C_K - C_{K'} = \int \chi_V(\mathbf{r}, \omega) d^2\mathbf{r} d\omega$$

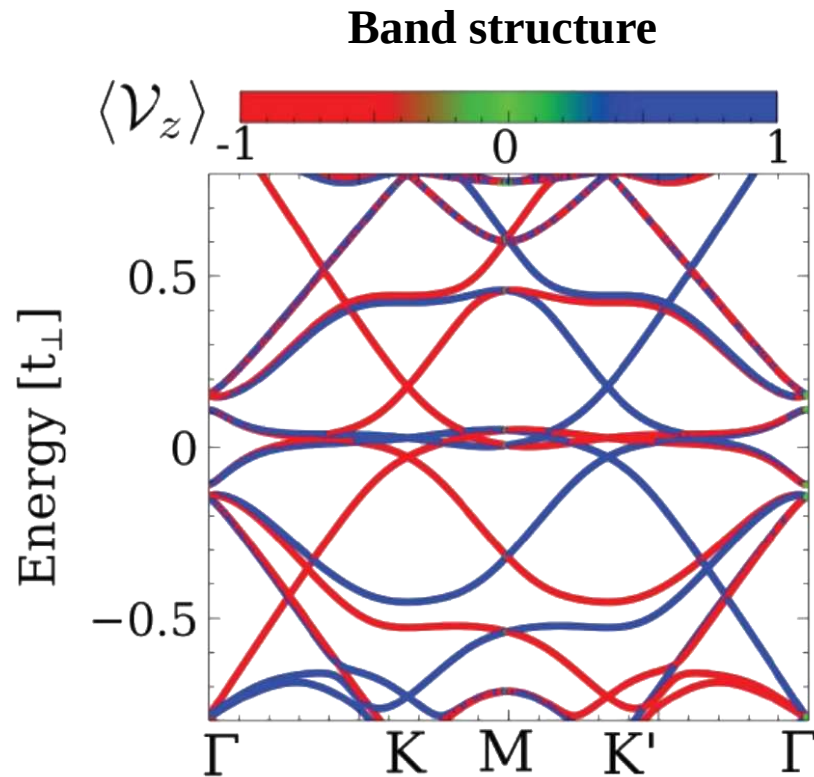
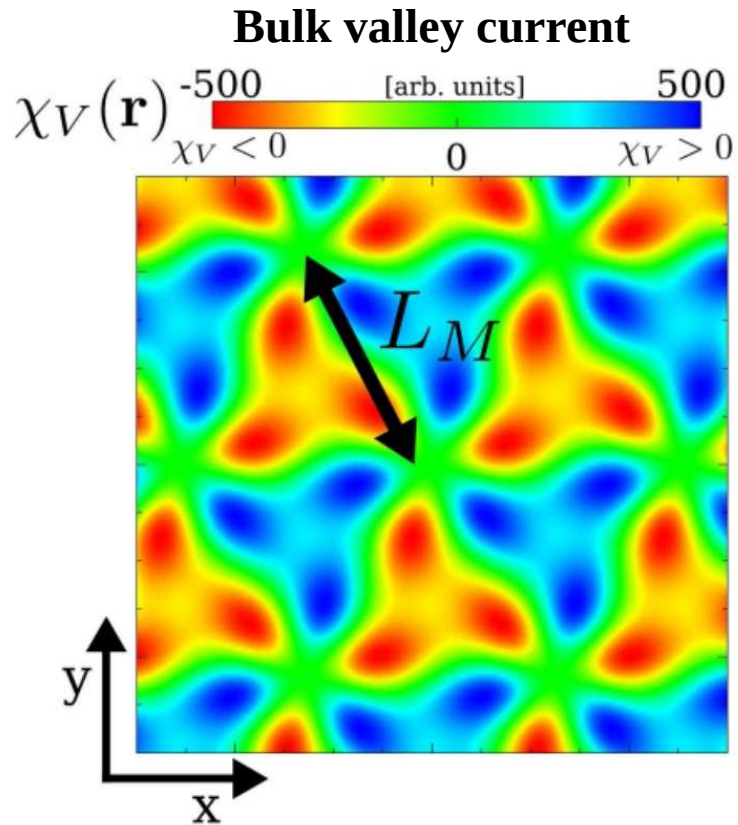
Spatially and frequency resolved valley Chern density

$$\chi_V(\mathbf{r}, \omega)$$

It represents bulk valley currents

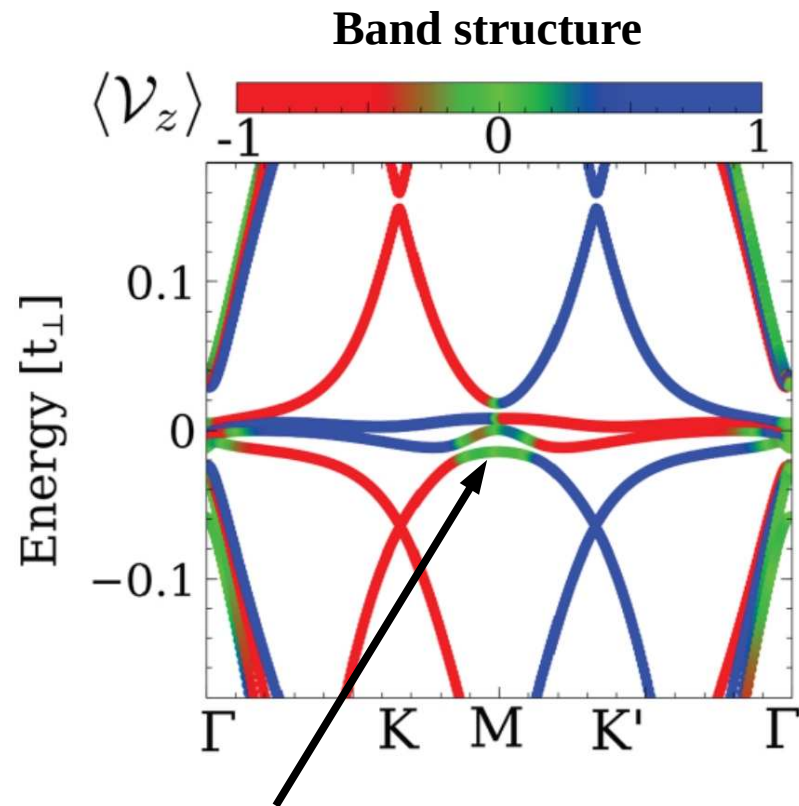
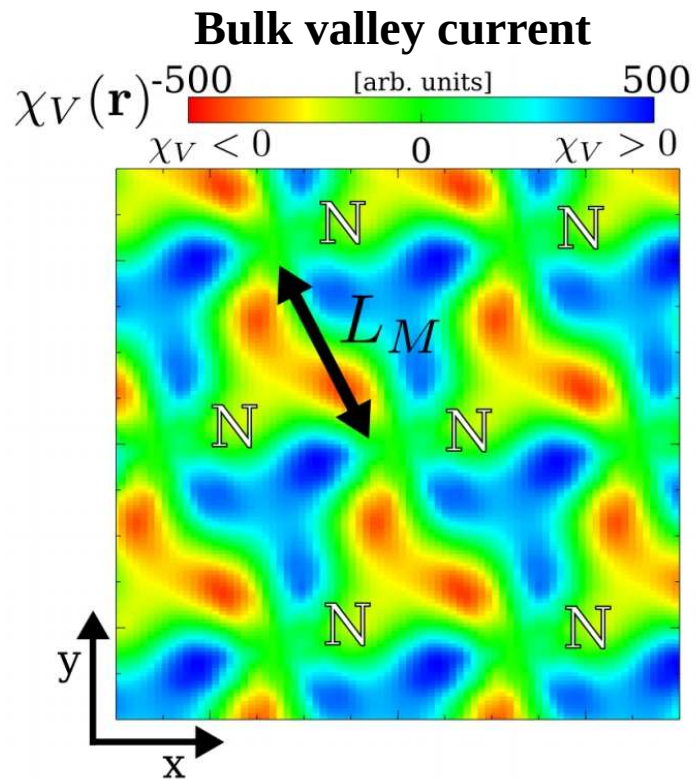


Internal topological valley currents in a twisted trilayer



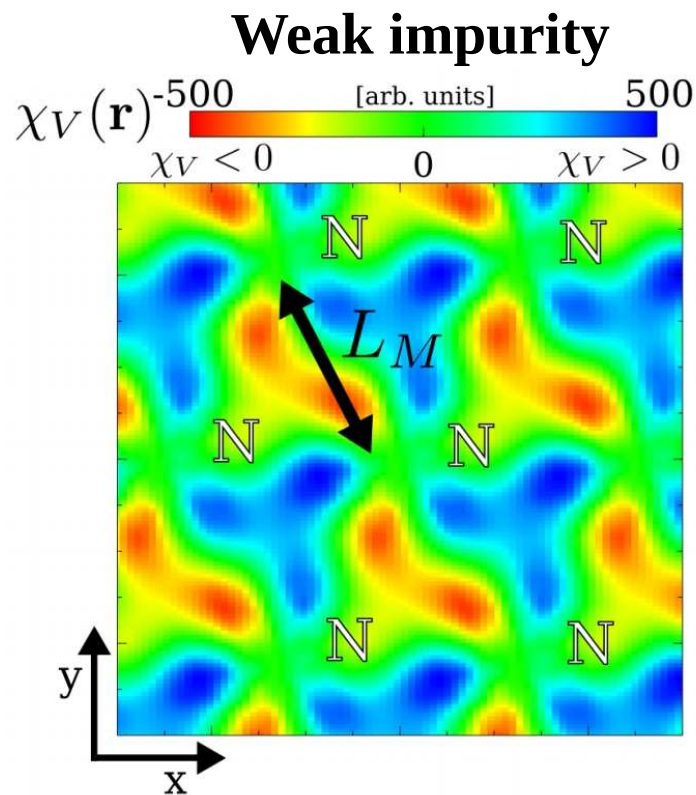
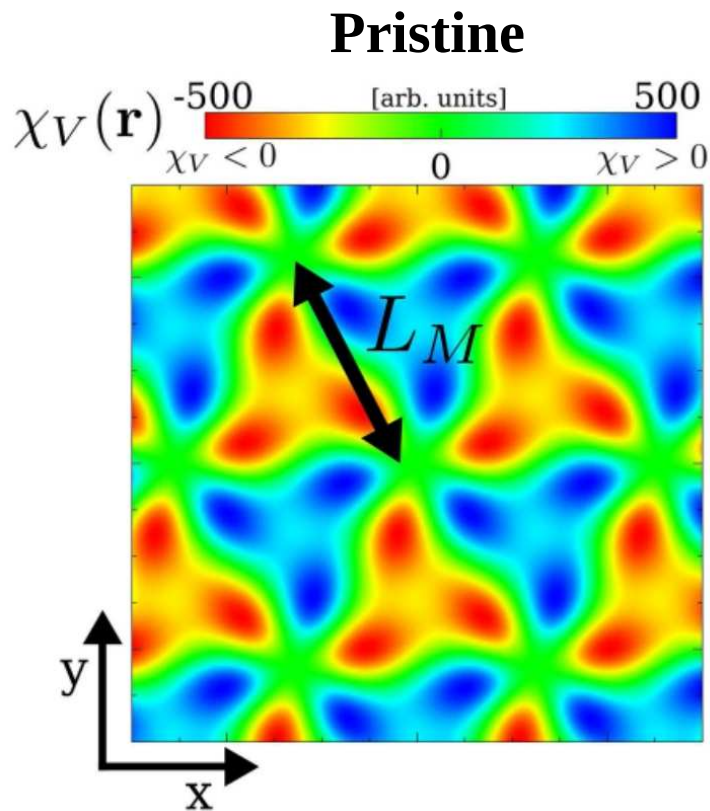
Internal topological valley currents emerge in the moire supercell

Weak impurity in a twisted graphene trilayer



Weak impurities give rise to valley mixing at low energies

Disruption of valley currents by an impurity

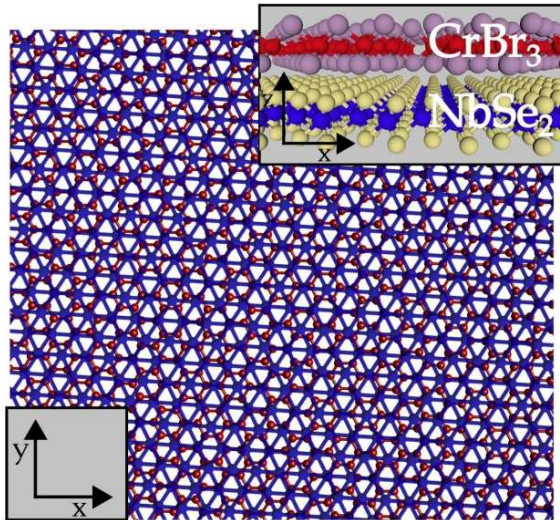


Weak impurities disrupt the valley currents in twisted graphene trilayers

Impurities in a twisted topological superconductor

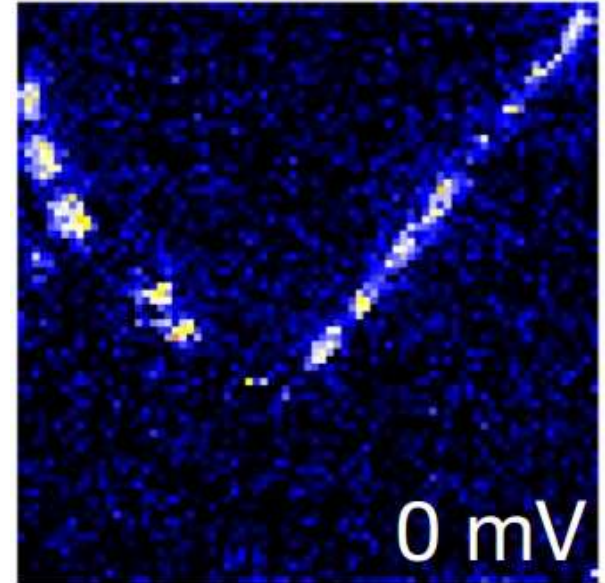
Topological superconductivity in CrBr₃/NbSe₂ heterostructure

CrBr₃/NbSe₂ heterostructure



Nature 588, 424–428 (2020)
Nano Lett. 2022, 22, 1, 328–333

Zero bias dI/dV

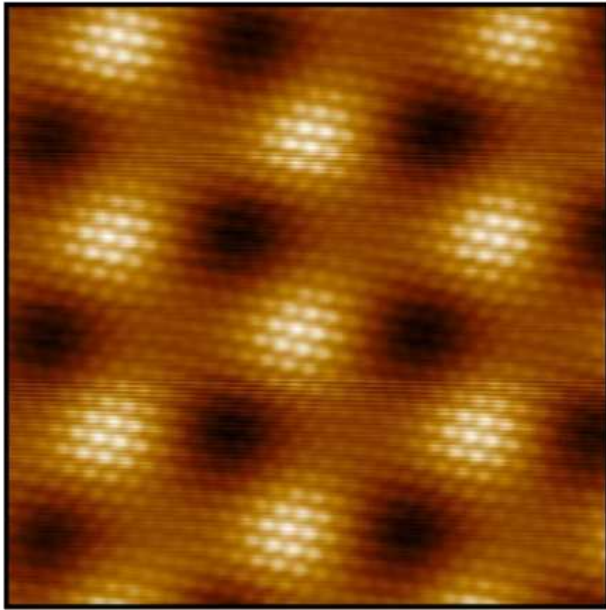


A topological superconducting state appears in CrBr₃/NbSe₂

$\text{CrBr}_3/\text{NbSe}_2$ heterostructure

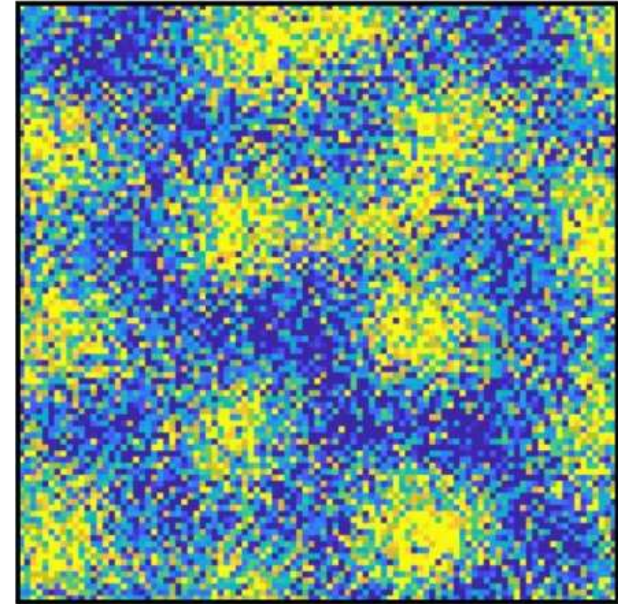
Topography

Height 0 Å 0.26 Å



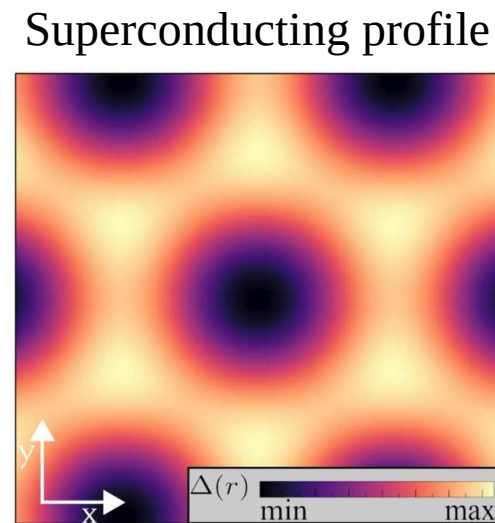
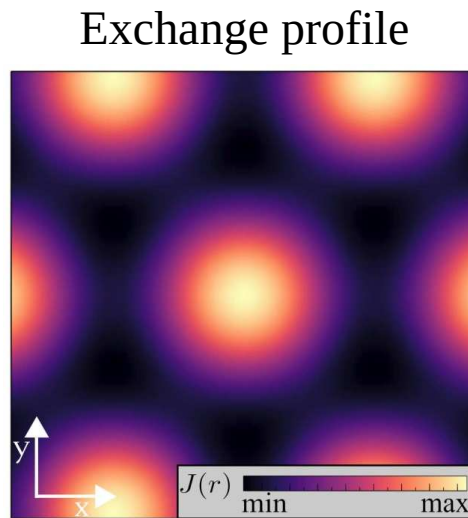
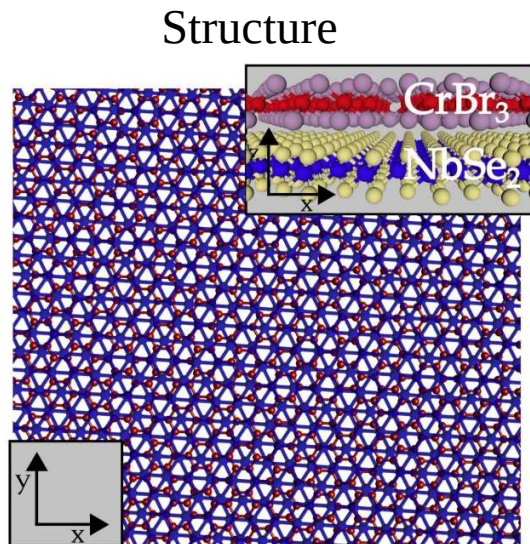
dIdV of the Yu-Shiba-Rusinov states

dI/dV (nS) 36 49



The low energy properties are affected by the moiré pattern

Hamiltonian for the pristine moire topological superconductor



$$\mathcal{H}_0 = \mathcal{H}_{\text{kin}} + \mathcal{H}_J + \mathcal{H}_R + \mathcal{H}_{\text{SC}}$$

Kinetic
energy

Exchange
coupling

Rashba
SOC

s-wave
superconductivity

Hamiltonian for the defective moire topological superconductor

Full Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{\text{imp}}$$

Local impurity in the moire system

$$\mathcal{H}_{\text{imp}} = w \sum_s c_{n,s}^\dagger c_{n,s}$$

Weak impurity

$$w \rightarrow 0$$

Strong impurity

$$w \rightarrow \infty$$

$$\mathcal{H}_0 = \mathcal{H}_{\text{kin}} + \mathcal{H}_J + \mathcal{H}_R + \mathcal{H}_{\text{SC}}$$

Kinetic
energy

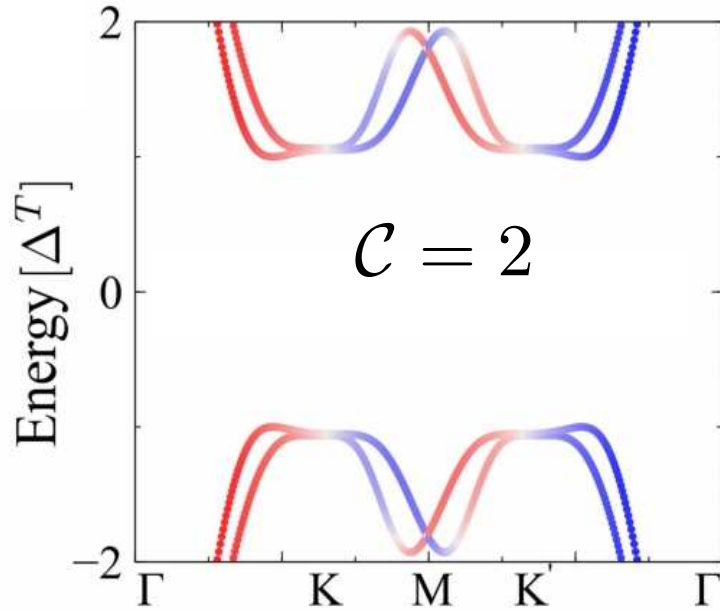
Exchange
coupling

Rashba
SOC

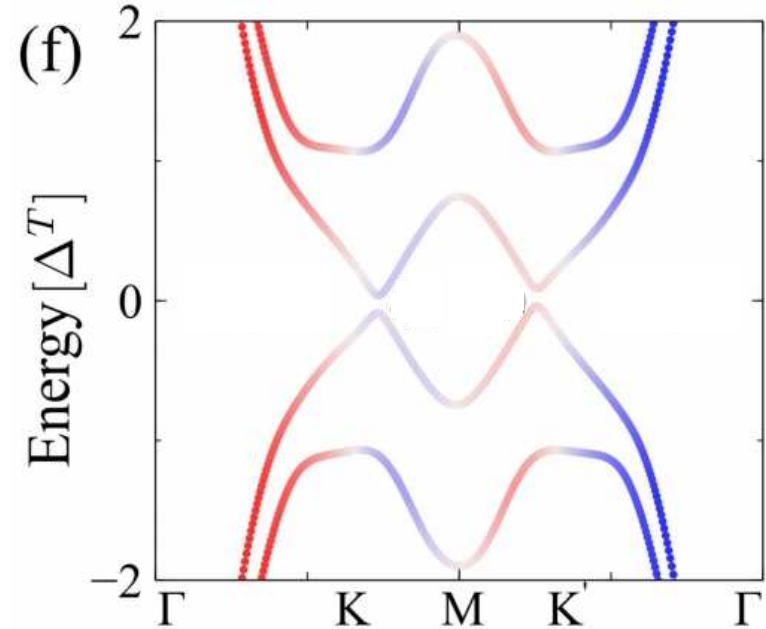
s-wave
superconductivity

Impurities in the moire topological superconductor

Band structure in the absence of impurity

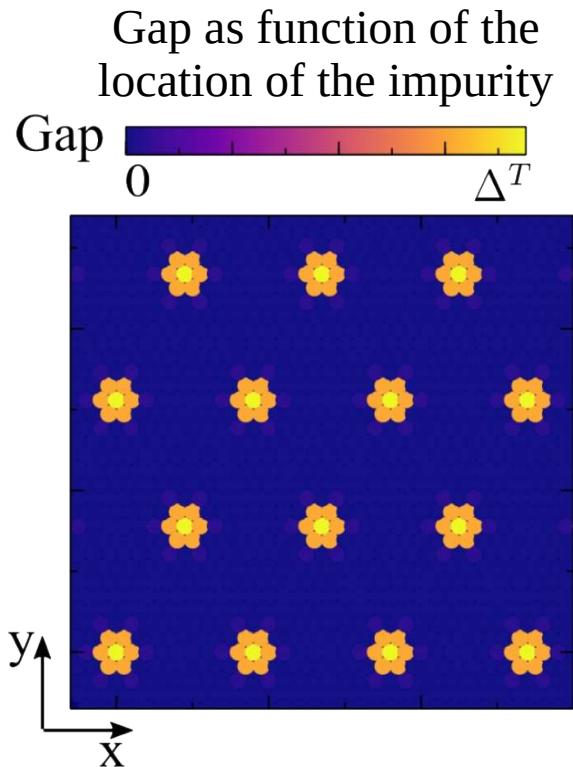


Band structure in the presence of an impurity

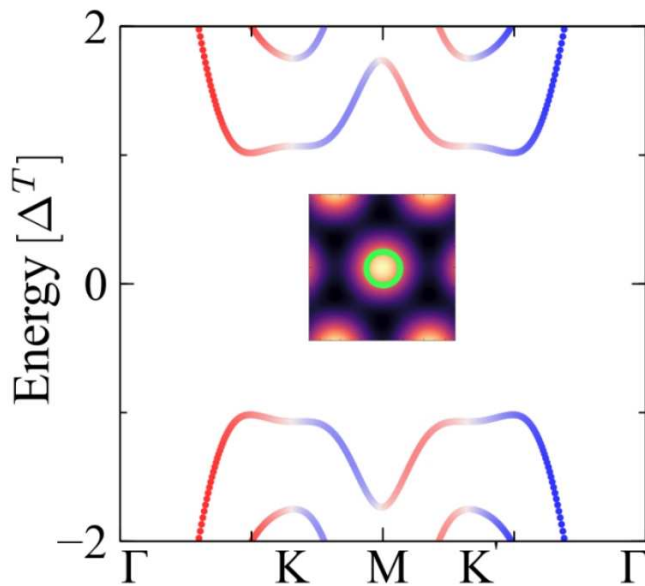


Strong local impurities decrease the size of a topological gap

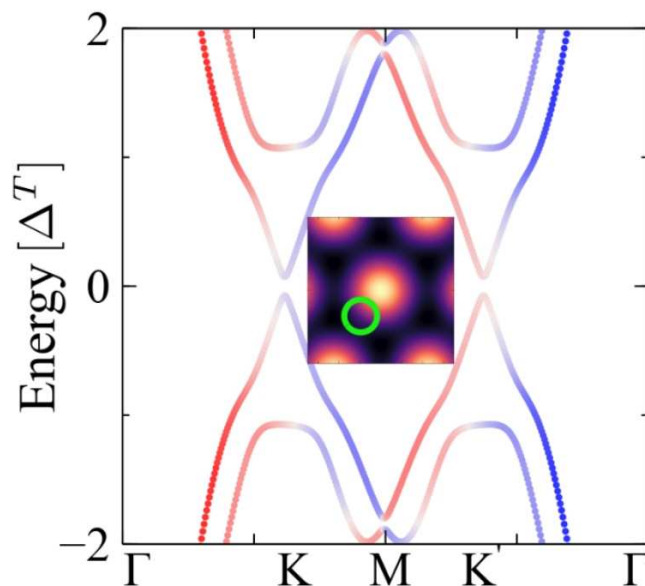
Gap as a function of the location of a strong impurity



Band structure for impurity at the center



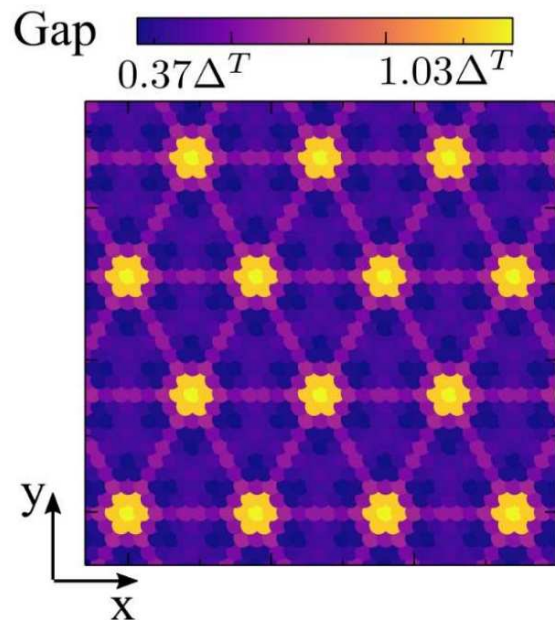
Band structure for impurity away from the center



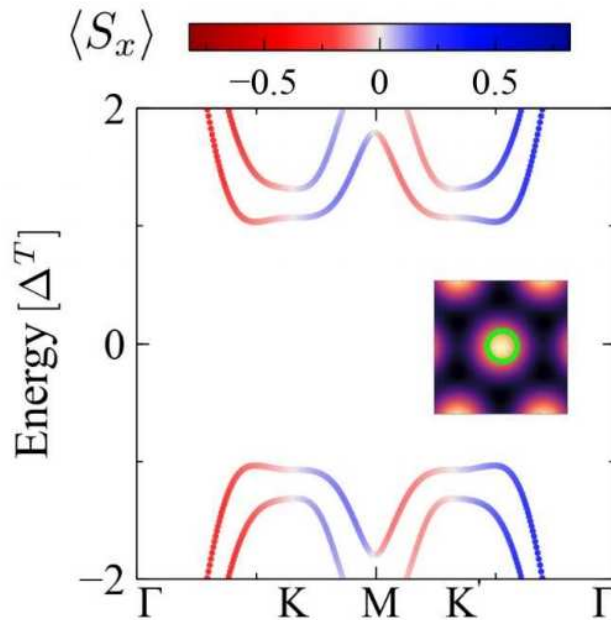
The location of the impurity strongly affects the resulting gap

Gap as a function of the location of a strong impurity

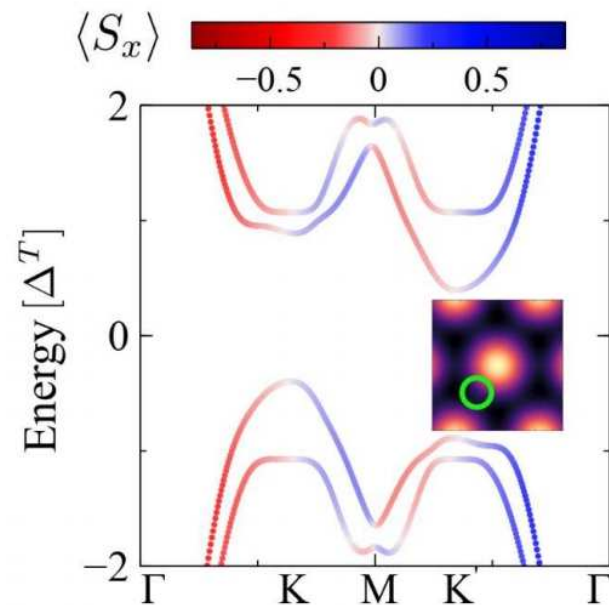
Gap as function of the location of the impurity



Band structure
for impurity at the center

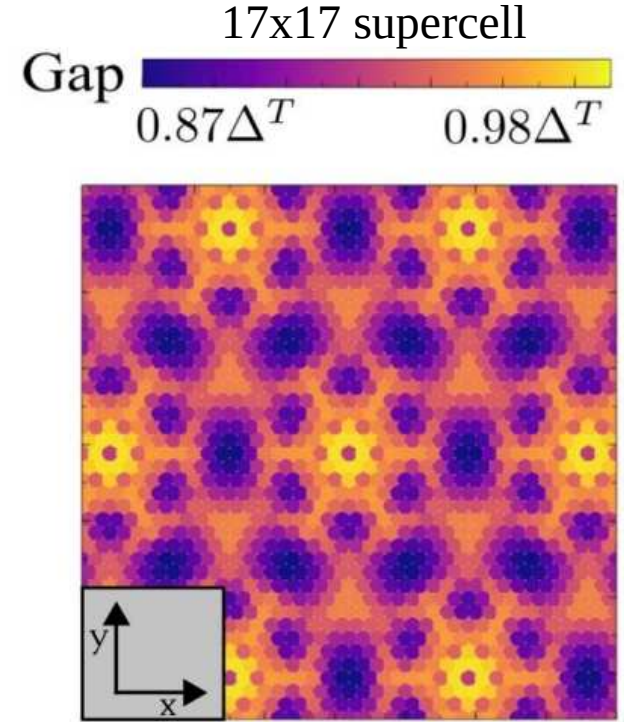
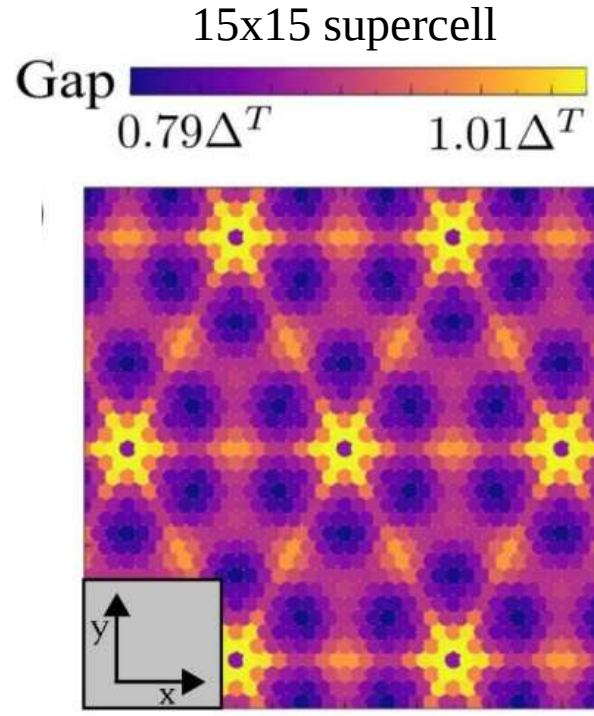
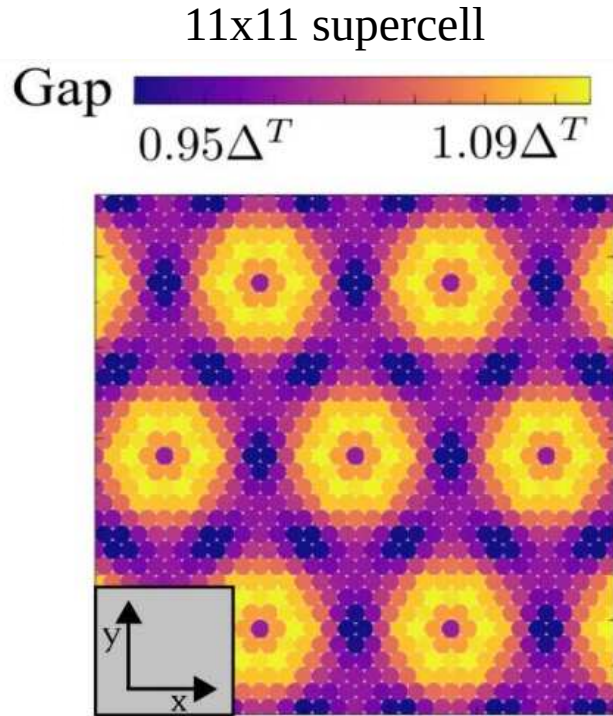


Band structure for
impurity away from the center



The location of the impurity strongly affects the resulting gap

Gap as function of location for different moire unit cells



Detrimental locations for impurities depend on the specific moire supercell

The single impurity limit

We now look at the single impurity regime

From the Green's function, we can compute the spatially resolved density of states at the energy of the YSR state

$$\rho(\mathbf{x}, \omega) = -\frac{1}{\pi} \sum_{s, \tau} \langle \mathbf{x}, s, \tau | \text{Im}(G_V(\omega)) | \mathbf{x}, s, \tau \rangle$$

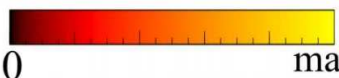
Dyson equation for a single impurity

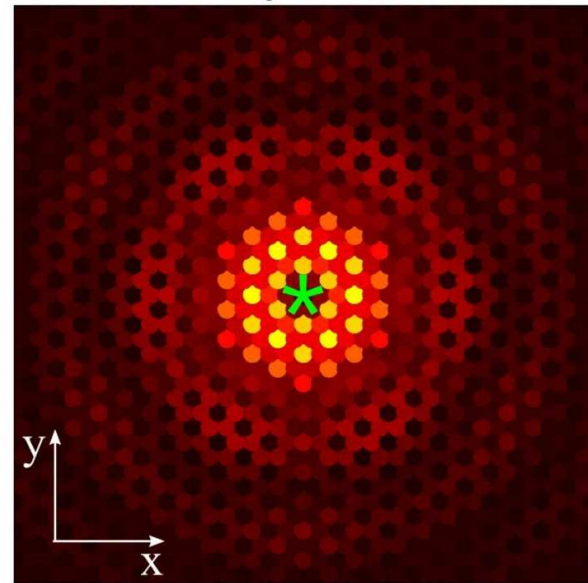
$$G_V(\omega) = [\omega - H_V - \Sigma(\omega) + i0^+]^{-1}$$

Selfenergy from the pristine regime

$$G_0(\omega) = [\omega - H_0 - \Sigma(\omega) + i0^+]^{-1}$$

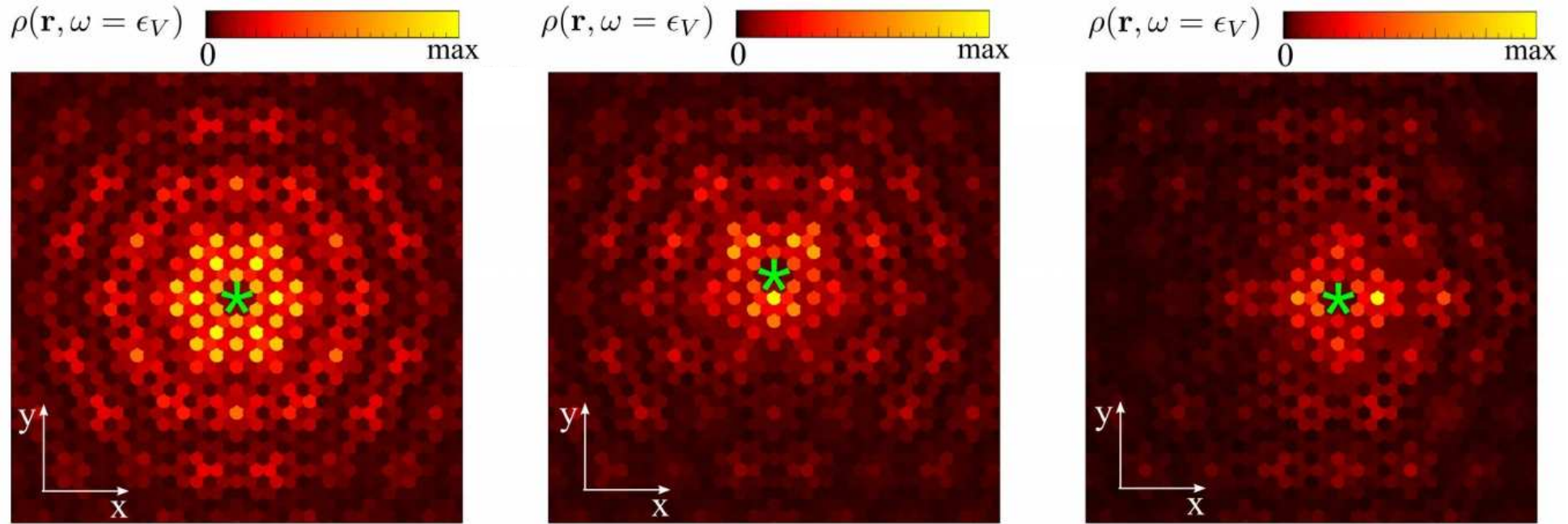
YSR without moire

$\rho(\mathbf{r}, \omega = \epsilon_V)$ 



Single impurity in a topological moire superconductor

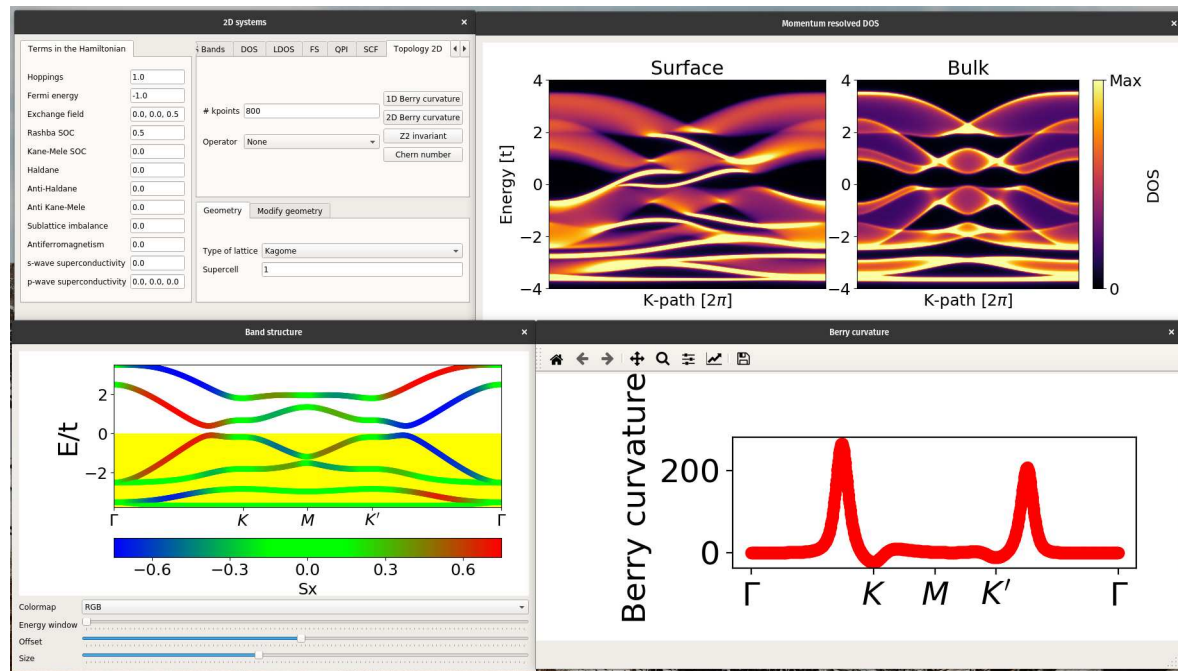
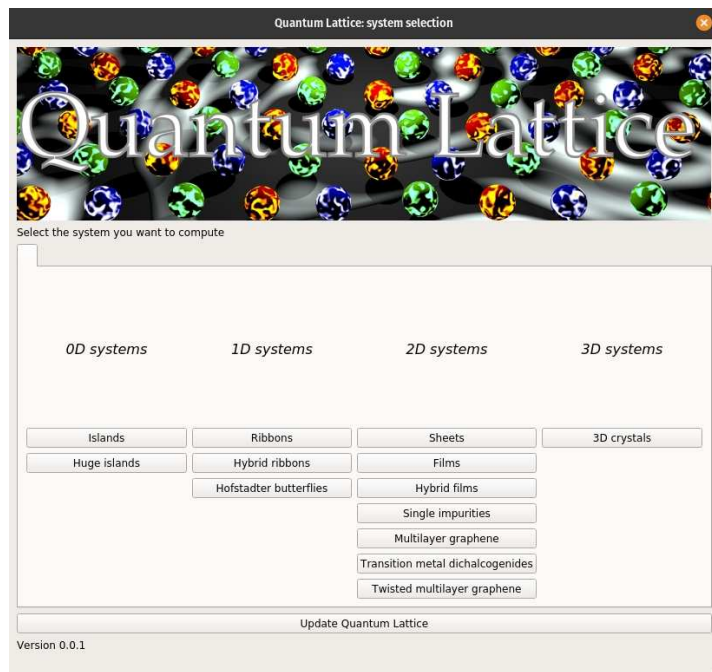
Wavefunction of the YSR state for different impurity locations



The YSR state wavefunction and the moire give rise to interference effects

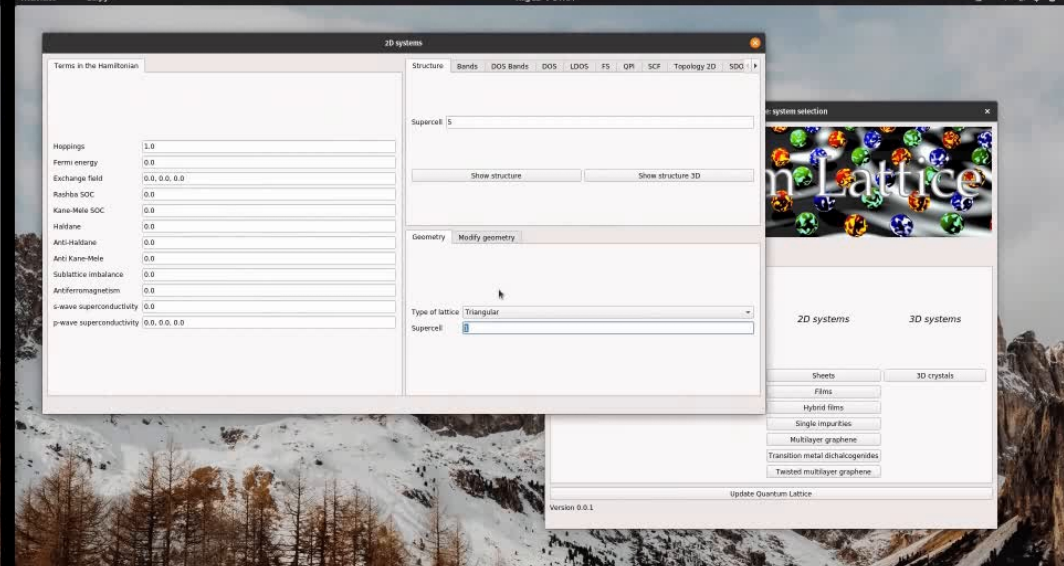
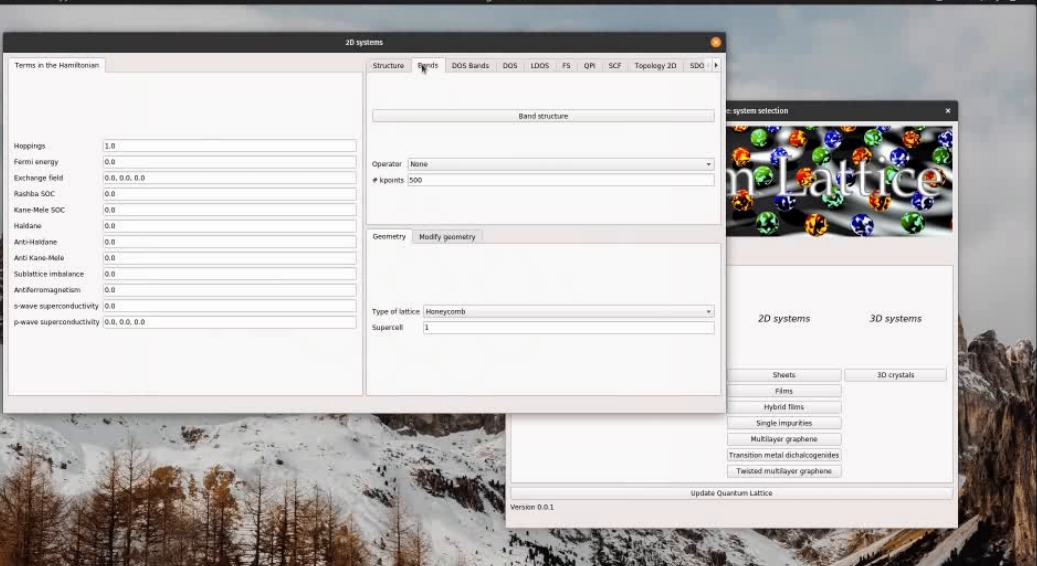
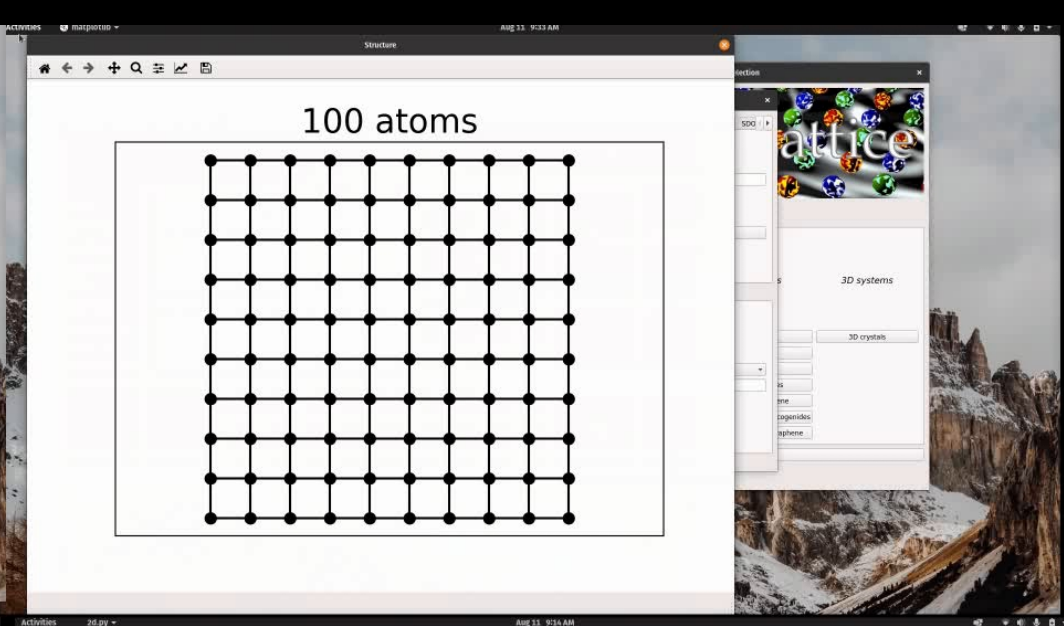
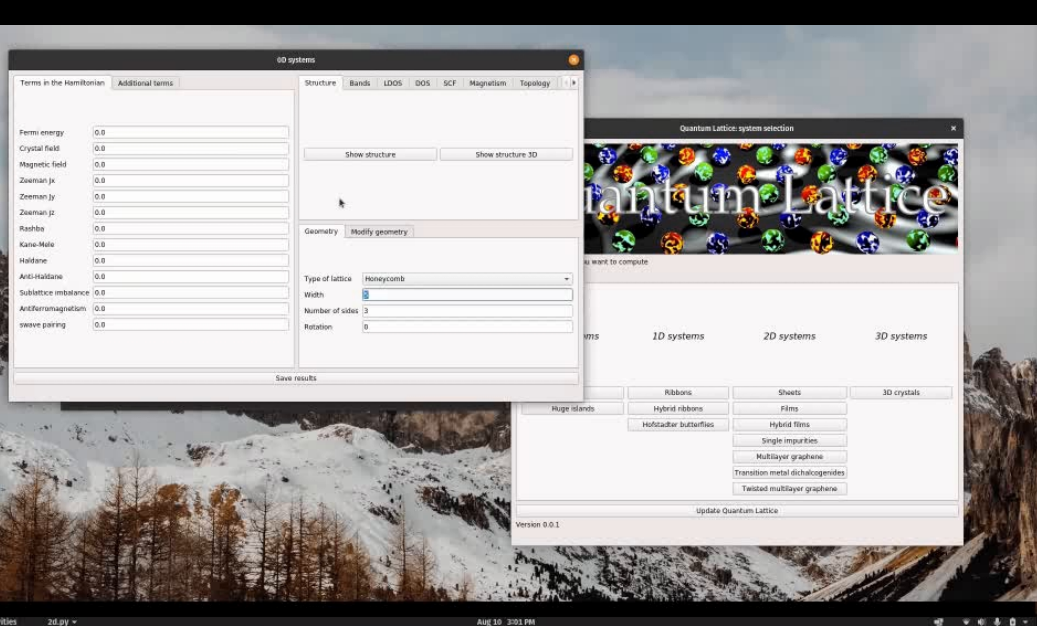
Computing electronic properties

Quantum Lattice: A user interface to compute electronic properties



Quantum Lattice: open source interactive interface for tight binding modeling

<https://github.com/joselado/quantum-lattice>



Hamiltonian

Interlayer

Electric bias

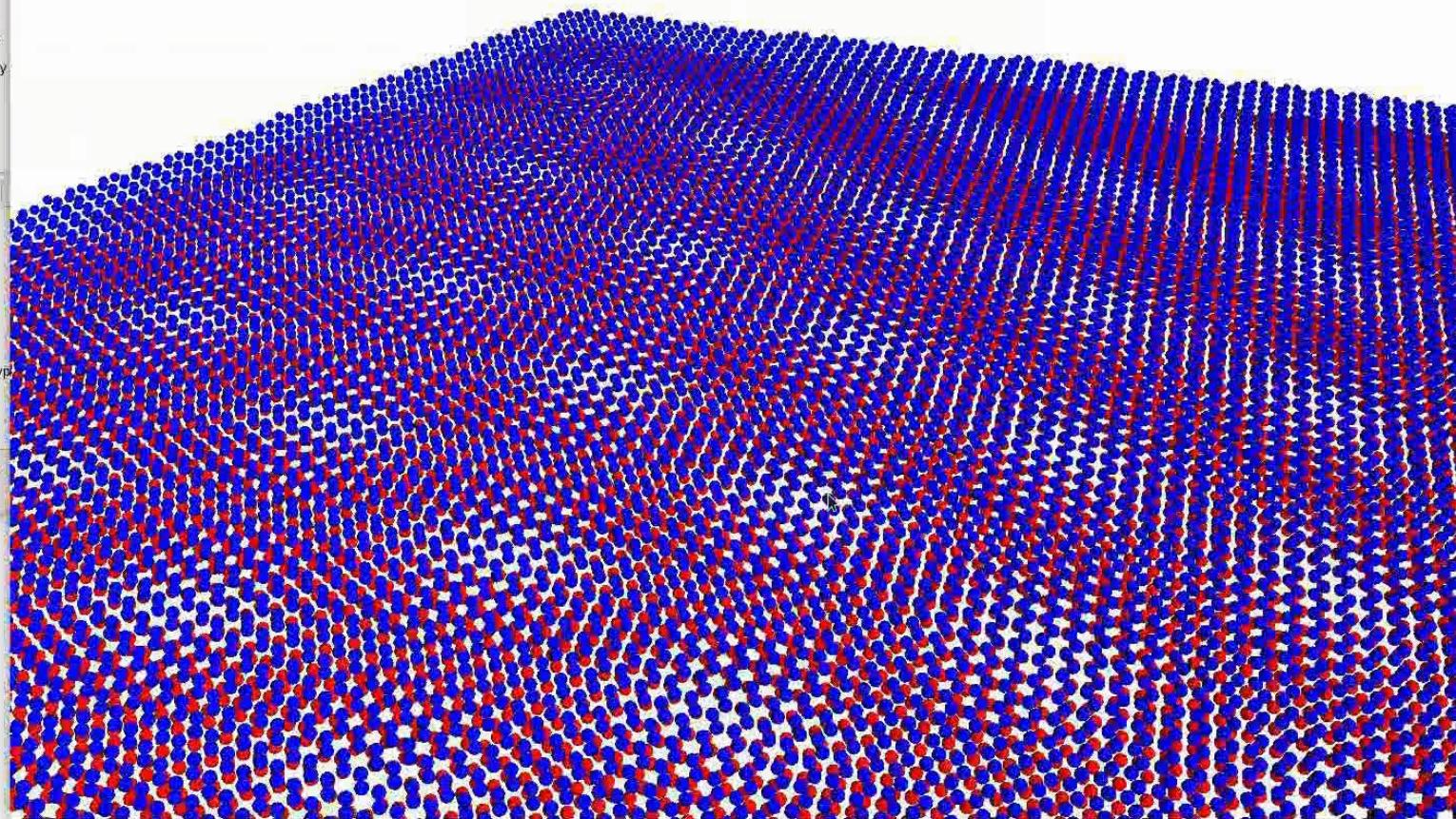
Fermi energy

Geometry

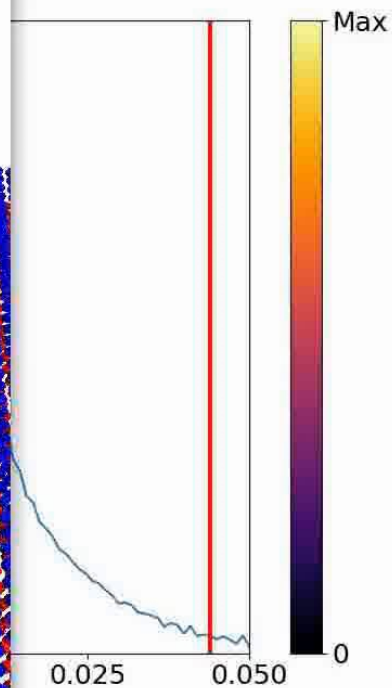
Cell size

Multilayer type

Parallelization

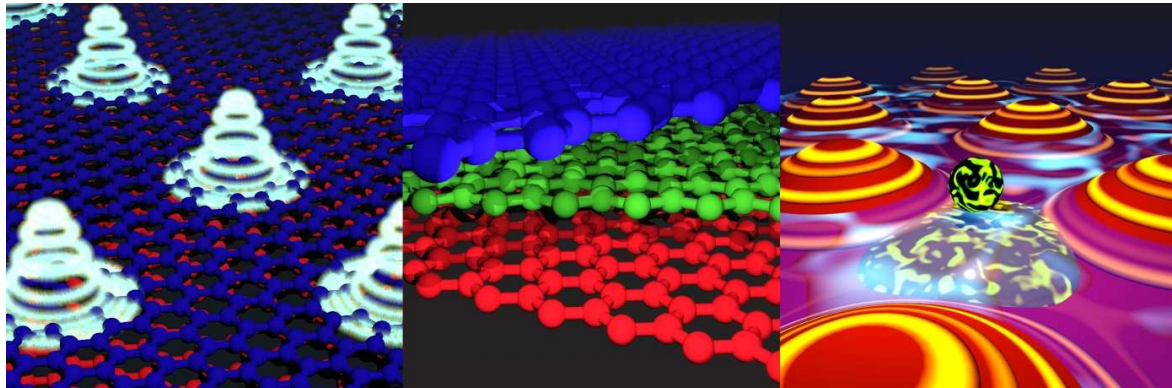


IPR



Take home

Impurities in twisted moire systems can have strong impacts in the low energy dispersion, creating resonant states and affecting topological and correlated states



Thank you!

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Phys. Rev. B 99, 245118 (2019)
Phys. Rev. Materials 3, 084003 (2019)
Phys. Rev. Research 2, 033357 (2020)
arXiv:2202.11003 (2022)

