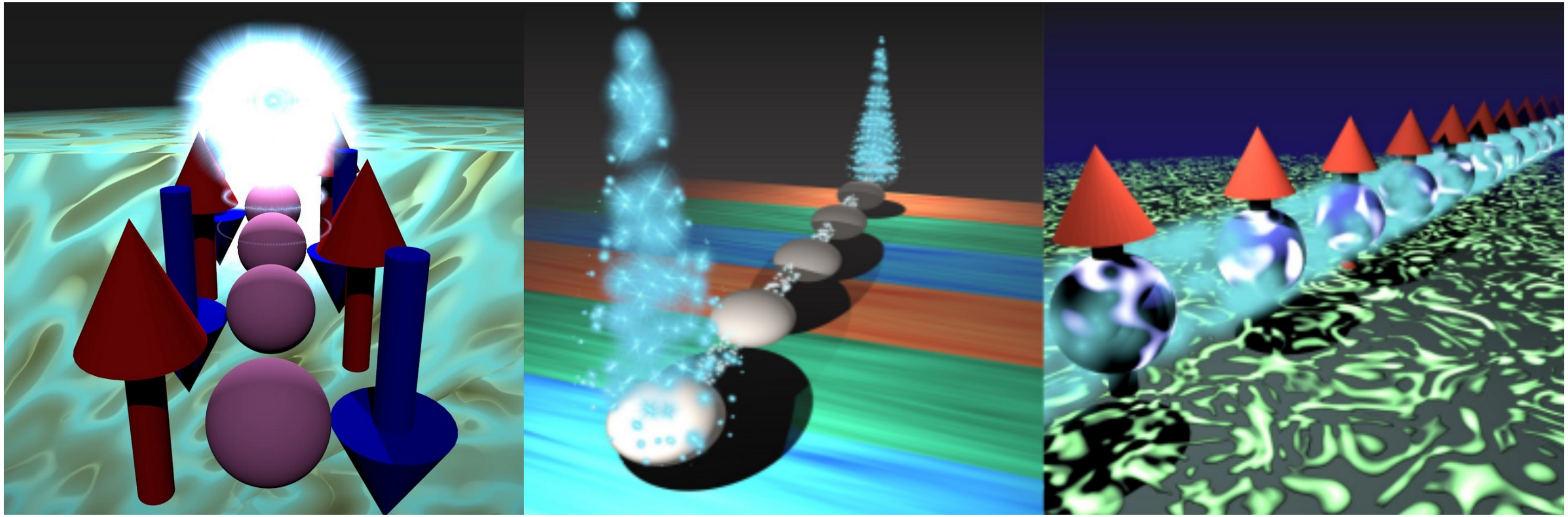
Dynamical excitations in interacting non-Hermitian and quasiperiodic topological many-body matter

Jose Lado

Department of Applied Physics, Aalto University, Finland



Phys. Rev. Research 2, 023347 (2020)

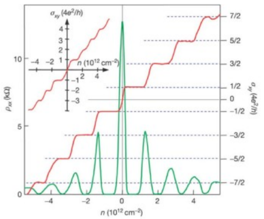
Phys. Rev. Research 1, 033009 (2019)
Phys. Rev. Research 3, 013265 (2021)

Phys. Rev. Research 4, L012006 (2022)
arXiv:2208.06425 (2022)

TOPCORE meeting, January 17th 2023, Amsterdam, Netherlands

Topological matter

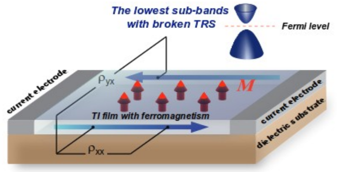
Quantum Hall effect



graphene

Nature 438, 197-200 (2005)

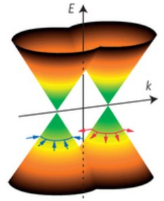
Quantum Anomalous Hall effect



Cr-doped $(Bi,Sb)_2Te_3$

Science 340.6129 (2013): 167-170

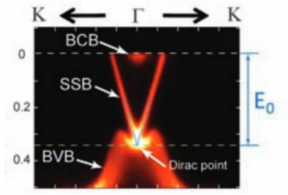
Weyl semimetal



TaAs

Nature Physics 11, 724–727 (2015)

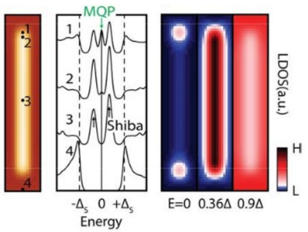
Quantum Spin Hall effect



Bi_2Se_3

Science 325.5937 (2009): 178-181

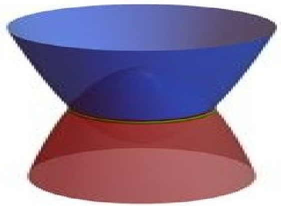
Topological superconductor



Fe@Pb

Science 346.6209 (2014): 602-607

Nodal line semimetals



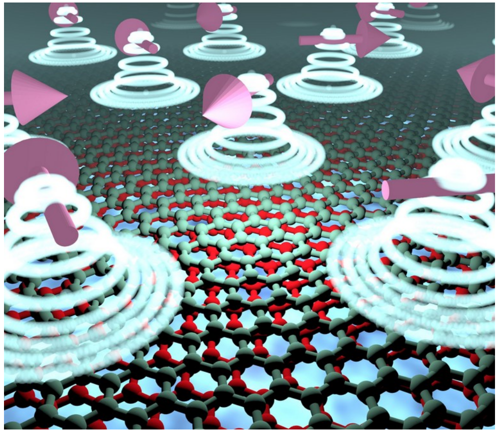
Mg_3Bi_2

Advanced Science 6.4 (2019): 1800897

All these states are described by effective single-particle Hamiltonians

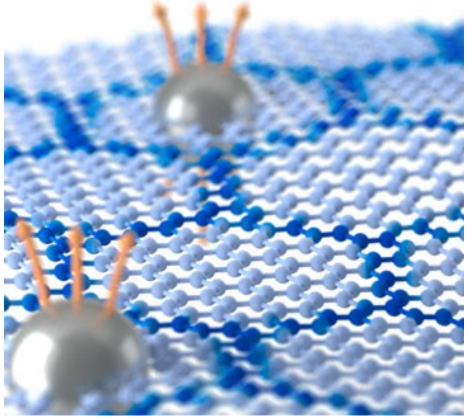
Many-body quantum matter

Quantum spin liquids



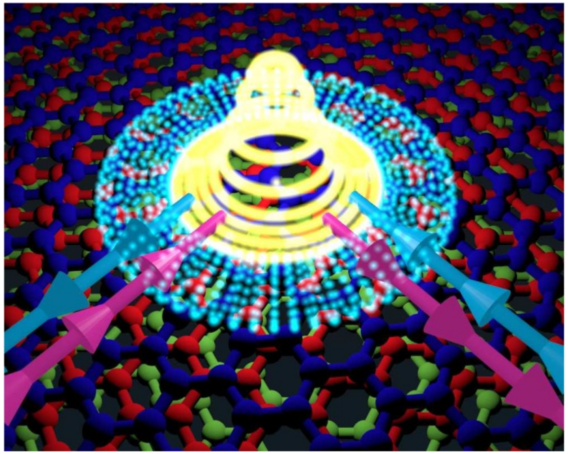
Reports on Progress in Physics 80.1 (2016): 016502.

Fractional Chern insulators



Rev. Mod. Phys. 71, S298 (1999)

Parafermions



Annual Review of Condensed Matter Physics 7, 119-139 (2016)

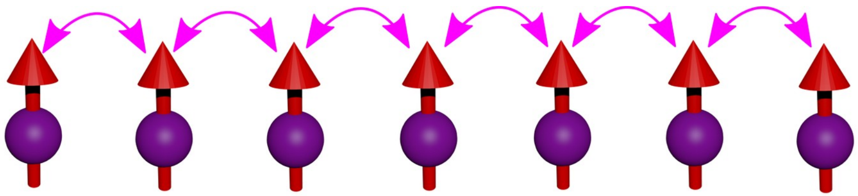
Many-body quantum matter represents an exciting critically open challenge

The problem of dimensionality

In a many-body problem, the size of our vectors grows as

2^L where L is the number of sites

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$



For a single-particle tight binding problem, we can reach up to 10^8 sites in a laptop

For a many-problem, we cannot even store states for systems bigger than $L = 30$ sites

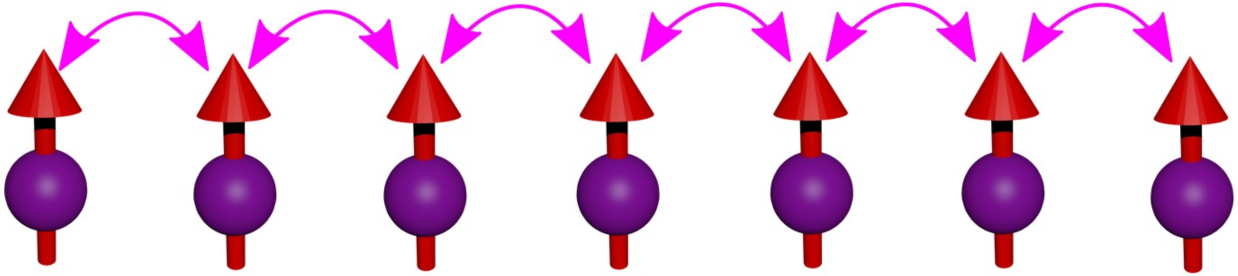
$$2^{30} \sim 10^9$$

How can be compute dynamical properties if we cannot even store the states?

Many-body dynamical correlators

One dimensional Heisenberg Hamiltonian

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$



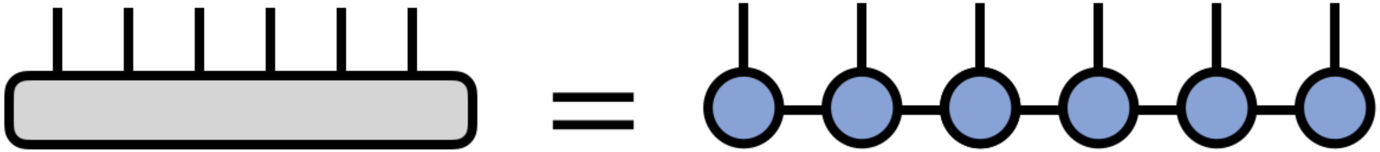
We want to compute the dynamical correlator

$$\mathcal{S}(N,\omega) = \langle GS | S^z_N \delta(\omega - \mathcal{H} + E_0) S^z_N | GS \rangle$$

State compression with tensor-networks

Given a many-body wavefunction, we can parametrice the components as

$$|\Psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left[A_1^{(s_1)} A_2^{(s_2)} \cdots A_N^{(s_N)}\right] |s_1 s_2 \ldots s_N\rangle$$



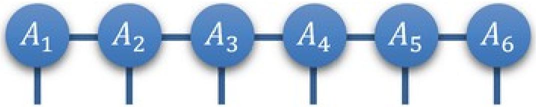
Matrix product state

The previous representation allows drastically reducing the memory required to store a state

Annals of Physics 326, 96 (2011)

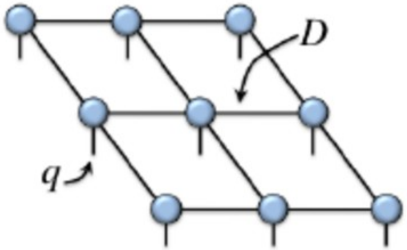
Many-body state compression

Matrix-product states



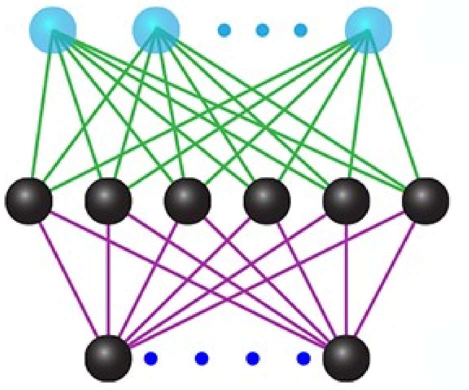
Phys. Rev. Lett. 69, 2863 (1992)

Projected entangled pair-states



Annals of Physics 326, 96 (2011)

Neural-network quantum states



Science 355.6325 (2017): 602-606.

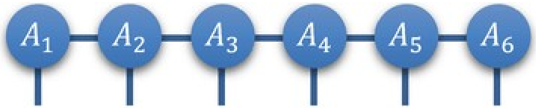
Compressed many-body states allow using kernel-polynomial methods for many-body systems

The kernel-polynomial tensor-network algorithm

Take a many-body Hamiltonian $$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$

Obtain a tensor-network representation of the many-body ground state

$$|GS\rangle =$$



And apply the kernel polynomial algorithm in the compressed representation

$$\bar{\chi}(\omega) = \langle \mathrm{GS}| S^z_N \delta(\omega - \bar{H}) S^z_N |\mathrm{GS}\rangle$$

$$\bar{\chi}(\omega) = \frac{1}{\pi\sqrt{1-\omega^2}} \left[\mu_0 + 2 \sum_{l=1}^{N_P} \mu_l T_l(\omega) \right]$$

$$\mu_l = \langle \mathrm{GS}| S^z_N T_l(\bar{H}) S^z_N |\mathrm{GS}\rangle$$

$$\mu_l = \langle \mathrm{GS}| S^z_N |w_l\rangle$$

$$|w_0\rangle = S^z_N |\mathrm{GS}\rangle,$$

$$|w_1\rangle = \bar{H}|w_0\rangle,$$

$$|w_{l+1}\rangle = 2\bar{H}|w_l\rangle - |w_{l-1}\rangle,$$

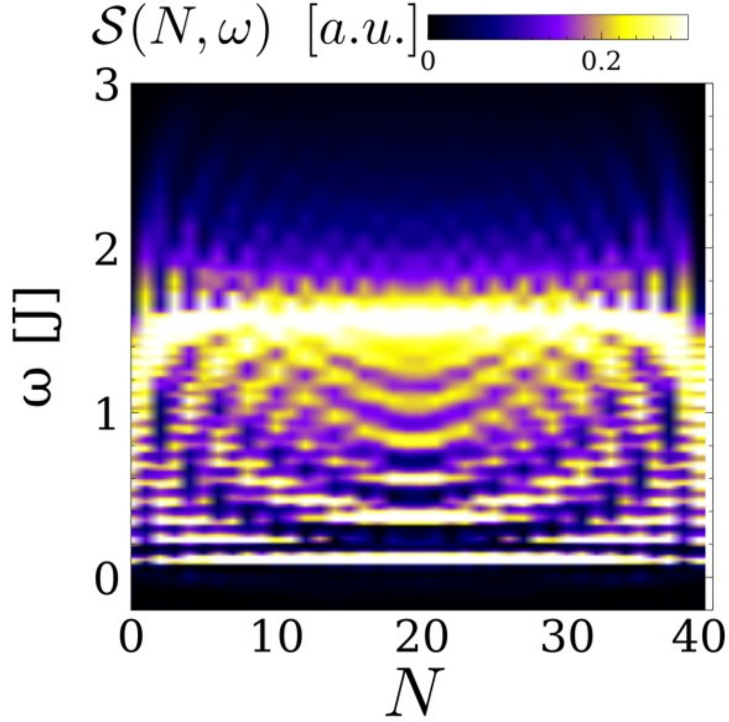
Rev. Mod. Phys. 78, 275 (2006)

Phys. Rev. Research 1, 033009 (2019)

Open-source implementation in *https://github.com/joselado/dmrgpy*

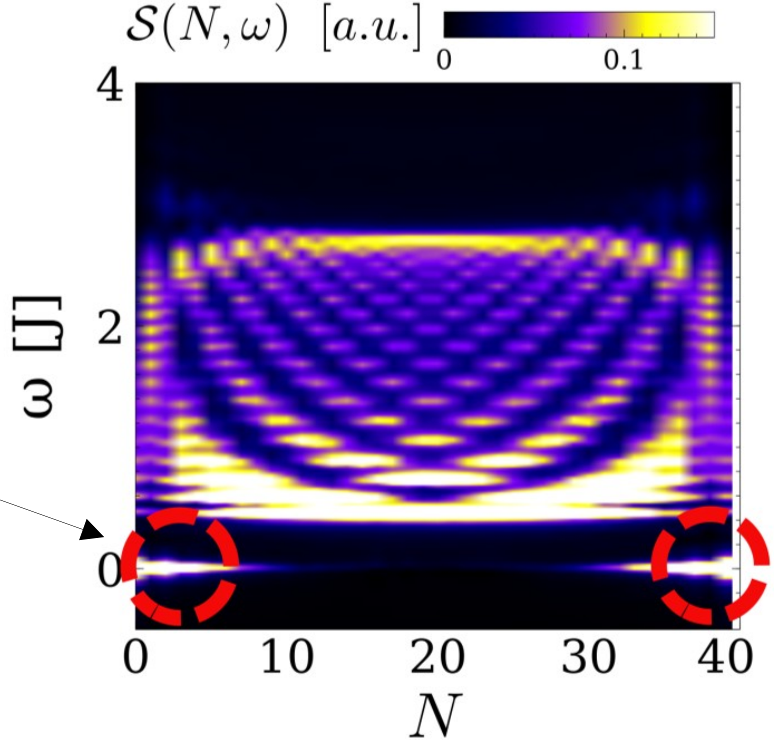
Dynamical structure factor of a Heisenberg model

S=1/2 chain



Edge modes

S=1 chain



Behind the scenes

Bound states at spin-liquid/superconductor interfaces	***Quasiperiodic many-body topology***	***Non-Hermitian many-body topology***

Bound states at spin-liquid/superconductor interfaces

Manfred Sigrist

Phys. Rev. Research 2, 023347 (2020)

Driven by interface physics

Quasiperiodic many-body topology

Malte Rösner

Oded Zilberberg

Phys. Rev. Research 3, 013265 (2021)
Phys. Rev. Research 1, 033009 (2019)

Driven by quasiperiodicity

Non-Hermitian many-body topology

Timo Hyart

Fei Song

Guangze Chen

Phys. Rev. Research 4, L012006 (2022)
arXiv:2208.06425 (2022)

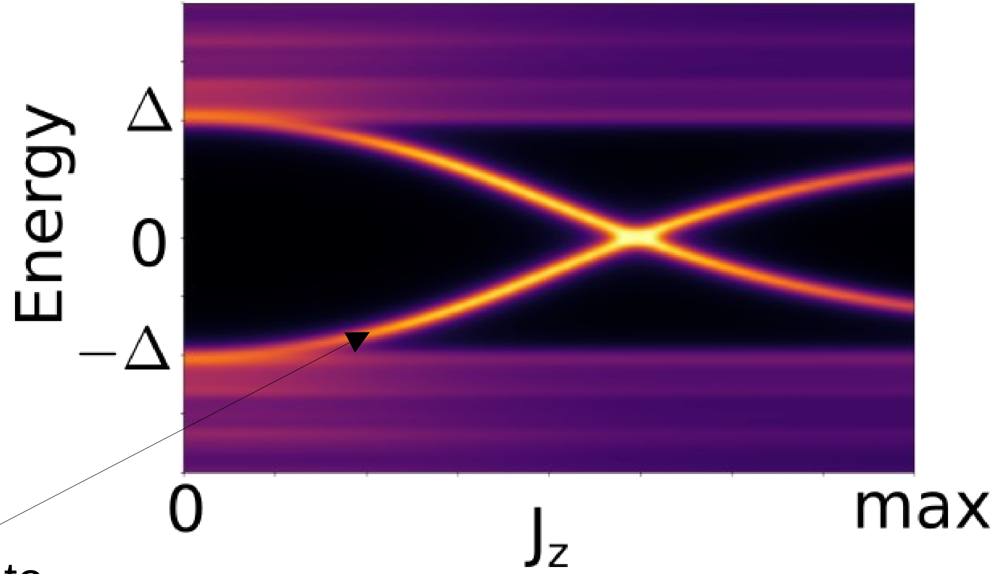
Driven by losses

Many-body excitations in spin-liquid/superconductor junctions

Magnetic impurities in superconductors

Magnetic impurity

$$H = J_z(c^\dagger_{0,\uparrow}c_{0,\uparrow} - c^\dagger_{0,\downarrow}c_{0,\downarrow})$$



Yu-Shiba-Rusinov state

An in-gap Yu-Shiba-Rusinov state appears due to local time-reversal symmetry breaking

Classical and quantum magnets

Stagger antiferromagnet (mean-field solution): breaks time-reversal

$$\mathcal{H} = \mathcal{H}_{\mathrm{kin}} + \mathcal{H}_{\mathrm{AF}} \qquad |GS\rangle = |\uparrow\downarrow\rangle$$

Quantum antiferromagnet (many-body solution): does not break time-reversal

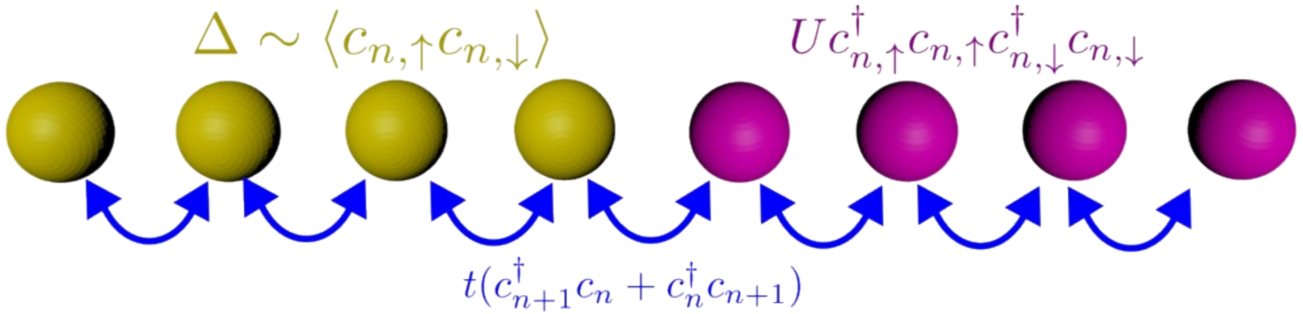
$$\mathcal{H} = \mathcal{H}_{\mathrm{kin}} + \mathcal{H}_{\mathrm{U}} \qquad |GS\rangle = |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

Hubbard interaction $\mathcal{H}_{\mathrm{U}} = \sum_n U c^\dagger_{n,\uparrow} c_{n,\uparrow} c^\dagger_{n,\downarrow} c_{n,\downarrow}$

What happens at interfaces between a quantum many-body 1D antiferromagnet and a superconductor?

Superconductor-quantum antiferromagnet junction

Superconductor **Quantum antiferromagnet**



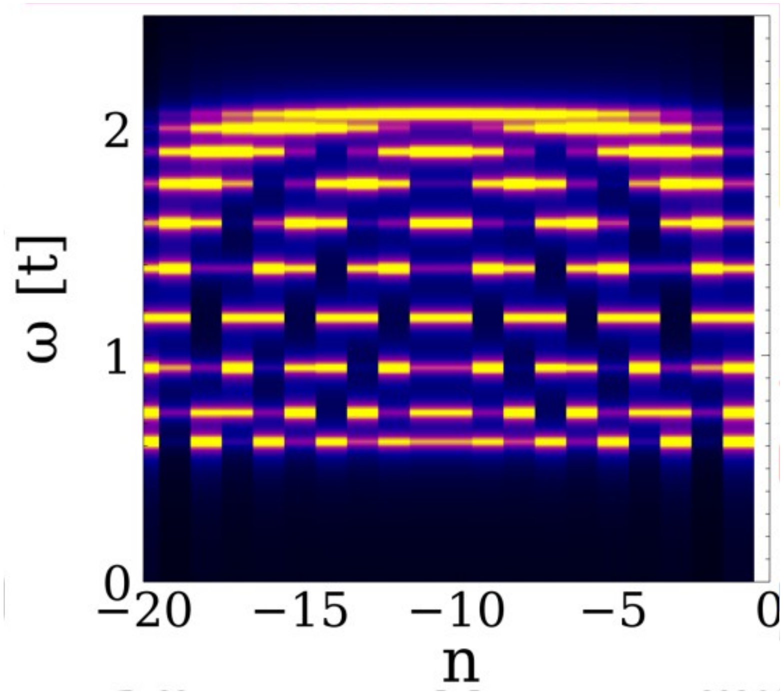
$$\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{SC}} + \mathcal{H}_{\text{int}}$$

<u>The ground state does not break time-reversal symmetry</u>

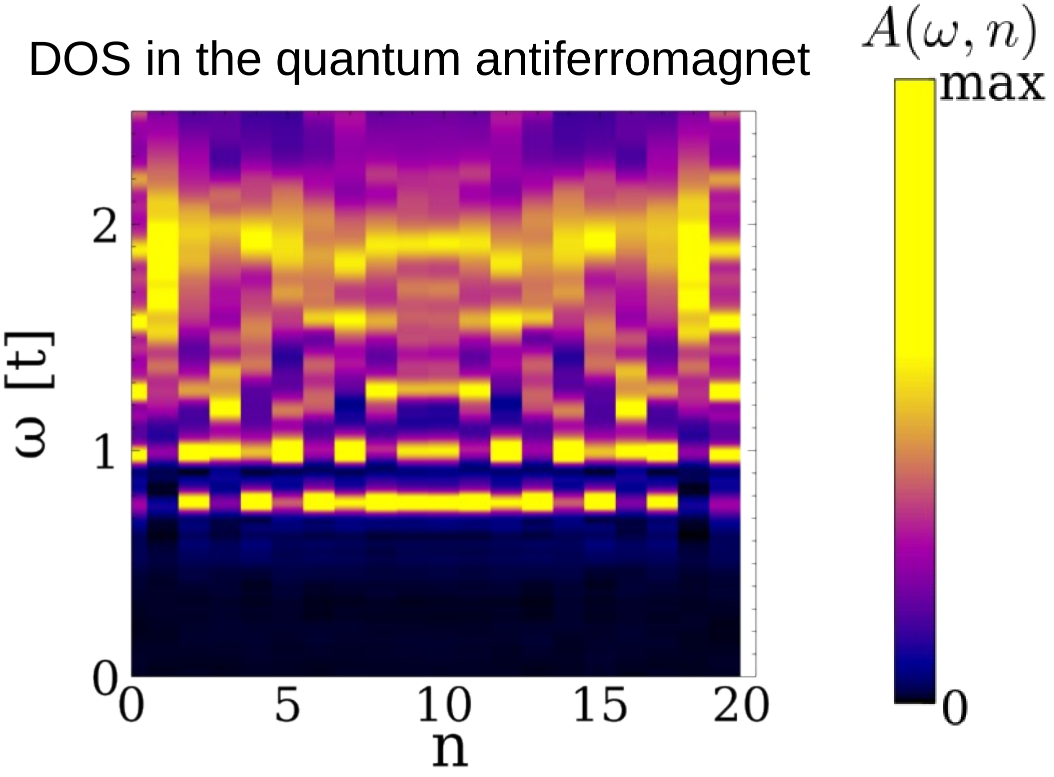
If it was a (time-reversal) symmetry broken antiferromagnet, there would be Yu-Shiba-Rusinov states

Many-body spectral function

DOS in the superconductor



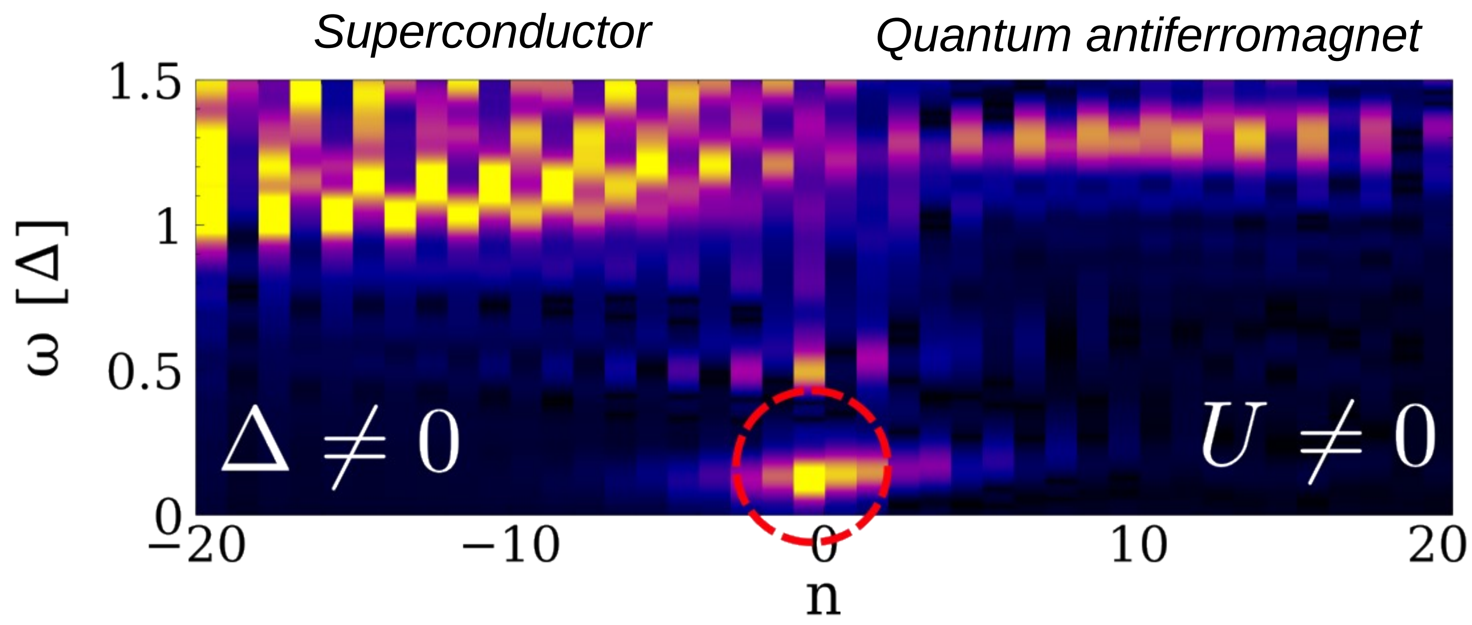
DOS in the quantum antiferromagnet



Both systems show an electronic gap when decoupled

In-gap modes at the SC-quantum AF interface

Superconductor-quantum antiferromagnet junction

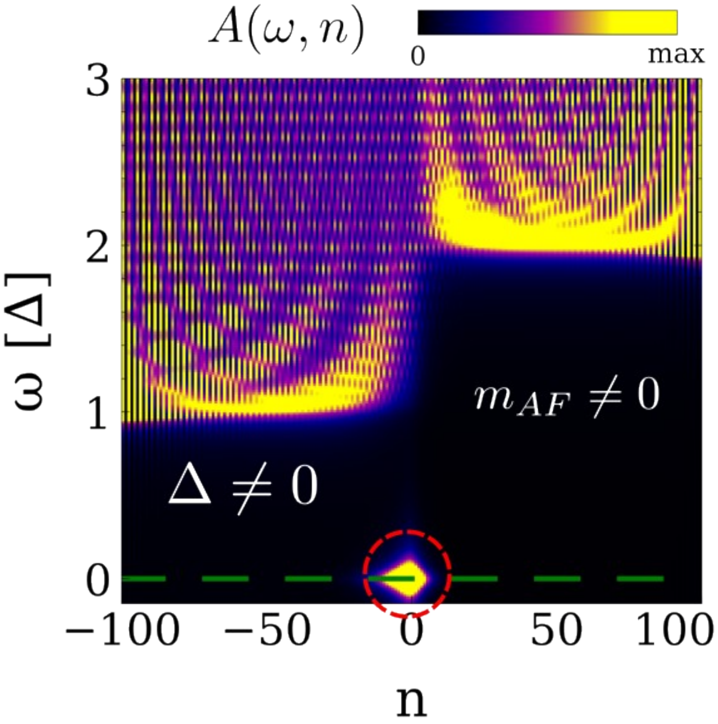


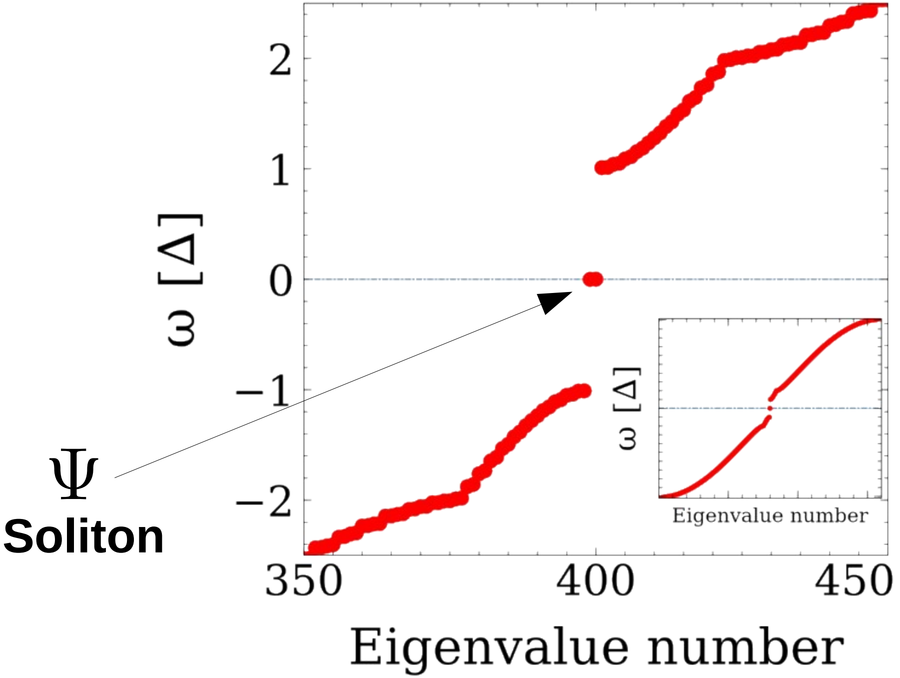
Solitonic in-gap modes appear between the superconductor and the quantum antiferromagnet

Single-particle interfacial zero modes

$$\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{SC}} + \mathcal{H}_{\text{AF}}$$

Single particle limit (stagger magnetization and no interactions)

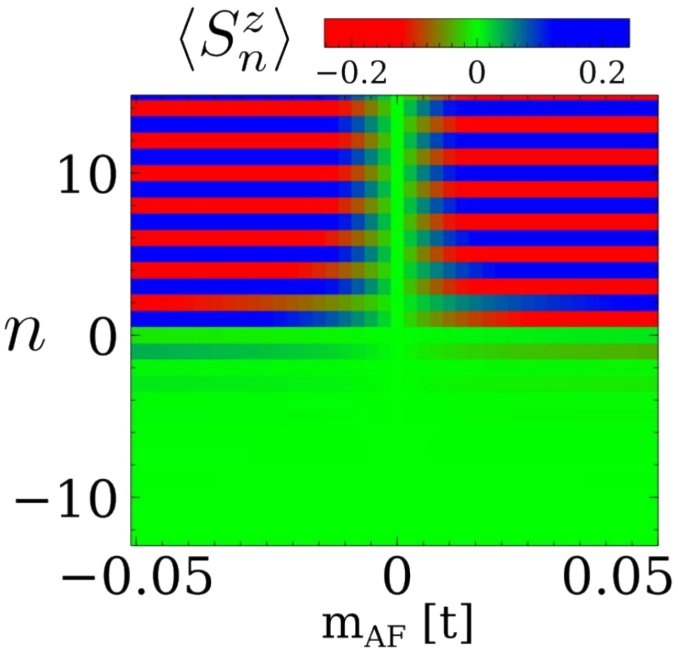




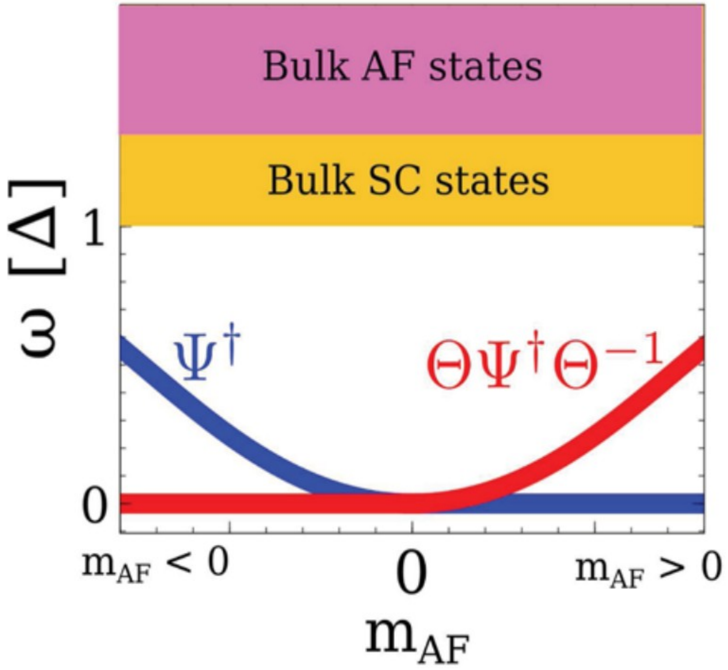
How are these modes connected to the many-body in-gap mode from before?

From many-body to the single-particle symmetry broken state

$$\mathcal{H} = \mathcal{H}_{\text{kin}} + \mathcal{H}_{\text{SC}} + \mathcal{H}_{\text{AF}} + \mathcal{H}_{\text{int}}$$

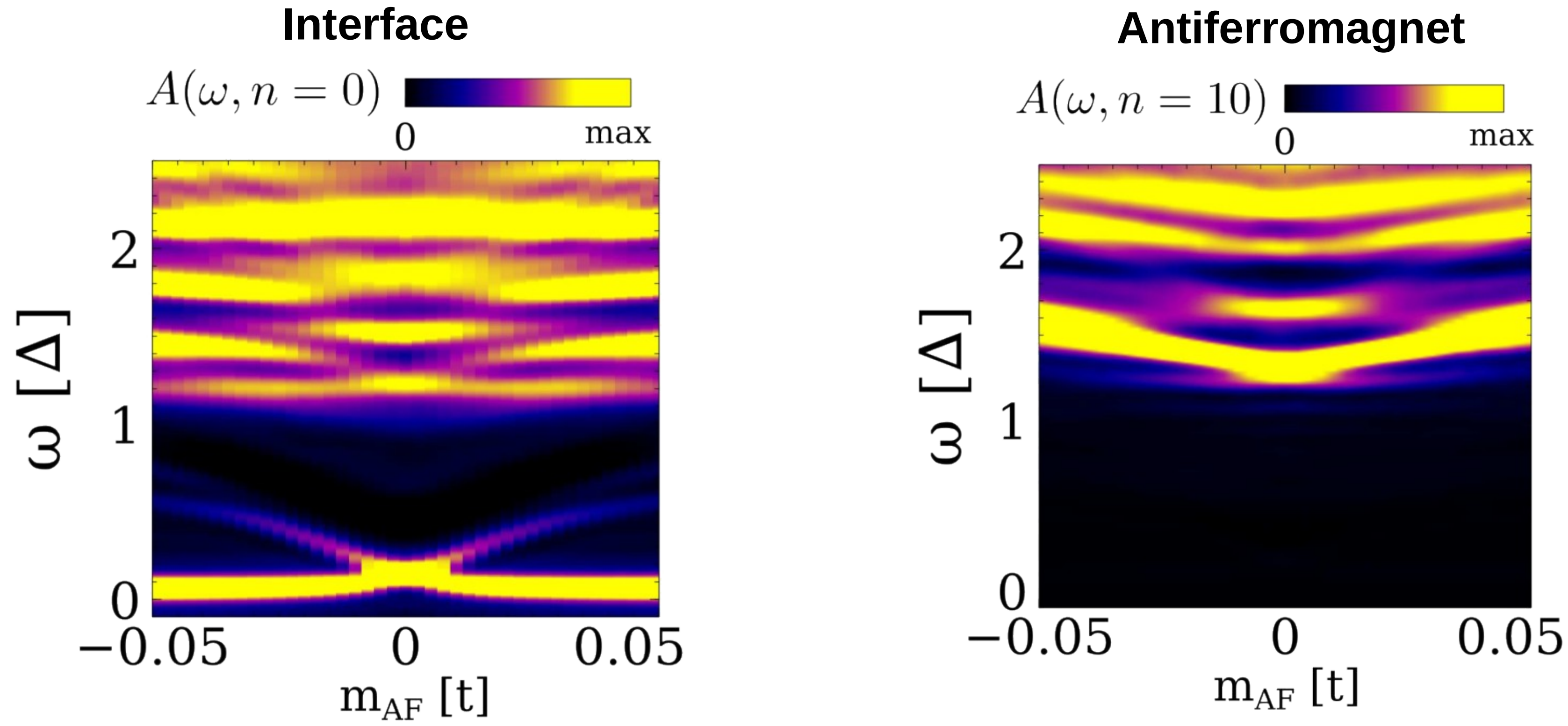


Sketch of the charge excitations



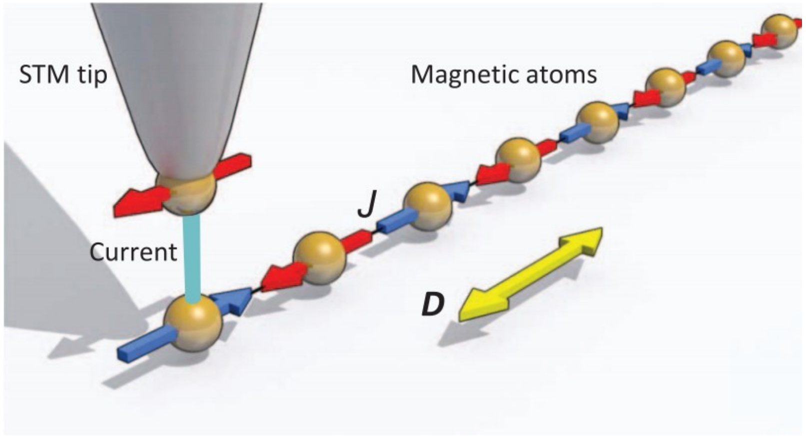
Switching on a magnetization pushes the interacting model to the symmetry broken state

From many-body to symmetry broken

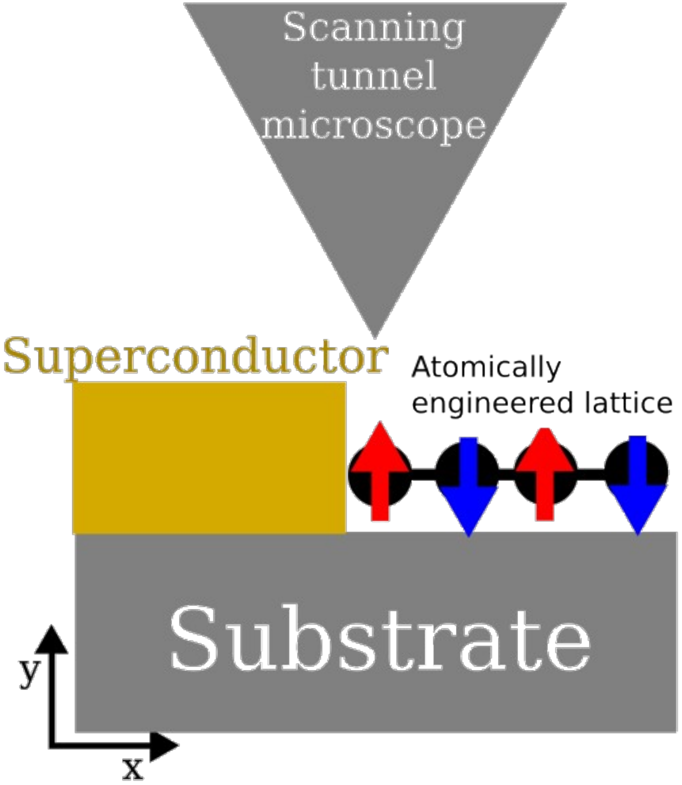


The solitonic single-particle mode transforms into the many-body in-gap mode

Experimental realization with atomically engineered lattices



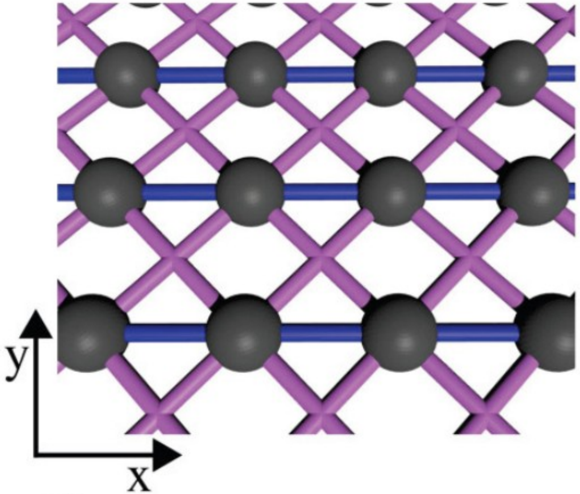
Science 335.6065 (2012): 196-199
Nature Physics 12, 656–660 (2016)
Rev. Mod. Phys. 91, 041001 (2019)



Coulomb modulated topology

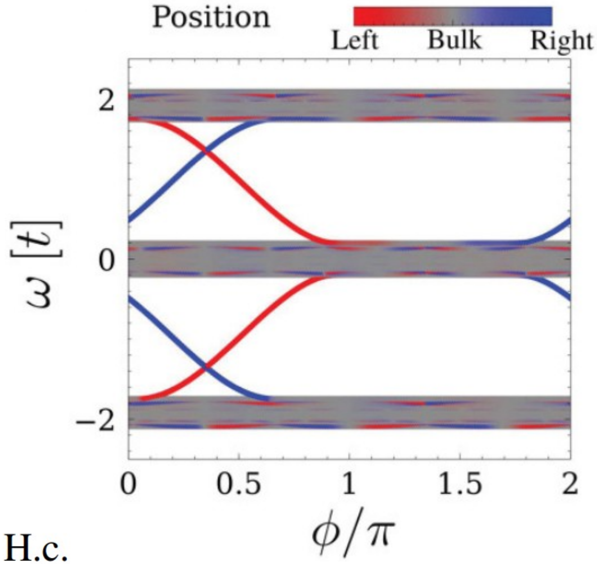
Quasiperiodic topology

Quantum Hall in a two-dimensional lattice



$\sim$

Spectrum of the associated quasiperiodic Hamiltonian



$$H = t \sum_N [1 + \lambda \cos(\alpha N + \phi)] c_N^\dagger c_{N+1} + \mathrm{H.c.}$$

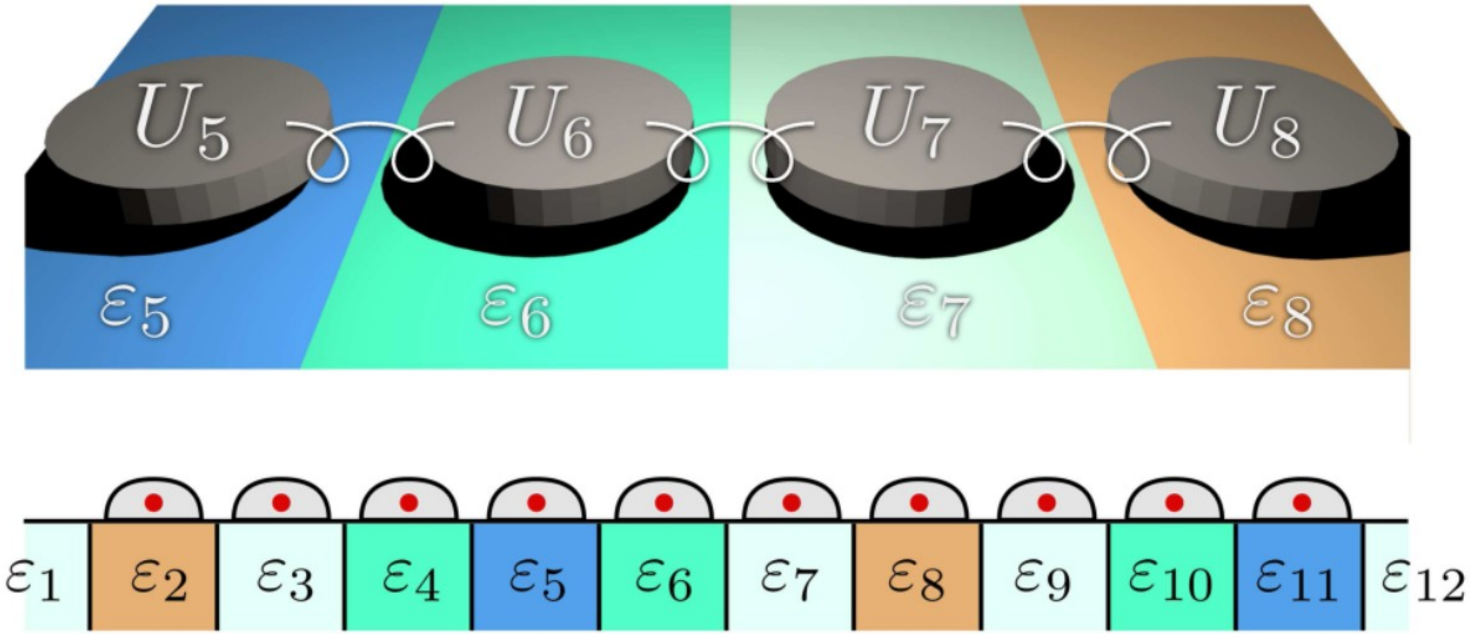
1D quasiperiodic models can be mapped to 2D quantum Hall states

This mapping is only valid in the non-interacting limit

Phys. Rev. Lett. 109, 116404 (2012)

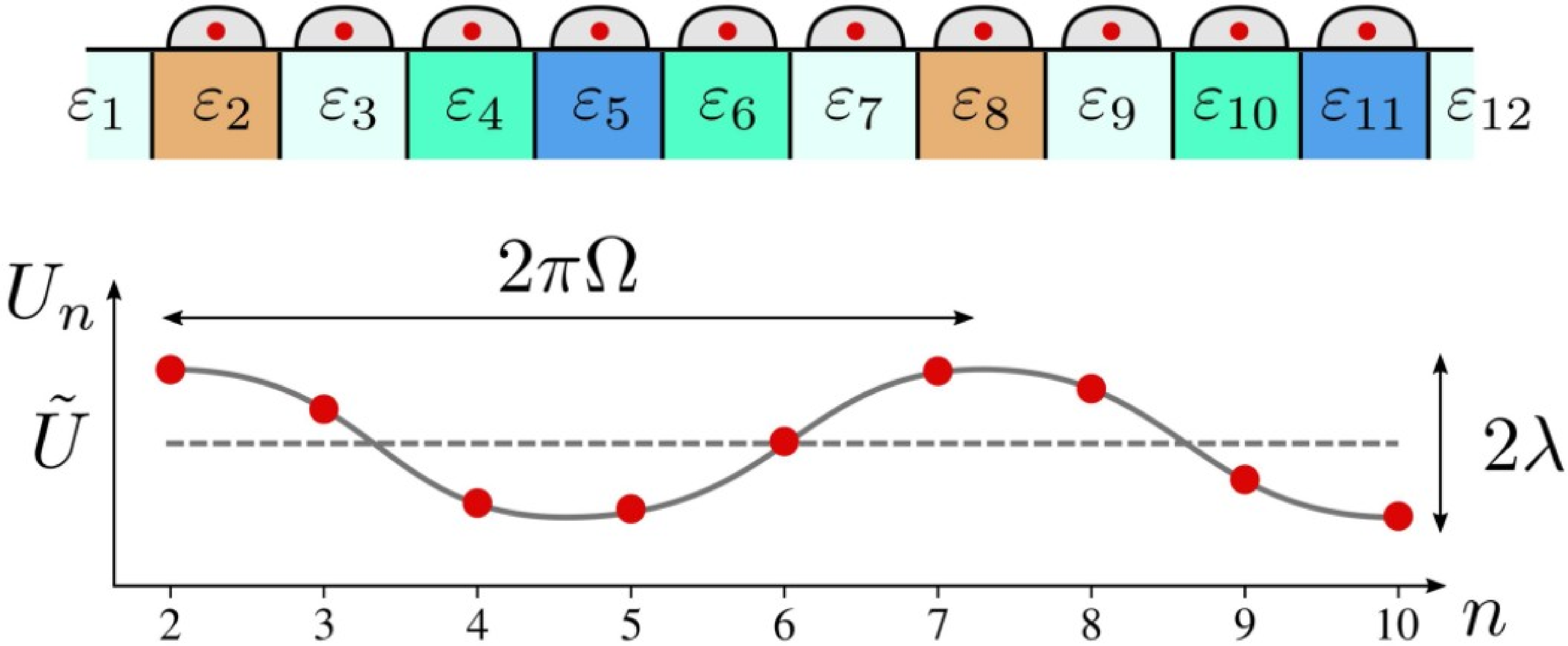
Many-body quasiperiodic interactions

Substrate engineering allow creating spatially dependent interactions



$$H = \sum_{n,s} t\, c^{\dagger}_{n,s} c_{n+1,s} + \sum_{n} U_n c^{\dagger}_{n,\uparrow} c_{n,\uparrow} c^{\dagger}_{n,\downarrow} c_{n,\downarrow} + h.c. \qquad U_n = \tilde{U}[1 + \lambda \cos(\Omega n + \phi)]$$

Many-body quasiperiodic interactions



$$H = \sum_{n,s} t\, c^\dagger_{n,s} c_{n+1,s} + \sum_n U_n c^\dagger_{n,\uparrow} c_{n,\uparrow} c^\dagger_{n,\downarrow} c_{n,\downarrow} + h.c. \qquad U_n = \tilde{U}[1 + \lambda\cos(\Omega n + \phi)]$$

Single particle Coulomb-engineered topology

Engineered interacting model

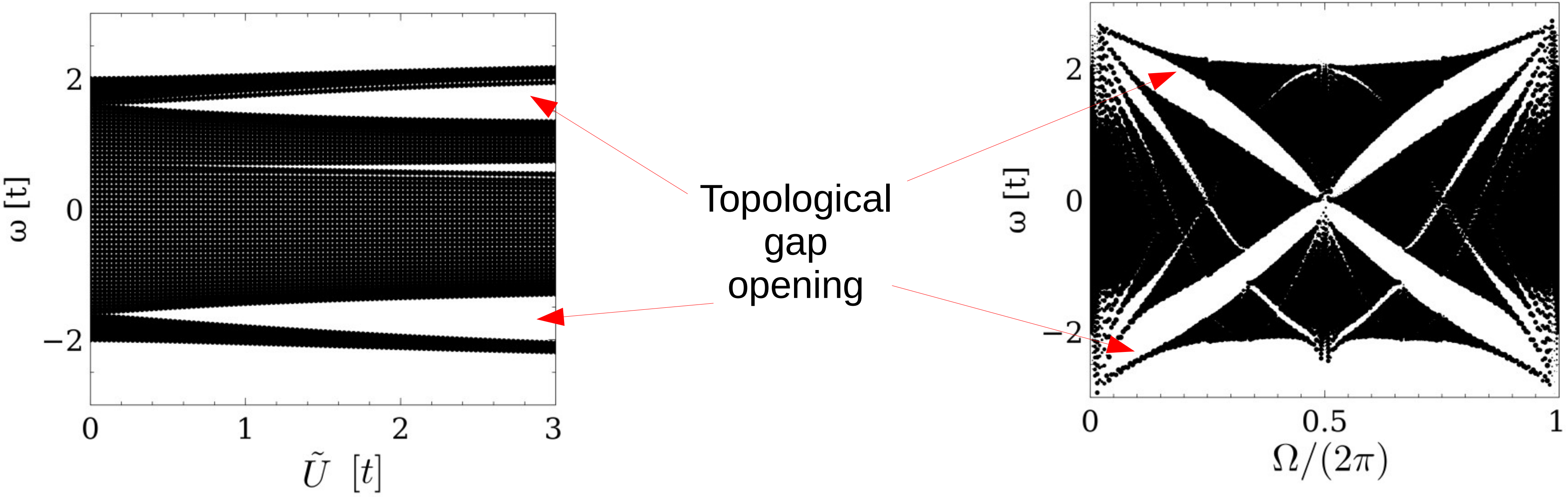
$$H = \sum_{n,s} t\, c^\dagger_{n,s} c_{n+1,s} + \sum_n U_n c^\dagger_{n,\uparrow} c_{n,\uparrow} c^\dagger_{n,\downarrow} c_{n,\downarrow} + h.c.$$

Modulated interactions

$$U_n = \tilde{U}[1 + \lambda \cos(\Omega n + \phi)]$$

Solved at the Hartree-Fock level

$$\sum_{s,s'} \langle \vec{\sigma}_{s,s'} c^\dagger_{n,s} c_{n,s'} \rangle = 0$$

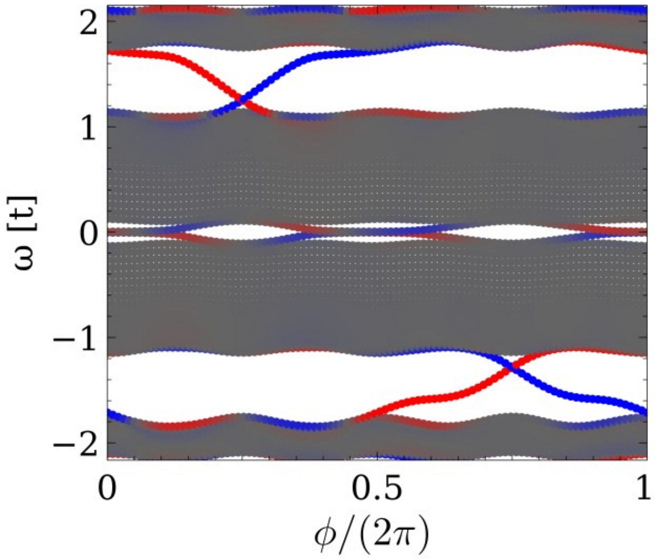


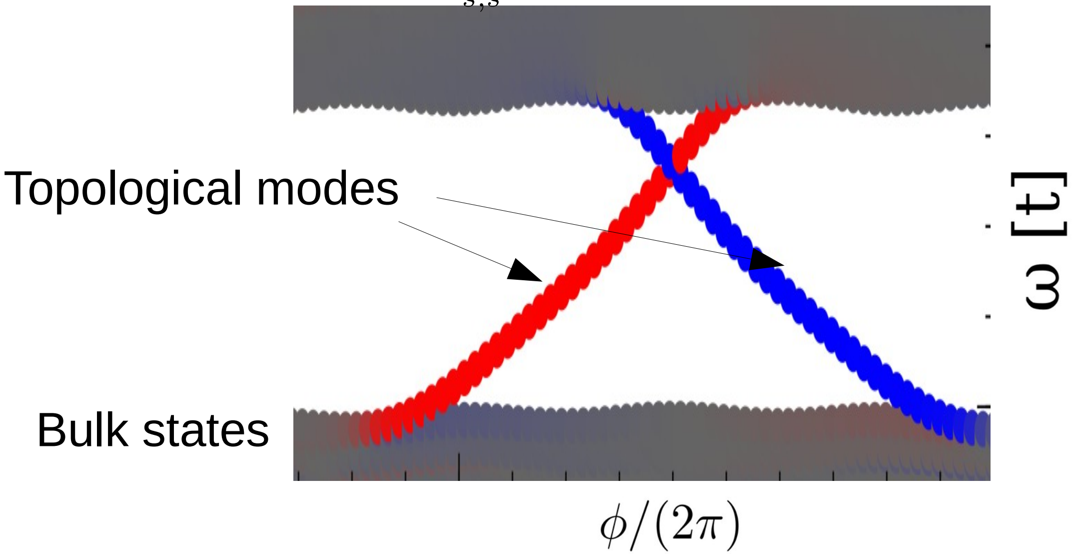
Single particle Coulomb-engineered topology

Engineered interacting model $$H = \sum_{n,s} t\, c^\dagger_{n,s} c_{n+1,s} + \sum_n U_n c^\dagger_{n,\uparrow} c_{n,\uparrow} c^\dagger_{n,\downarrow} c_{n,\downarrow} + h.c.$$

Modulated interactions $$U_n = \tilde{U}[1 + \lambda \cos(\Omega n + \phi)]$$

Solved at the Hartree-Fock level $$\sum_{s,s'} \langle \vec{\sigma}_{s,s'} c^\dagger_{n,s} c_{n,s'} \rangle = 0$$





Modulated local interactions create topological modes

Many-body topology

Let us focus in an interacting Hamiltonian with a trivial mean-field Hamiltonian

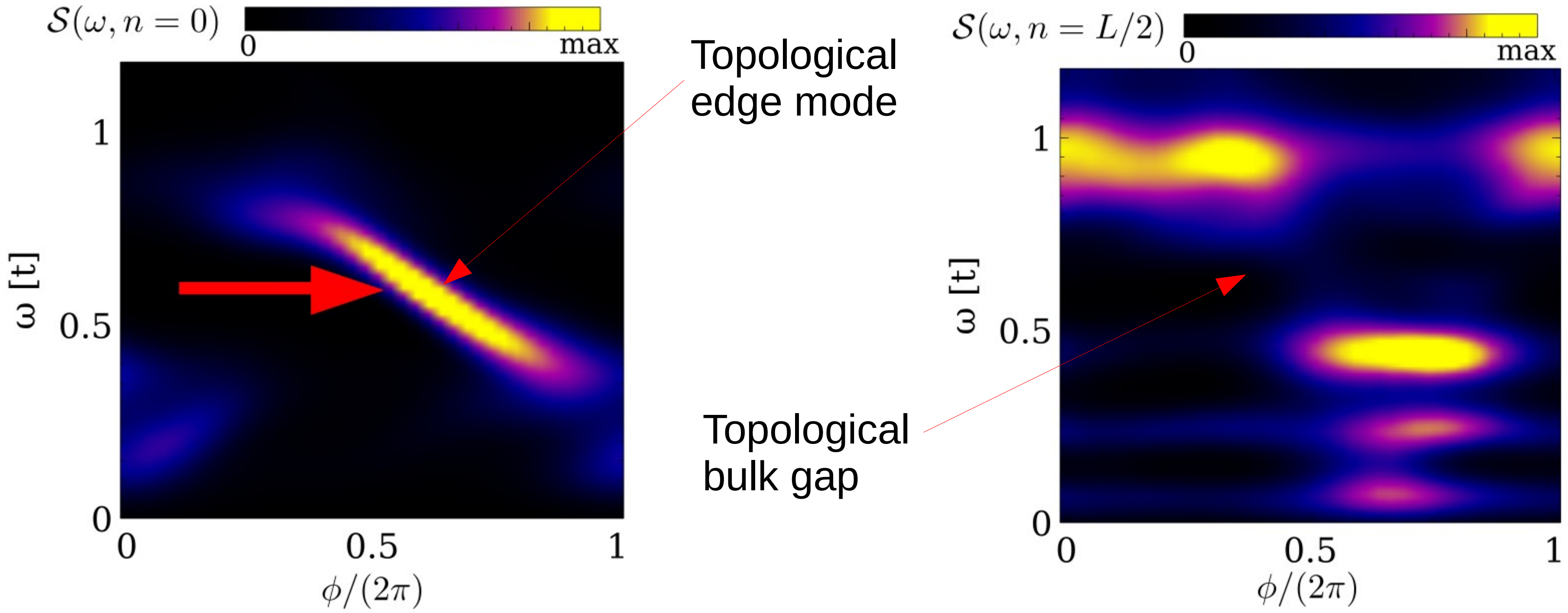
$$H = \sum_{n,s} t\, c^\dagger_{n,s} c_{n+1,s} + \sum_n U_n \left(c^\dagger_{n,\uparrow} c_{n,\uparrow} - \frac{1}{2} \right) \left(c^\dagger_{n,\downarrow} c_{n,\downarrow} - \frac{1}{2} \right) + h.c.$$

In the many-body setting, we will look at the dynamical spin excitation spectra

$$\mathcal{S}(\omega, n) = \langle GS | S^z_n \delta(\omega - H + E_{GS}) S^z_n | GS \rangle$$

Many-body topological modes

Edge spectral function **Bulk spectral function**

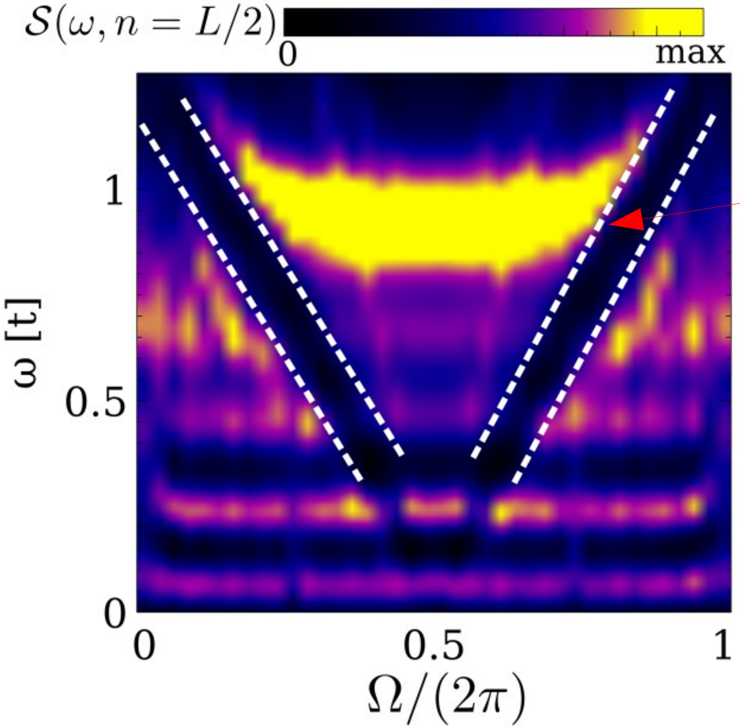


Topological modes protected by a many-body quasiperiodic Chern invariant

Phys. Rev. Research 1, 033009 (2020) *arXiv:2009.03752* *arXiv:2010.06171*

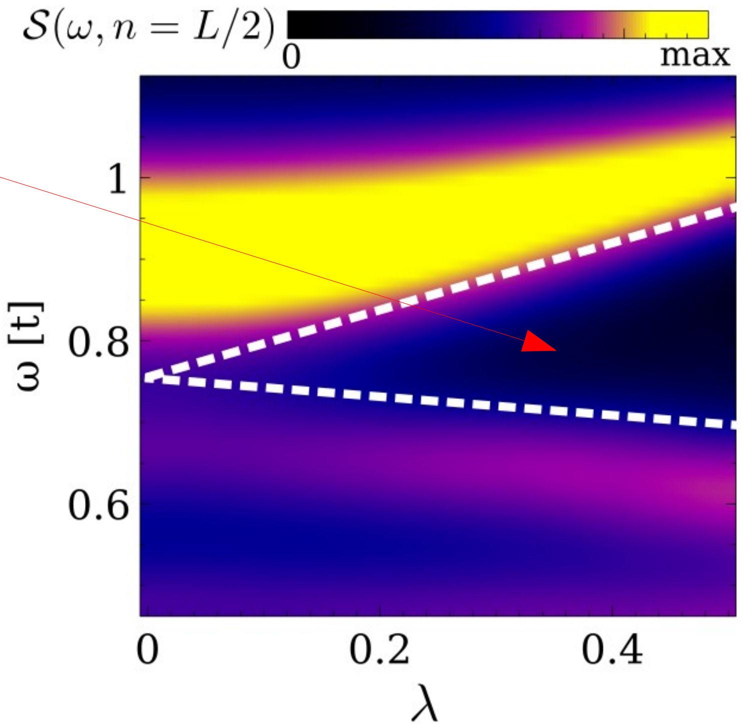
Many-body topological gaps

Bulk spectral function



Topological
bulk gap

Bulk spectral function



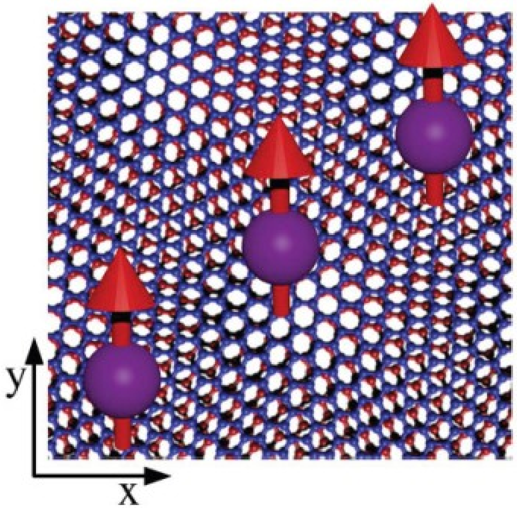
Topological gaps appear for generic modulation frequencies and strengths

Quasiperiodic spinon topology

Quasiperiodic spinon topology

Many-body spin Hamiltonians can be intrinsically quasiperiodic

For example, an atomic chain on top of a moire system

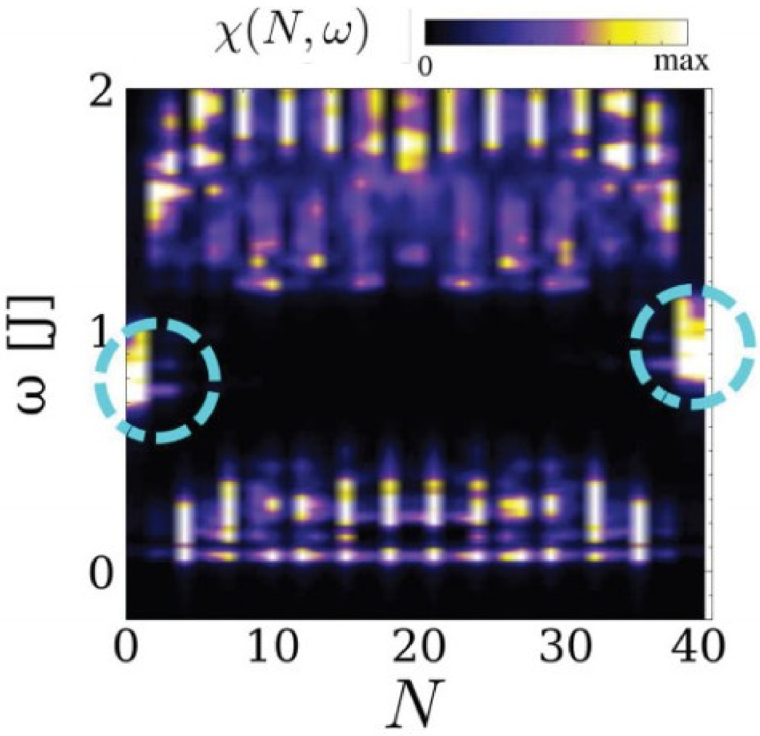


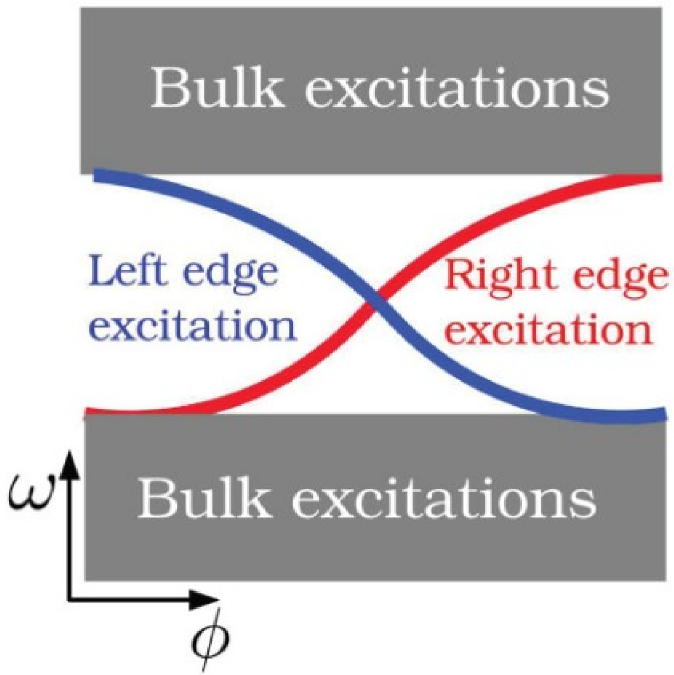
We will consider the generalized quasiperiodic Heisenberg model

$$H = J \sum_N [1 + \lambda \cos(\alpha N + \phi)] \vec{S}_N \cdot \vec{S}_{N+1}$$

with $S = 1/2, 1, 3/2, 2$

Topological quasiperiodic spinon for S=1/2

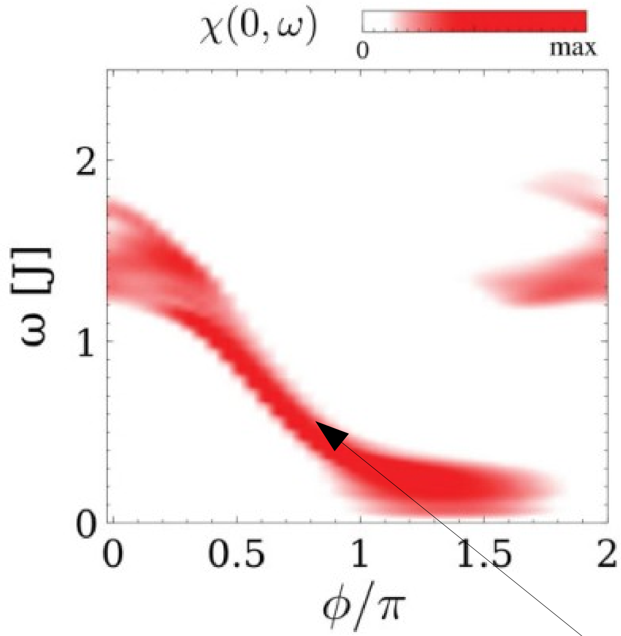




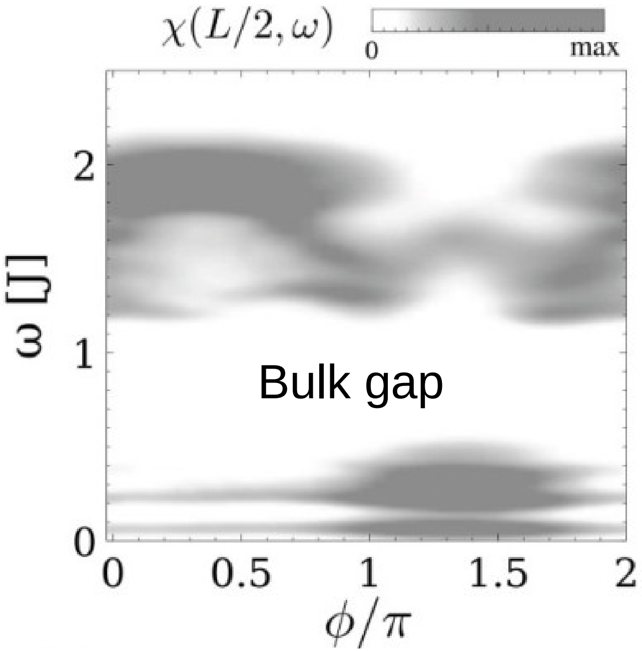
Quasiperiodic many-body interactions create topological spinon modes

Topological quasiperiodic spinons for S=1/2

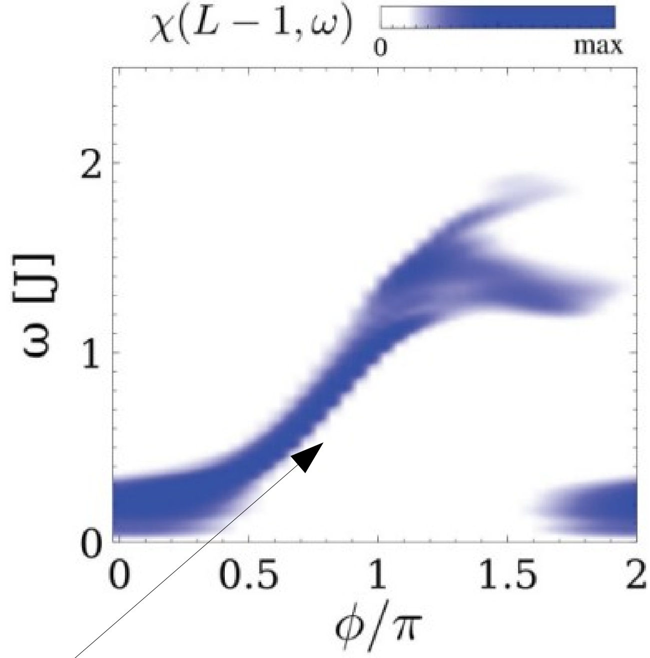
Left edge



Bulk



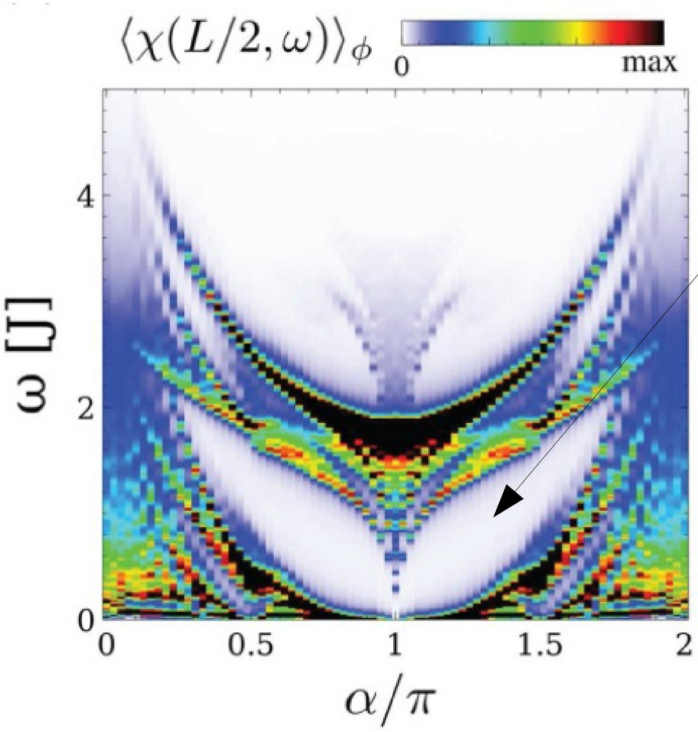
Right edge



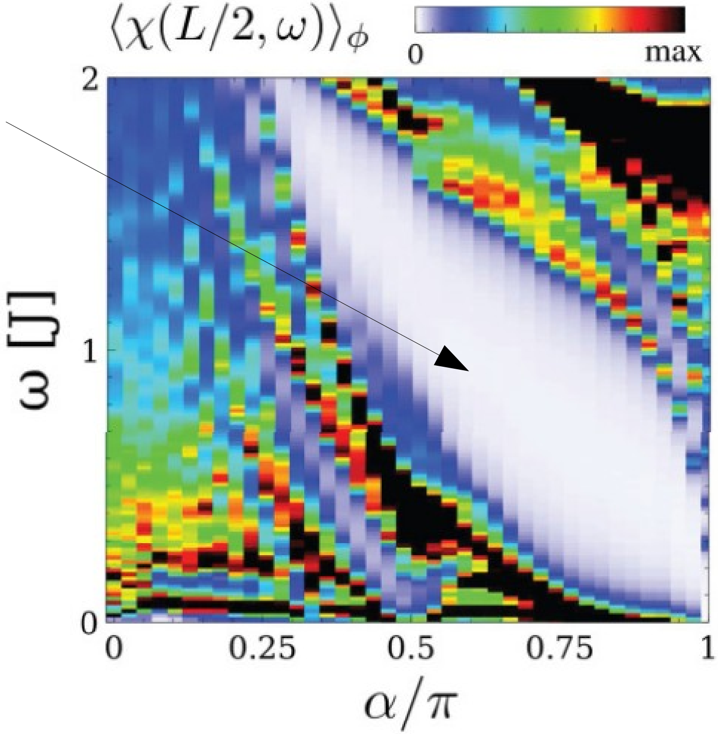
Topological spinon pumping modes

$$H = J\sum_N [1+\lambda\cos(\alpha N+\phi)]\vec{S}_N\cdot\vec{S}_{N+1}$$

Spinon Hofstadter spectra S=1/2



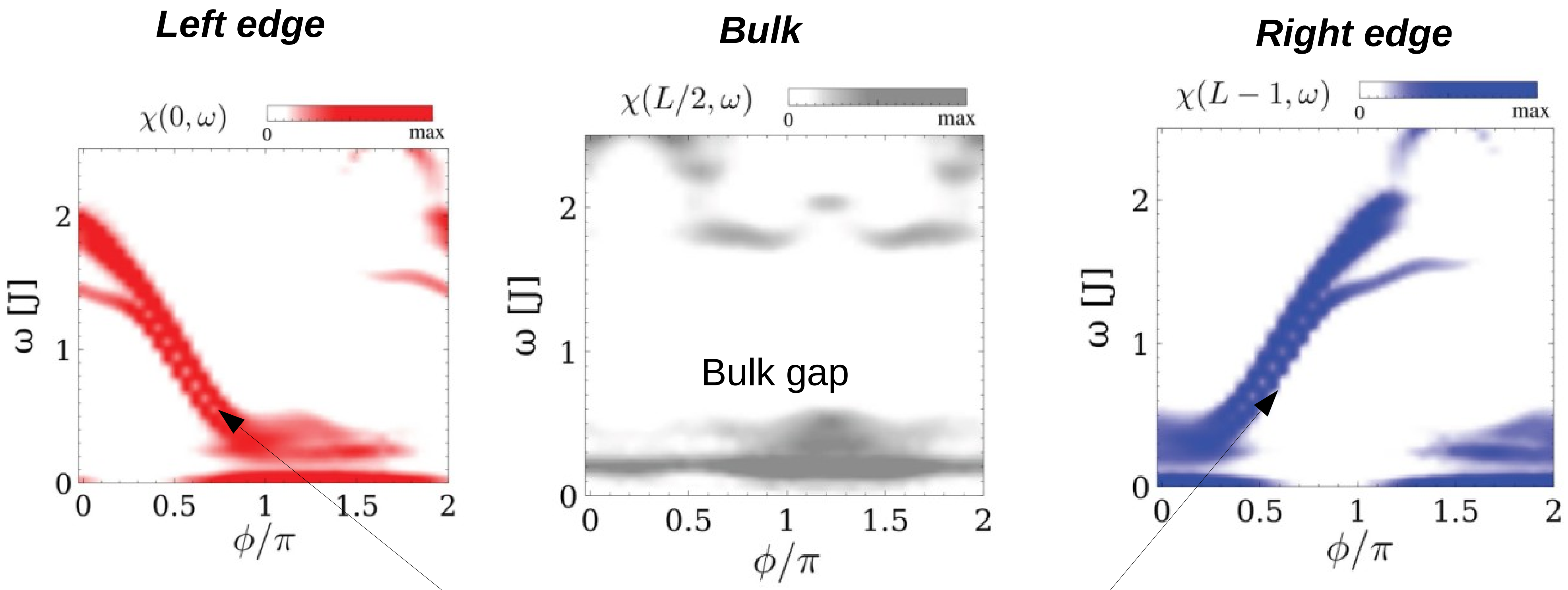
Topological gap



$$H = J\sum_N [1+\lambda\cos(\alpha N+\phi)]\vec{S}_N\cdot\vec{S}_{N+1}$$

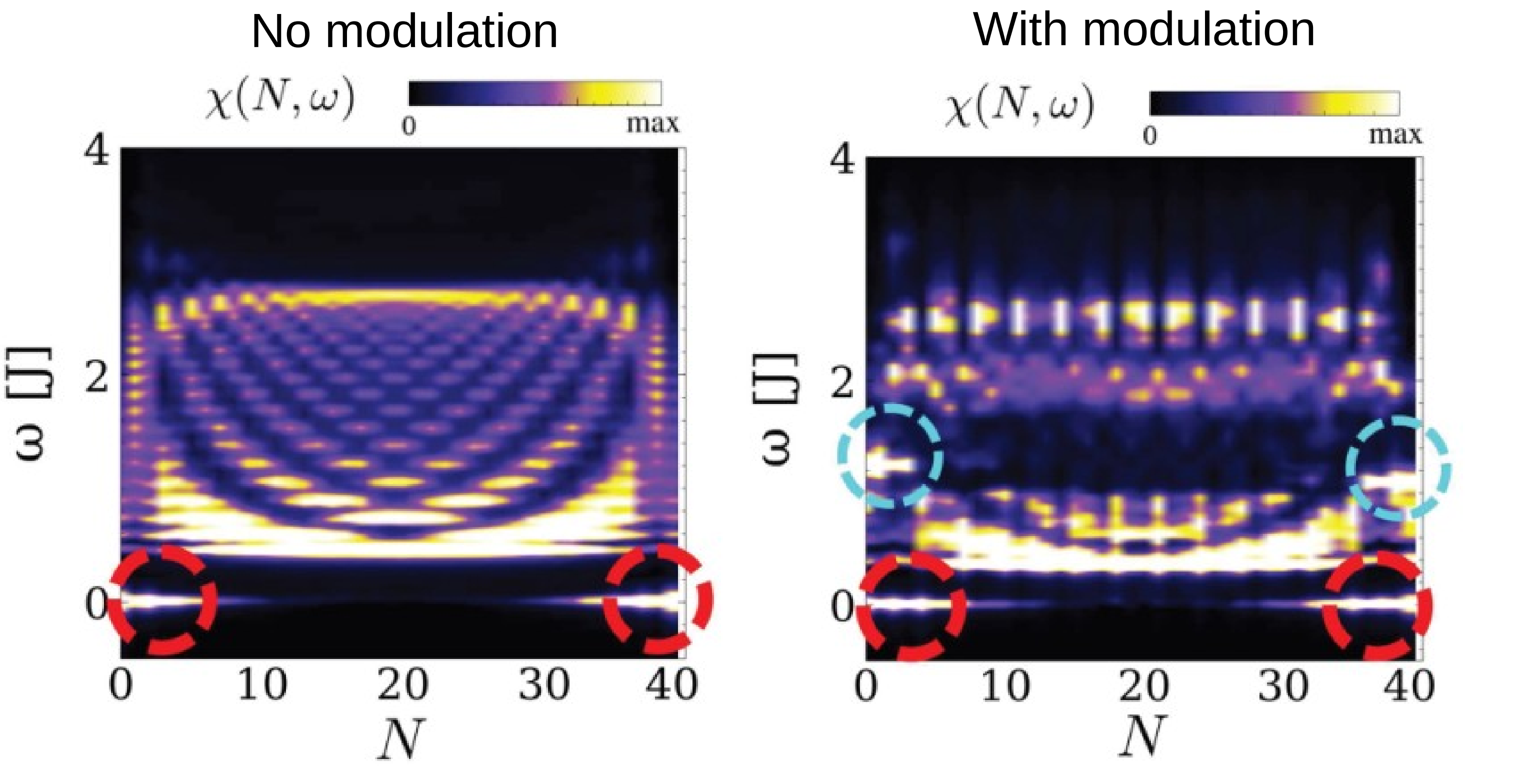
Topological gaps appear for arbitrary quasiperiodicity

Topological quasiperiodic spinons for S=1



$$H = J\sum_{N}[1+\lambda\cos(\alpha N+\phi)]\vec{S}_N\cdot\vec{S}_{N+1}$$

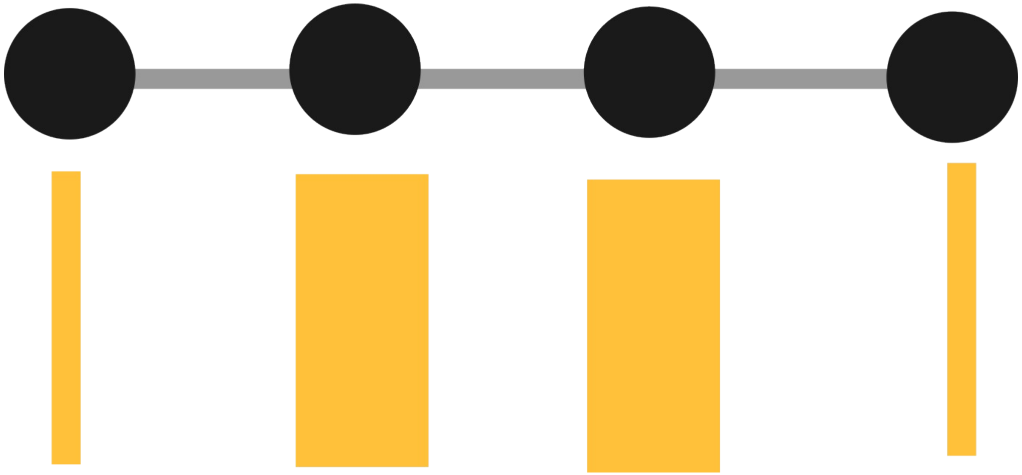
Topological quasiperiodic spinons for S=1



Quasiperiodicity creates additional topological modes away from E=0

Non-Hermitian topological many-body excitations

Non-Hermiticity in quantum dots



Quantum dots

Metallic leads

$$g_n = \mathrm{Im}(\Sigma_n)$$

Lead selfenergy

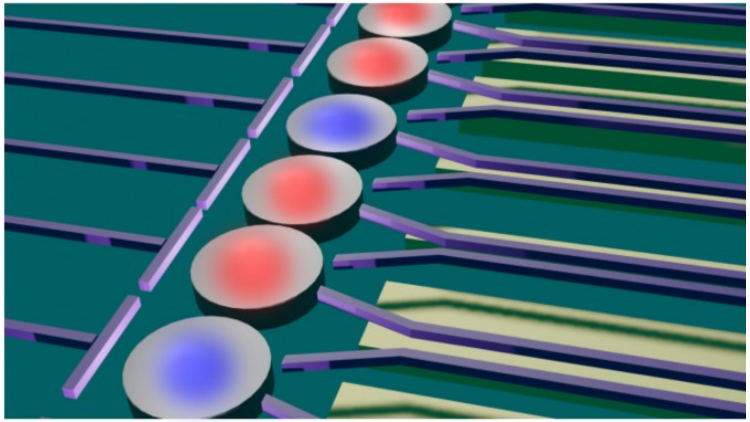
Hopping

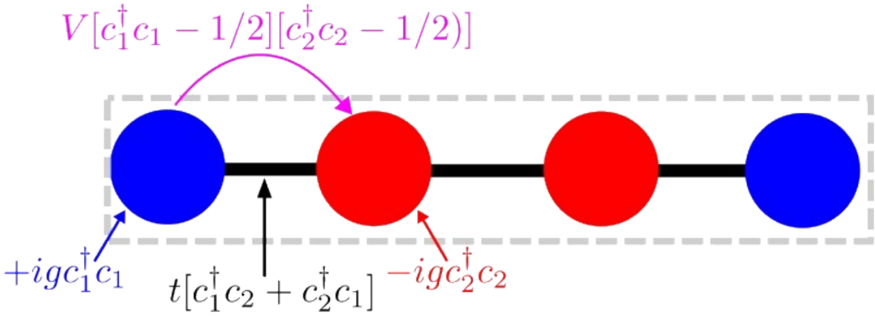
Imaginary onsite energy

$$H = c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n + i g_n c_n^\dagger c_n$$

<u>Coupling to metallic leads generates non-Hermitian terms in an effective Hamiltonian</u>

Topological excitations driven by modulated losses





Bloch Hamiltonian

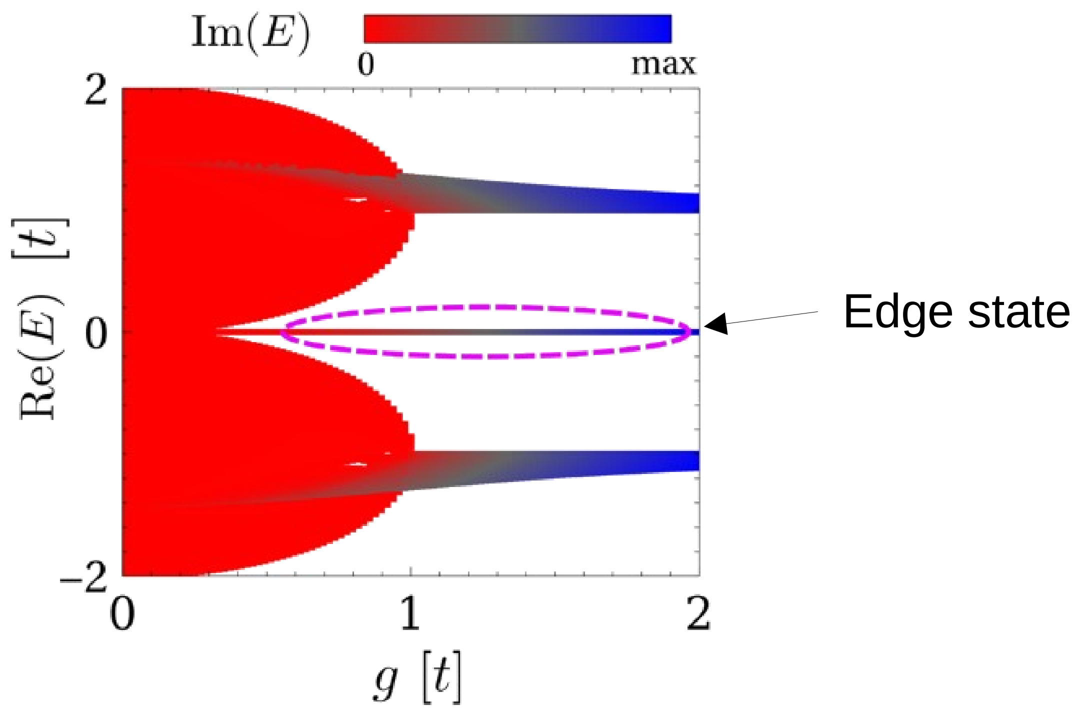
$$H_k = \begin{pmatrix} ig_1 & t & 0 & te^{-ik} \\ t & ig_2 & t & 0 \\ 0 & t & ig_3 & t \\ te^{ik} & 0 & t & ig_4 \end{pmatrix}$$

Losses coming from a self-energy

$$g_n = \mathrm{Im}(\Sigma_n)$$

Edge mode driven by losses

Non-interacting spectra



Does this topological mode survive the presence of many-body interactions?

Looking for a non-Hermitian edge mode

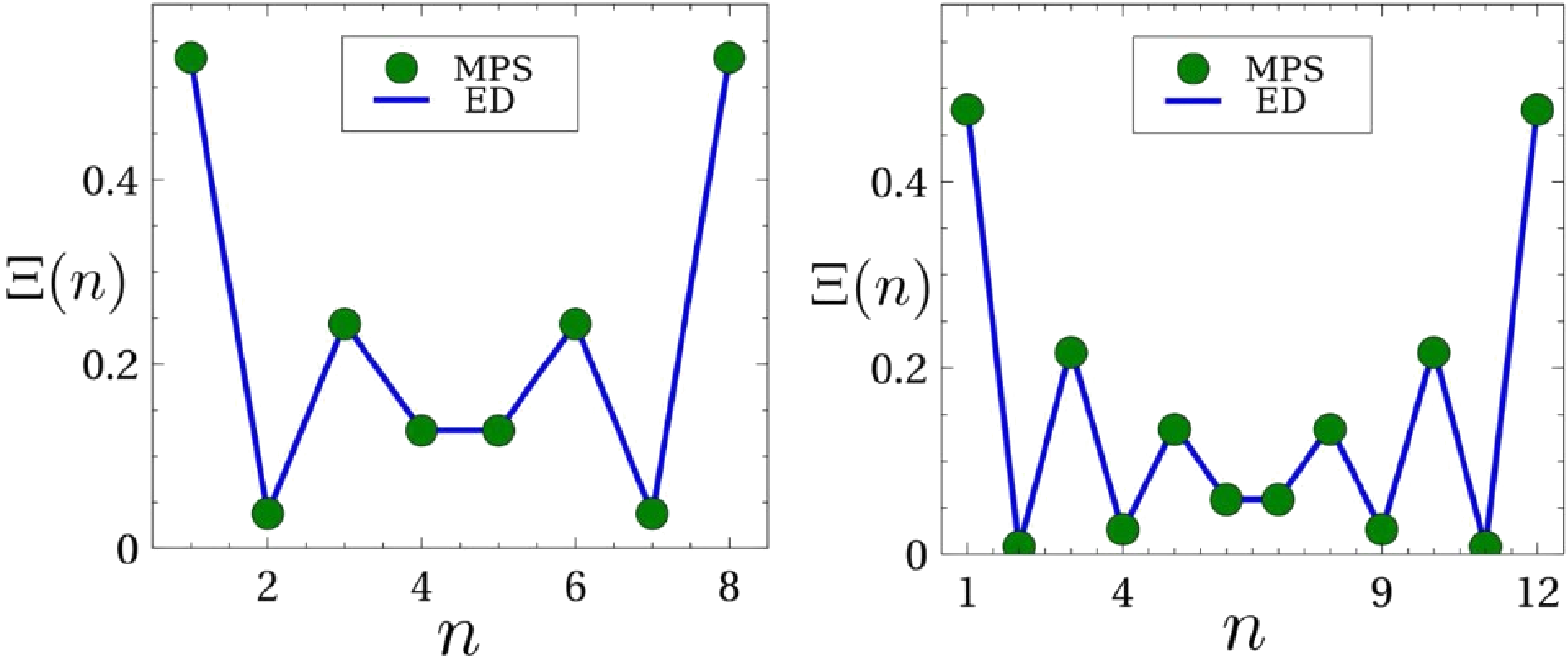
- As the Hamiltonian is non-Hermitian, DMRG cannot be used
- The ground state and excited is obtained using a non-Hermitian Krylov-Arnoldi iterative solver with matrix-product states
- The edge modes are imaged by the correlator

$$\Xi(n) = \sum_i |\langle \Psi_i | c_n | \Omega \rangle|^2.$$

Degenerate energy manifold

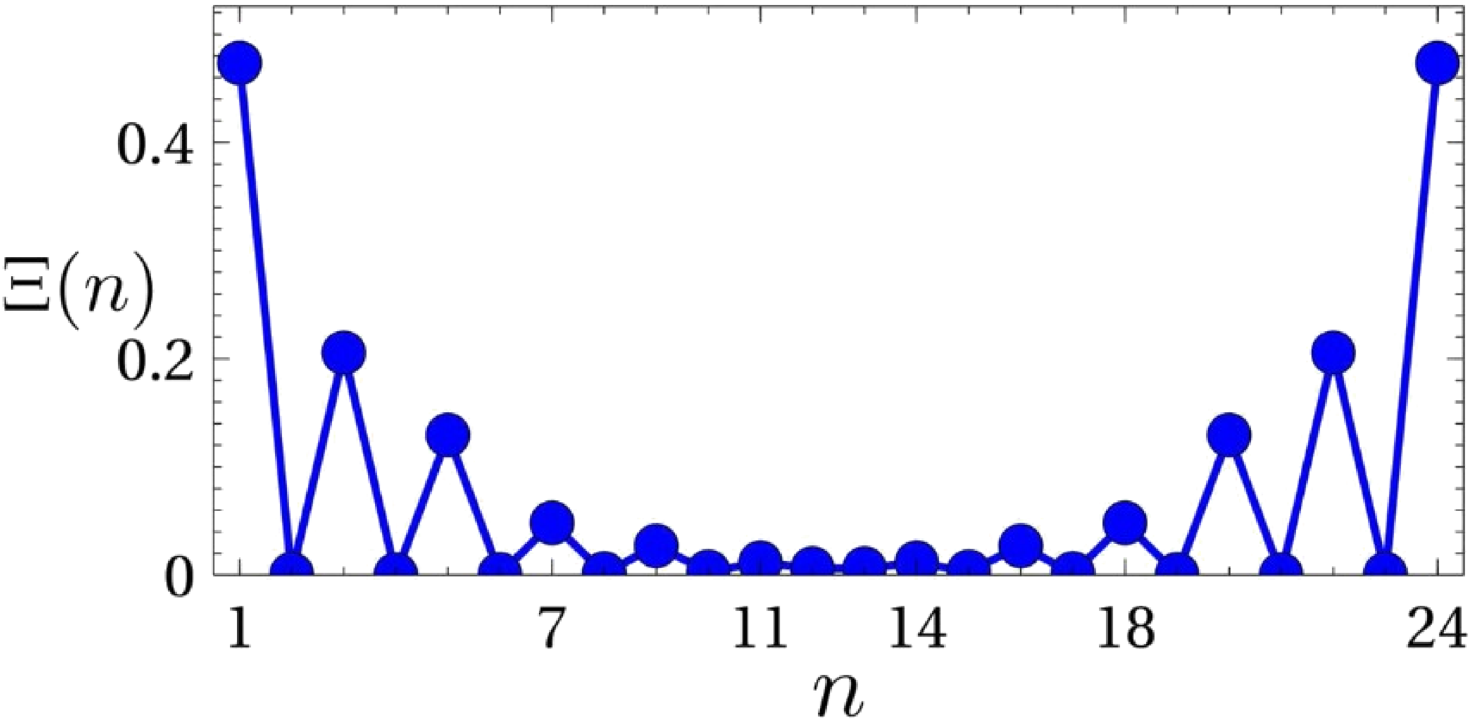
Ground state

Non-Hermitian edge modes



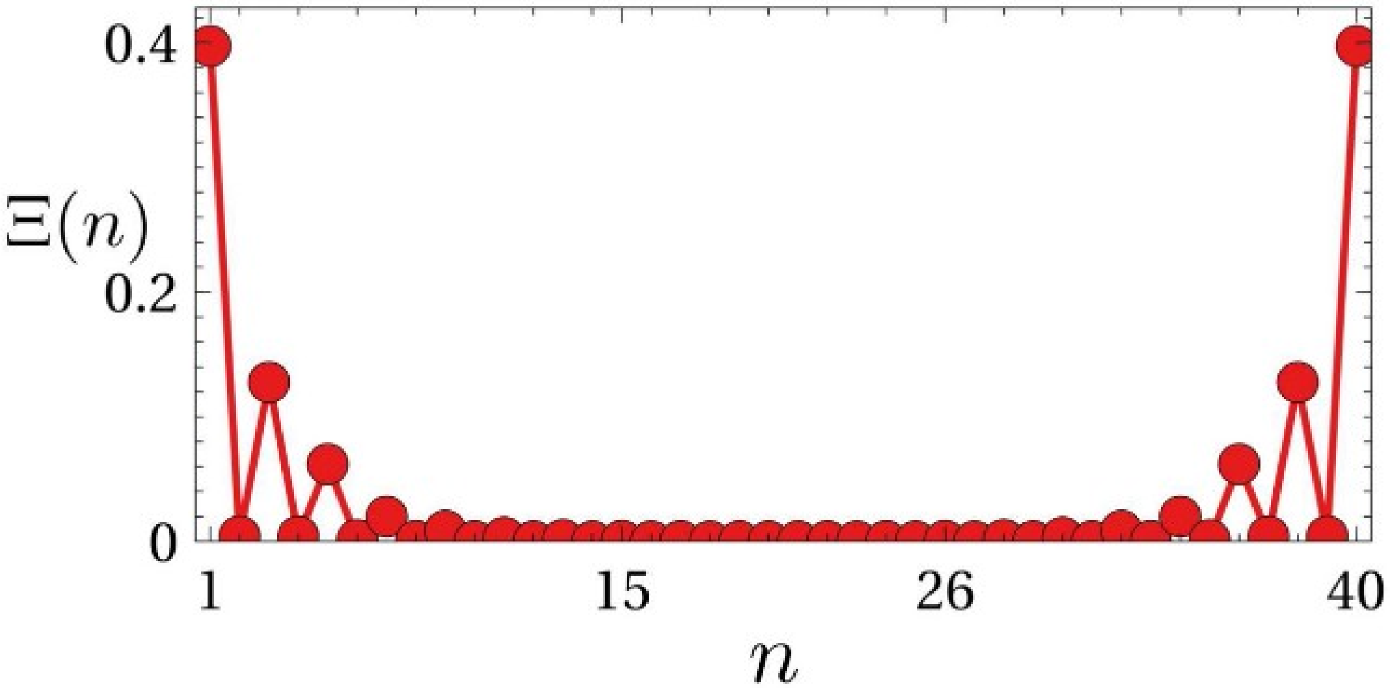
In short chains two overlapping edge modes appear in the correlator

Non-Hermitian edge modes



In long chains solved with MPS, the non-Hermitian mode becomes well localized at the edge

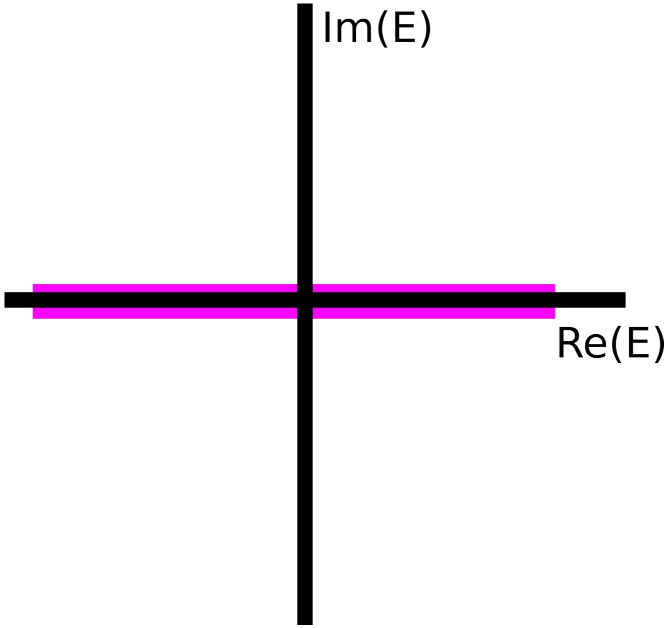
Non-Hermitian edge modes



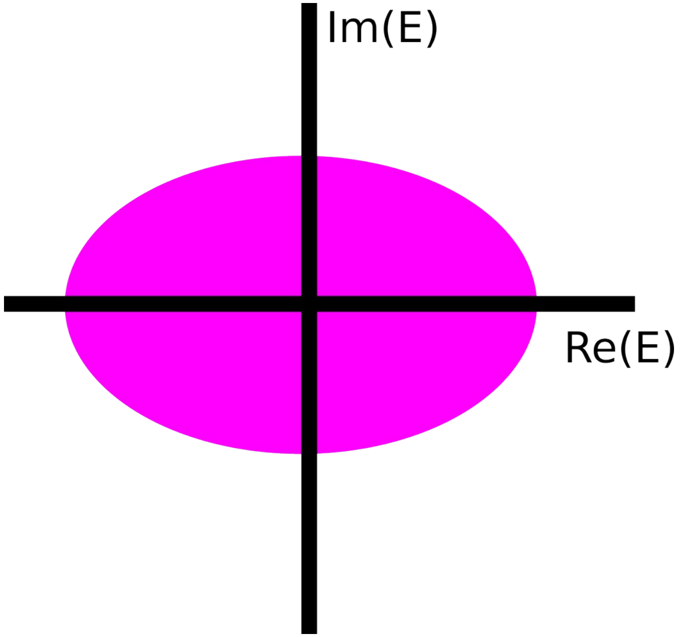
In long chains solved with MPS, the non-Hermitian mode becomes well localized at the edge

Dynamical correlators in non-Hermitian models

In the Hermitian case



In the non-Hermitian case



In the non-Hermitian case, dynamical correlators are defined in the complex energy plane

Dynamical correlator of a many-body non-Hermitian model

In a non-Hermitian model, the dynamical correlator cannot be compute with a usual Kernel polynomial expansion

$$\mathrm{S}(\omega, l) = \left\langle \mathrm{GS_L} \left| c_l \delta^2 \left(\omega + E_{\mathrm{GS}} - H\right) c_l^\dagger \right| \mathrm{GS_R} \right\rangle + \left\langle \mathrm{GS_L} \left| c_l^\dagger \delta^2 \left(\omega + E_{\mathrm{GS}} - H\right) c_l \right| \mathrm{GS_R} \right\rangle$$

$$\delta^2(\omega - H) = \sum_n \delta(\Re(\omega - E_n))\, \delta\left(\Im\left(\omega - E_n\right)\right) \left|\Psi_{\mathrm{R},n}\right\rangle \left\langle \Psi_{\mathrm{L},n}\right|$$

We define a Hermitrized Hamiltonian

$$\mathcal{H} = \begin{pmatrix} & \omega - H \\ \omega^* - H^\dagger & \end{pmatrix}$$

And perform a KPM expansion of

$$f(\omega) = \langle \psi_\mathrm{L} | \delta^2(\omega - H) | \psi_\mathrm{R} \rangle$$

$$f(\omega) = \frac{1}{\pi} \partial_{\omega^*} G(E = 0)$$

$$G(E) = \langle \mathrm{L} | (E - \mathcal{H})^{-1} | \mathrm{R} \rangle$$

Dynamical correlator of a topological non-Hermitian model

We take the following non-Hermitian model

$$\tilde{H} = \sum_{l=1}^{L-1} \left(\frac{J+\gamma}{2} c_l^\dagger c_{l+1} + \frac{J-\gamma}{2} c_{l+1}^\dagger c_l \right.$$

$$\left. + J_z (c_l^\dagger c_l - \frac{1}{2})(c_{l+1}^\dagger c_{l+1} - \frac{1}{2}) \right) + \sum_{l=1}^{L} i h_l^z (c_l^\dagger c_l - \frac{1}{2}).$$

Drives skin effect

Drives topology

Full dynamical correlator

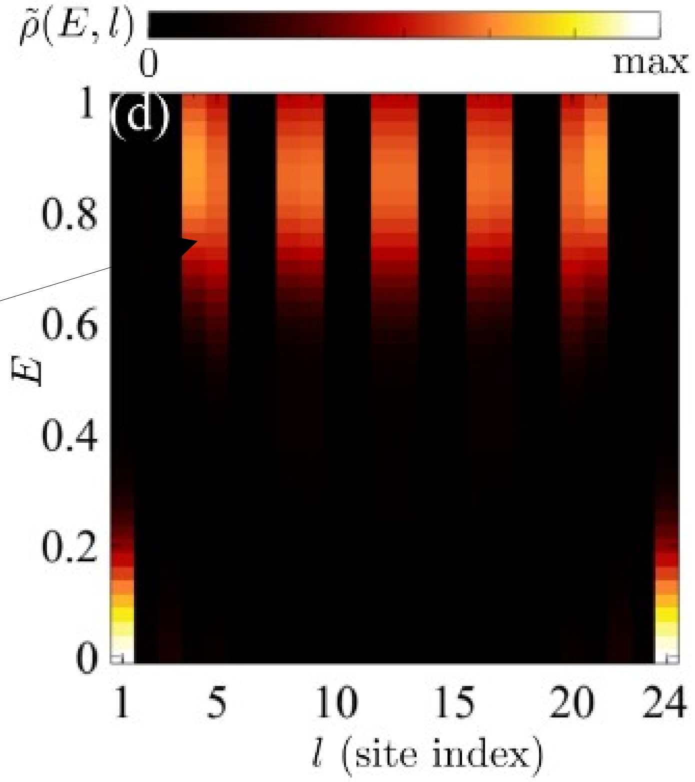
$$\mathrm{S}(\omega, l) = \left\langle \mathrm{GS_L} \middle| c_l \delta^2 \left(\omega + E_{\mathrm{GS}} - H\right) c_l^\dagger \middle| \mathrm{GS_R} \right\rangle + \left\langle \mathrm{GS_L} \middle| c_l^\dagger \delta^2 \left(\omega + E_{\mathrm{GS}} - H\right) c_l \middle| \mathrm{GS_R} \right\rangle$$

Integrated in the imaginary axis

(equivalent to the Hermitian one)

$$\tilde{\rho}(E \in \mathbb{R}, l) = \left| \int \mathcal{S}(E + iy, l) dy \right|$$

Non-Hermitian topological modes without skin effect

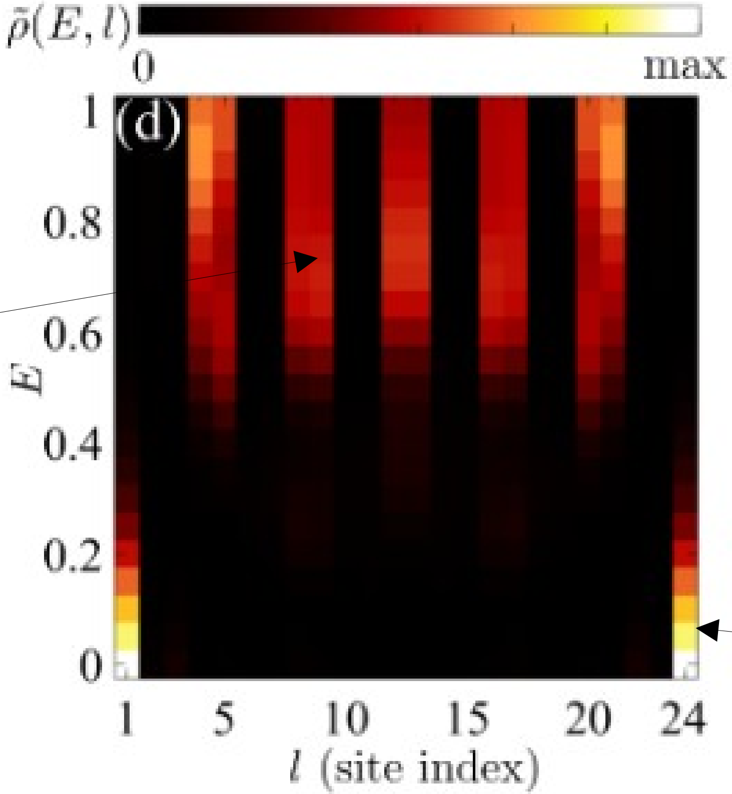


Bulk modes

Edge modes

The non-Hermitian dynamical correlator allows visualizing the topological modes

Non-Hermitian topological modes with skin effect



Bulk modes

Edge modes

The non-Hermitian dynamical correlator allows visualizing the topological modes

Open-source software for interacting and topological matter

Open-source software for tensor-network kernel polymonial algorithms

dmrgpy

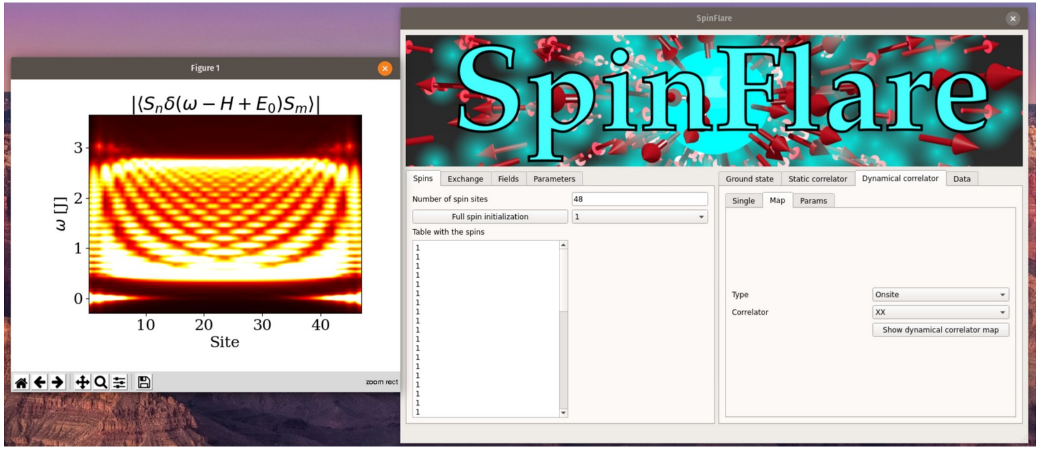
https://github.com/joselado/dmrgpy

```
from dmrgpy import spinchain
spins = ["S=1" for i in range(40)] # S=1 chain
sc = spinchain.Spin_Chain(spins) # create spin chain object
h = 0 # initialize Hamiltonian
for i in range(len(spins)-1):
  h = h + sc.Sx[i]*sc.Sx[i+1]
  h = h + sc.Sy[i]*sc.Sy[i+1]
  h = h + sc.Sz[i]*sc.Sz[i+1]
sc.set_hamiltonian(h)
sc.get_dynamical_correlator(name=(sc.Sz[0],sc.Sz[0]))
```

Generic Python library for tensor-network kernel polynomial algorithms for spins, fermions, parafermions

spinflare

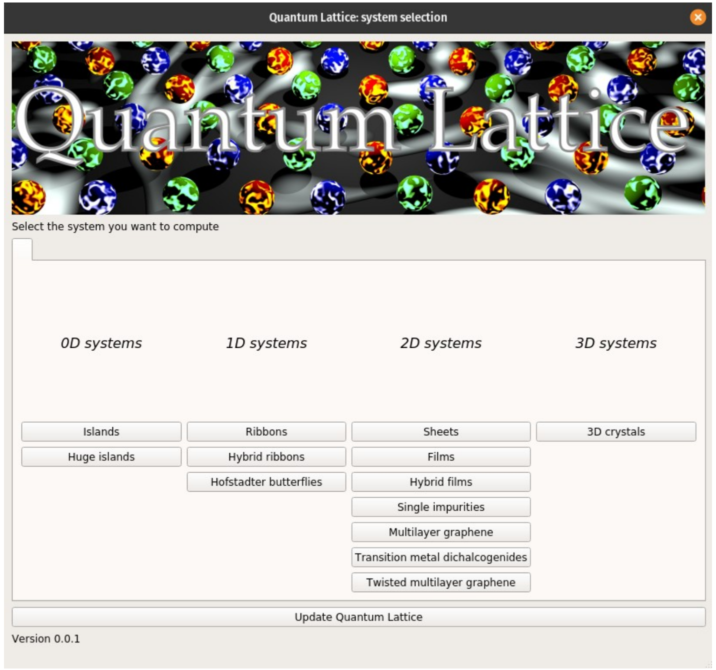
https://github.com/joselado/spinflare

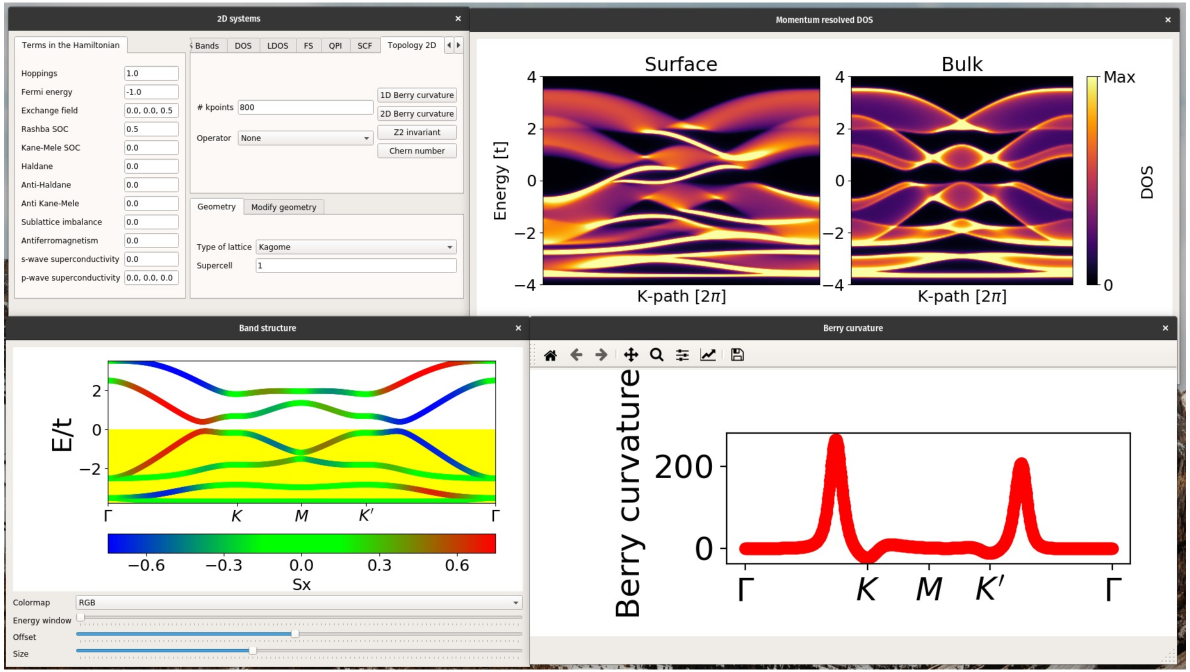


User friendly interface to compute excitations in quantum Heisenberg models

Based on ITensor

Quantum Lattice: A user interface to compute electronic properties

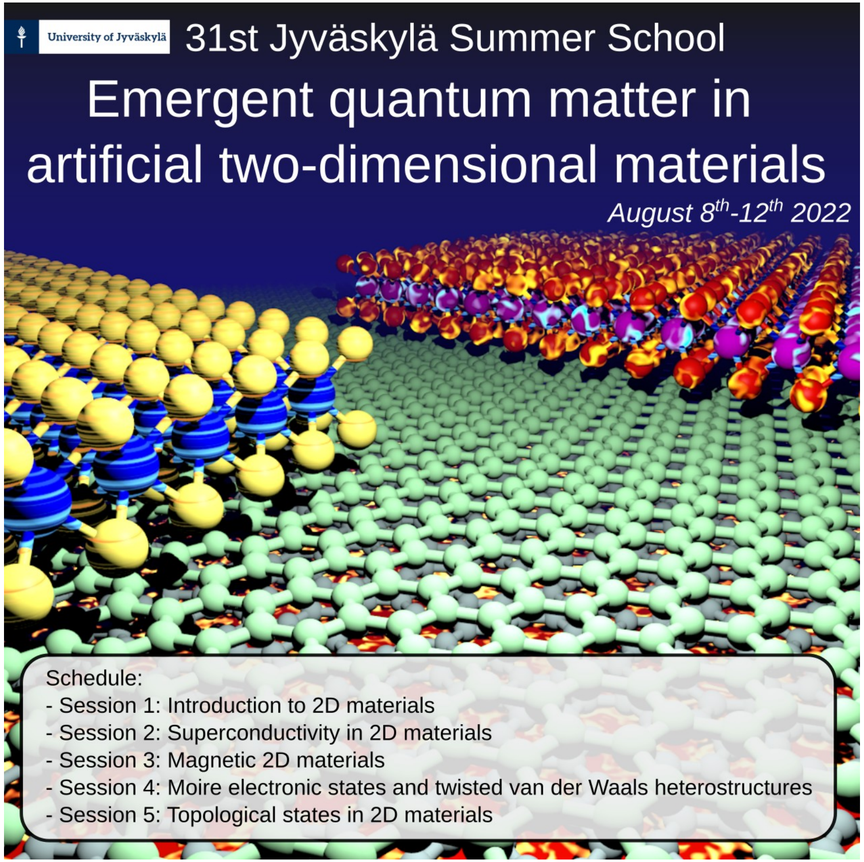




Quantum Lattice: open source interactive interface for tight binding modeling

https://github.com/joselado/quantum-lattice

Topological and correlated matter in 2D materials school



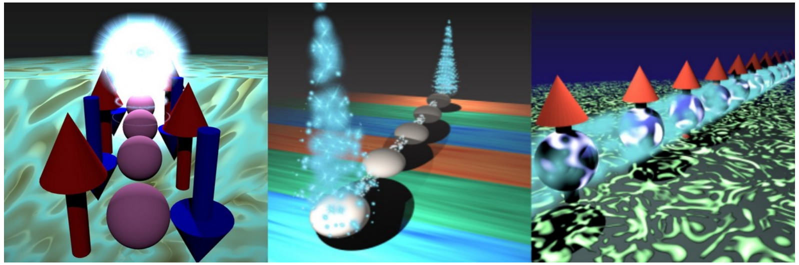
Recordings available at

https://www.youtube.com/channel/UCgnB-4CcqRQnvTi7P0cUakQ

- Session 1: Introduction to 2D materials
- Session 2: Superconductivity in 2D materials
- Session 3: Magnetic 2D materials
- Session 4: Moire electronic states and twisted van der Waals heterostructures
- Session 5: Topological states in 2D materials

Take home

Tensor-network kernel polynomial algorithms allow computing dynamical excitations in quasiperiodic and non-Hermitian quantum many-body systems



Thank you!

Phys. Rev. Research 2, 023347 (2020)
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Phys. Rev. Research 4, L012006 (2022)
arXiv:2208.06425 (2022)

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