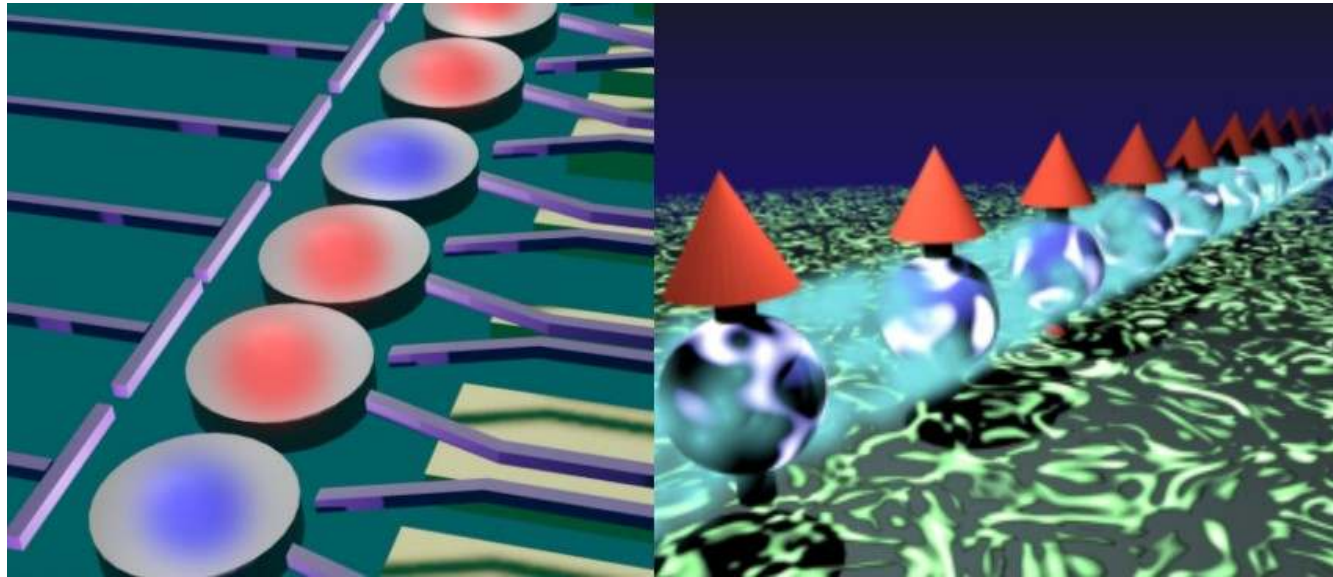


Static and dynamic topological modes in interacting non-Hermitian systems with tensor networks

Jose Lado

Department of Applied Physics, Aalto University, Finland



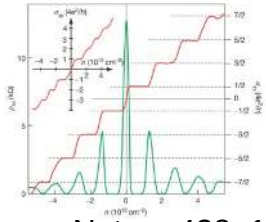
Phys. Rev. Research 4, L012006 (2022)

Phys. Rev. Lett. 130, 100401 (2023)

Non-Hermitian Physics: From classical to quantum, From theory to the lab
June 28th 2023

Topological matter

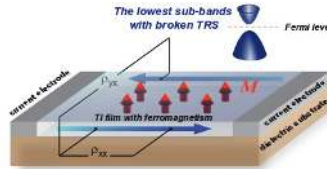
Quantum Hall effect



graphene

Nature 438, 197-200 (2005)

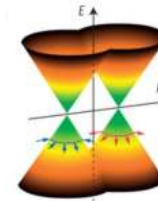
Quantum Anomalous Hall effect



Cr-doped
(Bi,Sb)₂Te₃

Science 340.6129 (2013): 167-170

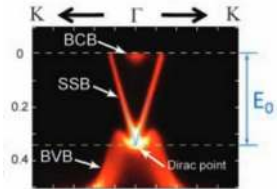
Weyl semimetal



TaAs

Nature Physics 11, 724–727 (2015)

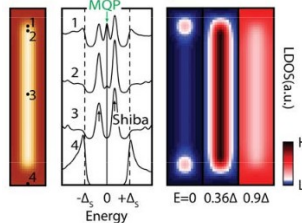
Quantum Spin Hall effect



Bi₂Se₃

Science 325.5937 (2009): 178-181

Topological superconductor



Fe@Pb

Science 346.6209 (2014): 602-607

Nodal line semimetals



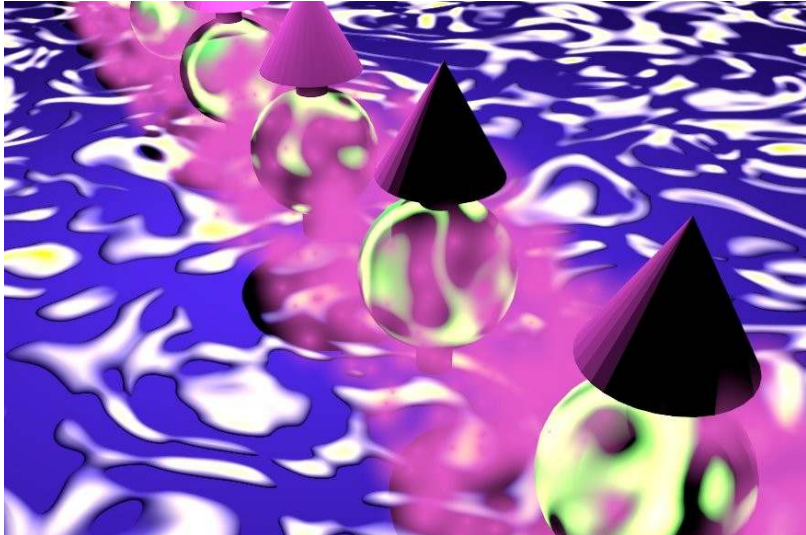
Mg₃Bi₂

Advanced Science 6.4 (2019): 1800897

All these states are described by effective isolated single-particle Hamiltonians

Two major challenges in topological matter

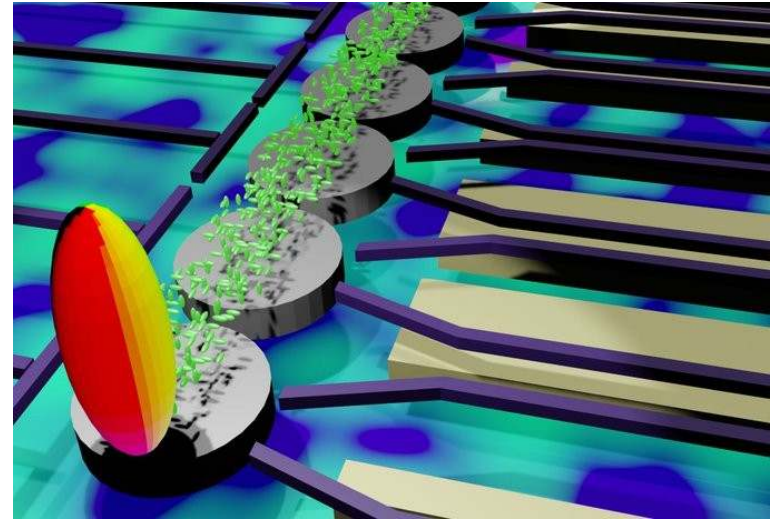
Many-body interactions



Interacting topological matter

Rep. Prog. Phys. 81 116501 (2018)

Coupling to the environment

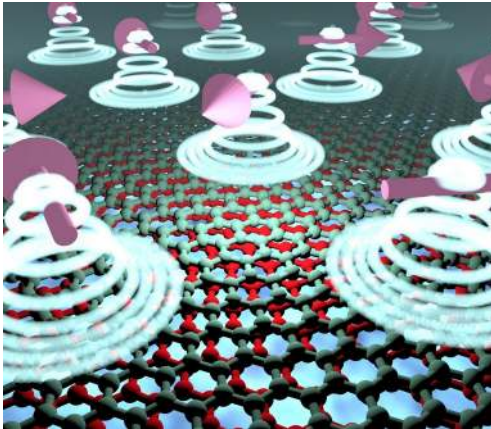


Non-Hermitian topological matter

Rev. Mod. Phys. 93, 015005 (2021)

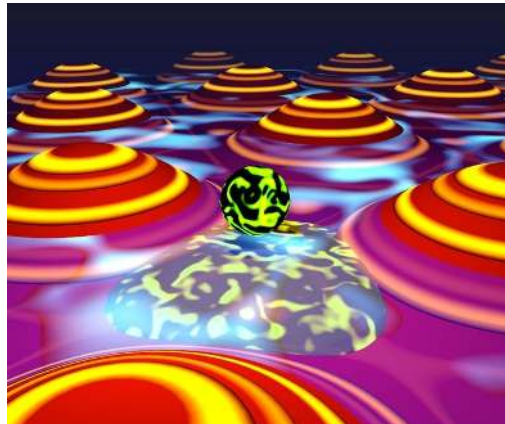
Many-body quantum matter

Quantum spin liquids



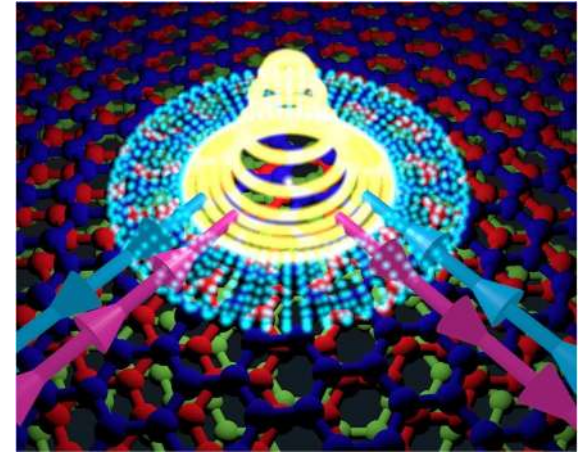
Reports on Progress in Physics 80.1 (2016): 016502.

Fractional Chern insulators



Rev. Mod. Phys. 71, S298 (1999)

Parafermions

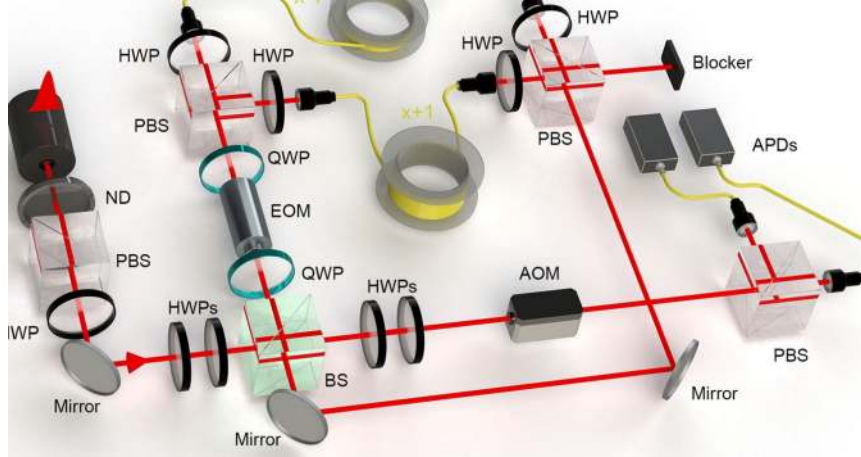


Annual Review of Condensed Matter Physics 7, 119-139 (2016)

Many-body quantum matter represents an exciting critically open challenge

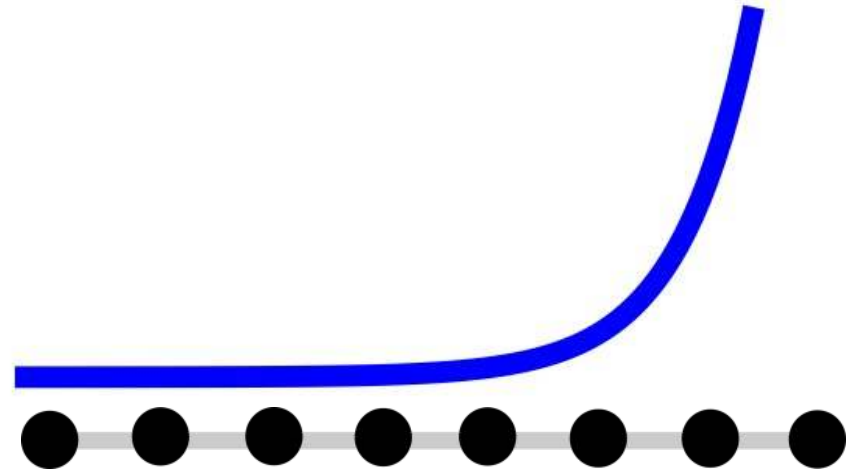
Non-Hermitian quantum matter

Non-Hermitian topology



Nat Comm 13, 3229 (2022)

Non-Hermitian skin effect

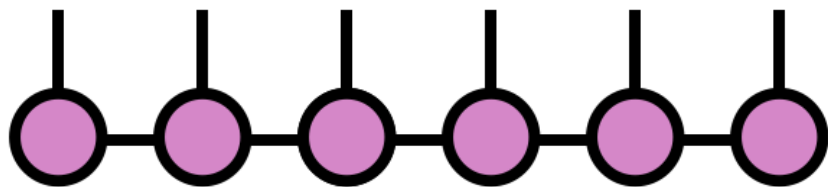


Phys. Rev. Lett. 129, 070401 (2022)

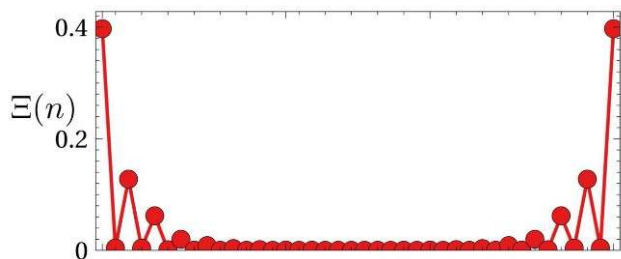
Non-Hermitian quantum matter opens exciting new possibilities in emergent phenomena

Plan for today

Tensor networks for
many-body matter

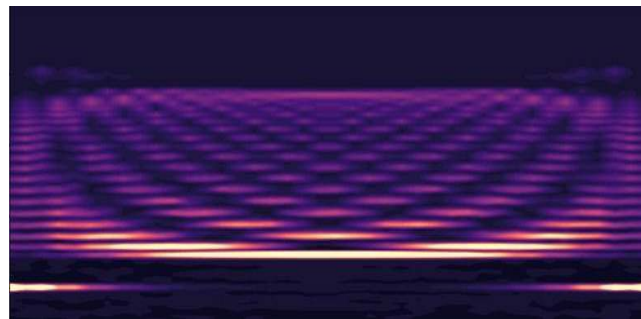


Edge modes from interacting
non-Hermitian topology

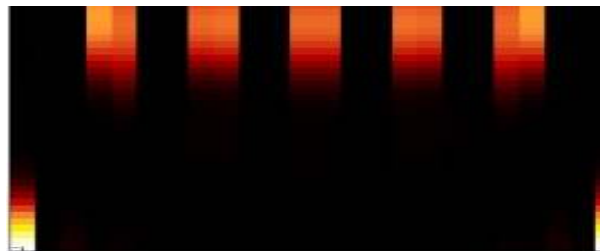


Phys. Rev. Research 4, L012006 (2022)

Dynamical correlators for
interacting many-body edge modes



*Dynamical correlators for
non-Hermitian many-body topology*



Phys. Rev. Lett. 130, 100401 (2023)

Behind the scenes

Edge modes from interacting non-Hermitian topology

Timo Hyart



Phys. Rev. Research 4, L012006 (2022)

Dynamical correlators for non-Hermitian many-body topology

Fei Song

Guangze Chen



Phys. Rev. Lett. 130, 100401 (2023)

Tensor networks for quantum many-body Hamiltonians

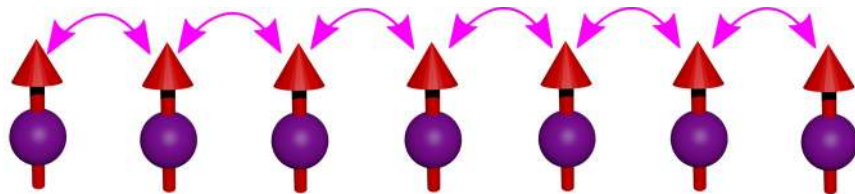
The problem of dimensionality

In a many-body problem, the size of our vectors grows as

$$2^L$$

where L is the number of sites

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$



For a single-particle tight binding problem, we can reach up to 10^8 sites in a laptop

For a many-problem, we cannot even store states for systems bigger than $L = 30$ sites

$$2^{30} \sim 10^9$$

The quantum many-body problem

Let us go back to a simple many-body problem

$$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

A typical wavefunction is written as

$$|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$$

We need to determine in total 2^L coefficients

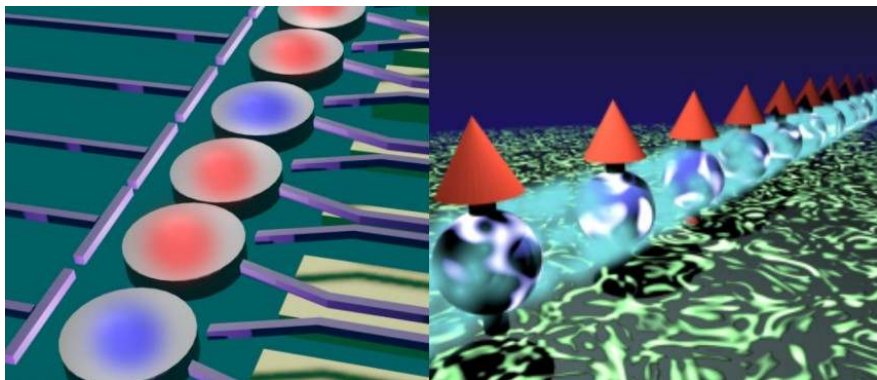
Is there an efficient way of storing so many coefficients?

The fundamental idea of tensor-networks

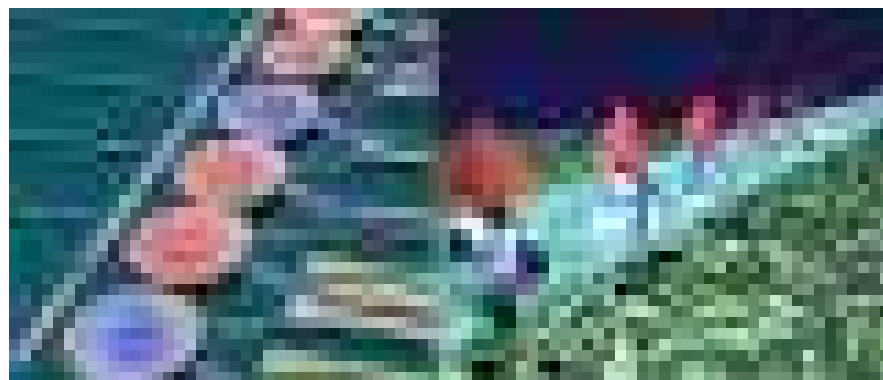
A many-body wavefunction is a very high dimensional object

$$|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$$

Tensor-networks allow “compressing” all that information in a very efficient way



“True wavefunction”



“Tensor-network wavefunction”

The matrix-product state ansatz

For this wavefunction $|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$

Let us imagine to propose a parametrization in this form

$$c_{s_1, s_2, \dots, s_L} = M_1^{s_1} M_2^{s_2} \dots M_L^{s_L}$$

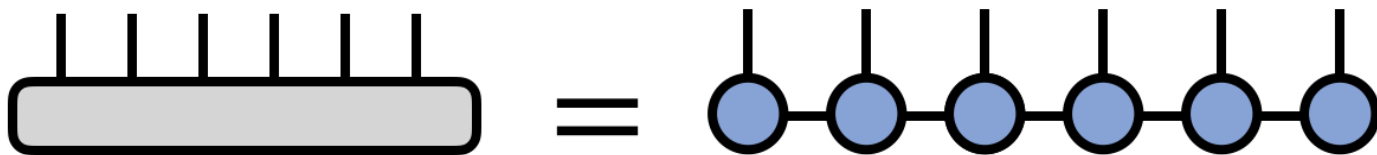
dimension 2^L dimension $\sim Lm^2$

(m dimension of the matrix)

State compression with tensor-networks

Given a many-body wavefunction, we can parametrize the components as

$$|\Psi\rangle = \sum_{\{s\}} \text{Tr} \left[M_1^{(s_1)} M_2^{(s_2)} \cdots M_N^{(s_N)} \right] |s_1 s_2 \cdots s_N\rangle$$

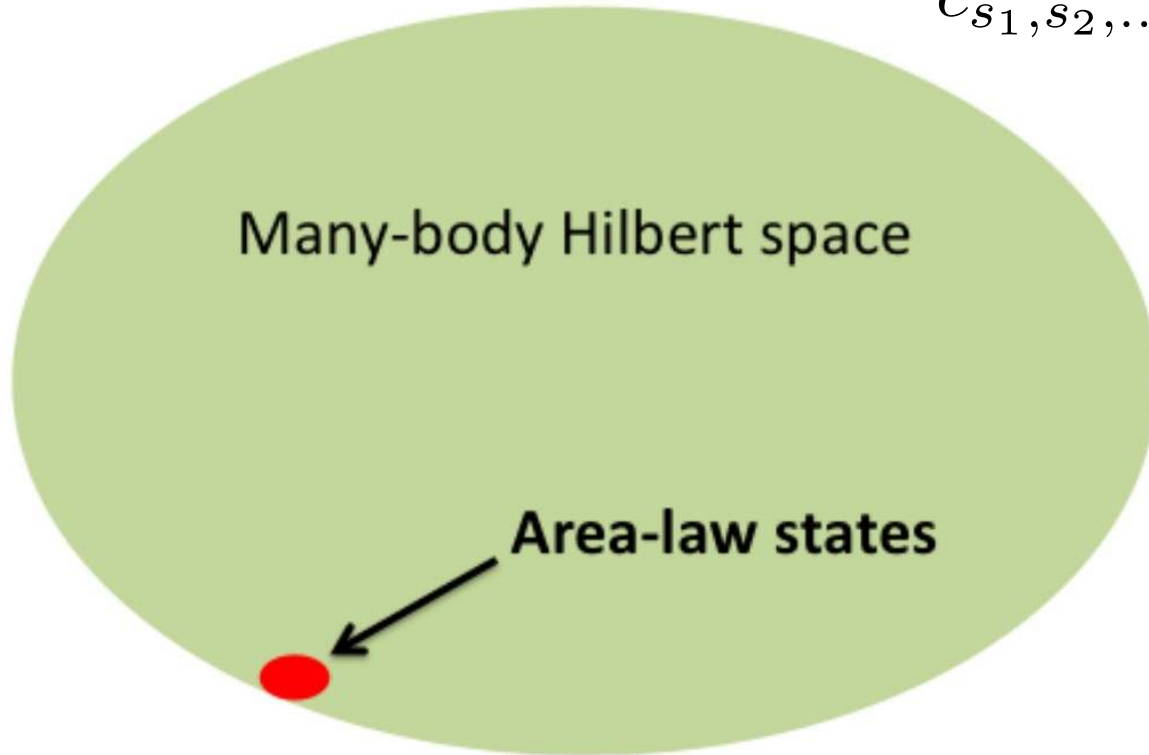


Matrix product state

The previous representation allows drastically reducing the memory required to store a state

MPS as a parametrization of area law states

$$c_{s_1, s_2, \dots, s_L} = M_1^{s_1} M_2^{s_2} \dots M_L^{s_L}$$



The matrix-product state ansatz

- This ansatz enforces a maximum amount of entanglement entropy in the state $S \sim \log m$
- One-dimensional (Hermitian) problems have ground states that can be captured with this ansatz

$$c_{s_1, s_2, \dots, s_L} = M_1^{s_1} M_2^{s_2} \dots M_L^{s_L}$$

This ansatz can be generalized for time-evolution, excited states, or typical thermal states

Matrix-product states for non-Hermitian systems

MPS are well controlled for Hermitian Hamiltonians, but will/should they work for non-Hermitian systems?

In general, this is not known yet

In practical cases, they do!

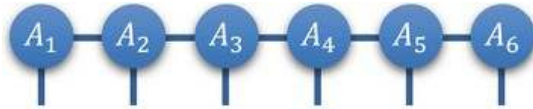
How do we find the ground states of non-Hermitian Hamiltonians with MPS?

We can use MPS as a variational manifold and find them iteratively

(Arnoldi-Krylov algorithm)

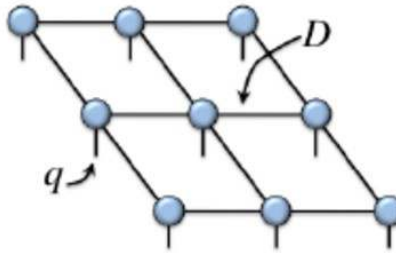
Many-body state compression

Matrix-product states



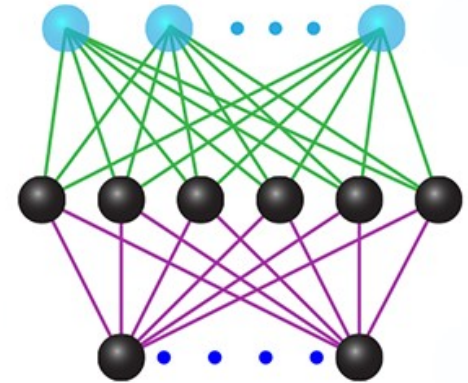
Phys. Rev. Lett. 69, 2863 (1992)

Projected entangled pair-states



Annals of Physics 326, 96 (2011)

Neural-network quantum states



Science 355.6325 (2017): 602-606.

Other compressed many-body states could be potentially used for non-Hermitian interacting many-body systems

Dynamical excitations in quantum many-body systems

Why dynamical correlators

For a single particle model

L eigenstates/eigenenergies

Hilbert space

One particle excitations

For a many-body model

2^L eigenstates/eigenenergies

Fock space

One, two, three... particle excitations

How do we identify single particle excitations in a many-body Hamiltonian?

A single particle Hamiltonian can be solved in any of the two spaces

$$H = \sum_{ij} t_{ij} c_i^\dagger c_j$$

Why dynamical correlators

Take a many-body Hamiltonian $\mathcal{H}$

With ground state $\mathcal{H}|\Omega\rangle = E_0|\Omega_0\rangle$

And excited states $\mathcal{H}|\Psi_n\rangle = E_n|\Psi_n\rangle$

A way to “filter” single particle excitations is by defining

$$A(\omega, l) = \sum_n \delta(\omega - (E_n - E_0)) |\langle \Psi_n | c_l^\dagger | \Omega \rangle|^2$$

Dirac delta function

Picks energy
differences ω

Only non-zero if they
differ by one electron

This is a dynamical correlator

Why dynamical correlators

Dynamical correlator (for single particle electron excitations)

$$A(\omega, n) = \langle \Omega | c_n \delta(\omega - (\mathcal{H} - E_0)) c_n^\dagger | \Omega \rangle$$

for fermionic models

Dynamical correlator (for spin excitations)

$$A(\omega, n) = \langle \Omega | S_n^+ \delta(\omega - (\mathcal{H} - E_0)) S_n^- | \Omega \rangle$$

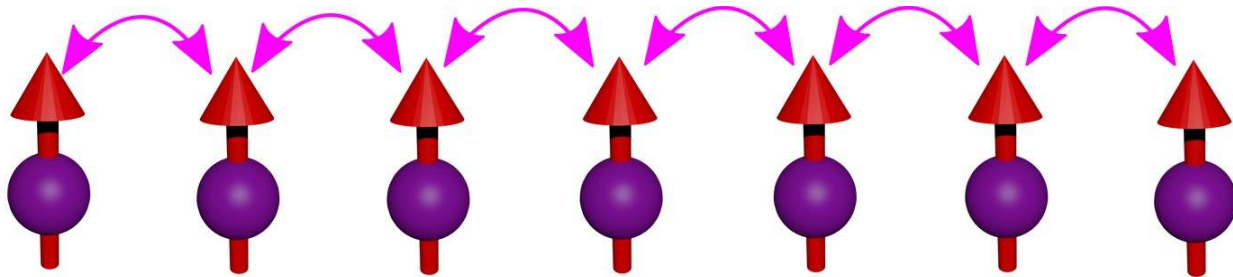
for spin models

Both can be related by a fermion-spin mapping (Jordan-Wigner transformation)

Many-body dynamical correlators

One dimensional Heisenberg Hamiltonian

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$



We want to compute the dynamical correlator

$$\mathcal{S}(N, \omega) = \langle GS | S_N^z \delta(\omega - \mathcal{H} + E_0) S_N^z | GS \rangle$$

The kernel-polynomial tensor-network algorithm

Take a many-body Hamiltonian

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$

Obtain a tensor-network representation of the many-body ground state

$$|GS\rangle = \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}$$

And apply the kernel polynomial algorithm in the compressed representation

$$\bar{\chi}(\omega) = \langle GS | S_N^z \delta(\omega - \bar{H}) S_N^z | GS \rangle$$
$$\bar{\chi}(\omega) = \frac{1}{\pi \sqrt{1 - \omega^2}} \left[\mu_0 + 2 \sum_{l=1}^{N_p} \mu_l T_l(\omega) \right]$$

Rev. Mod. Phys. 78, 275 (2006)

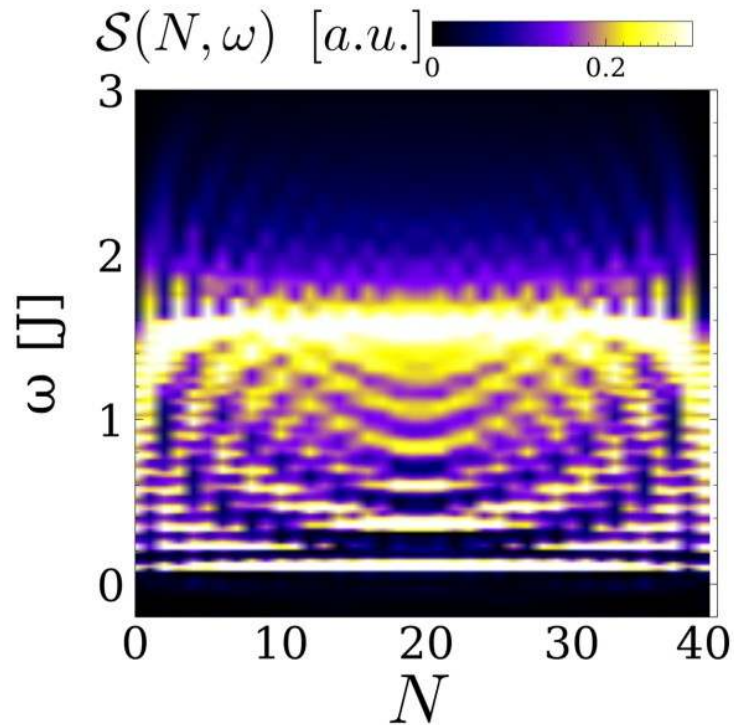
$$\mu_l = \langle GS | S_N^z T_l(\bar{H}) S_N^z | GS \rangle \quad |w_0\rangle = S_N^z |GS\rangle,$$
$$\mu_l = \langle GS | S_N^z |w_l\rangle \quad |w_1\rangle = \bar{H} |w_0\rangle,$$
$$|w_{l+1}\rangle = 2\bar{H} |w_l\rangle - |w_{l-1}\rangle,$$

Phys. Rev. Research 1, 033009 (2019)

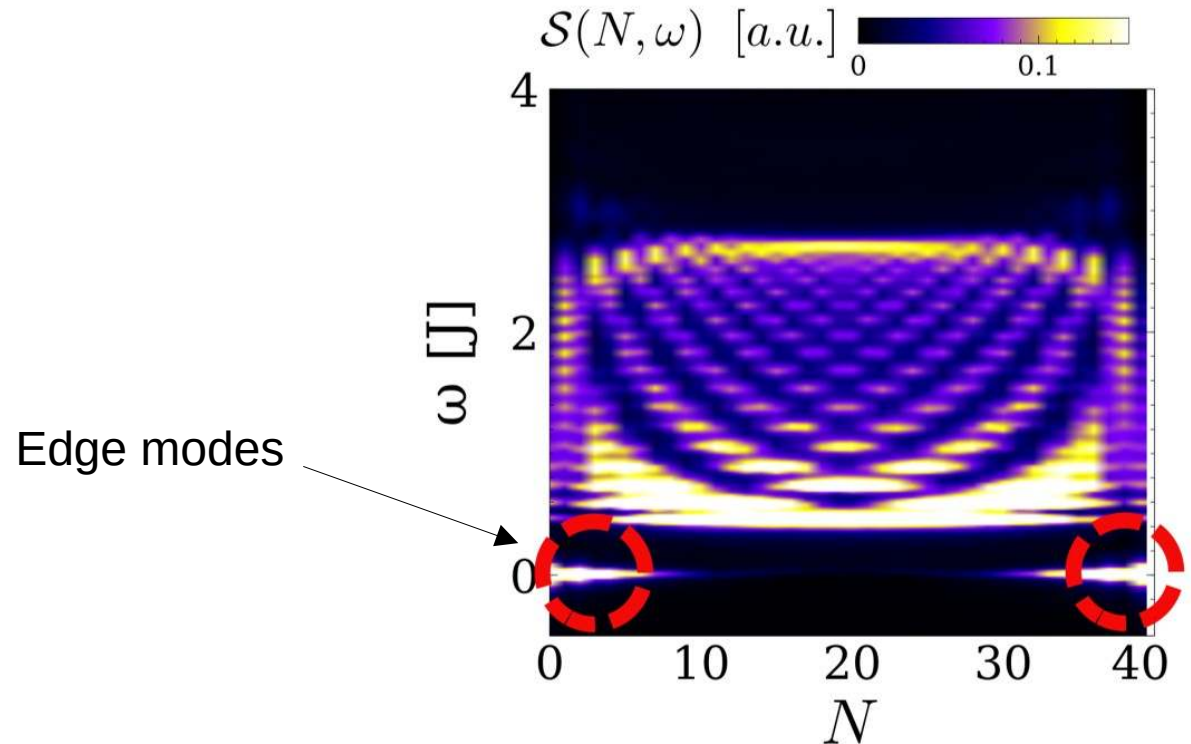
Open-source implementation in <https://github.com/joselado/dmrgpy>

Dynamical structure factor of a Heisenberg model

S=1/2 chain



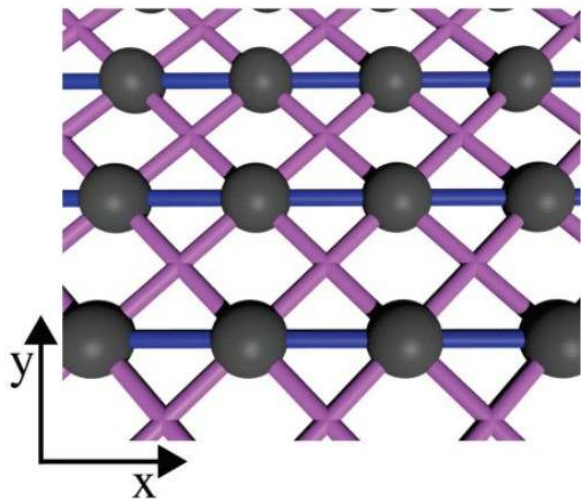
S=1 chain



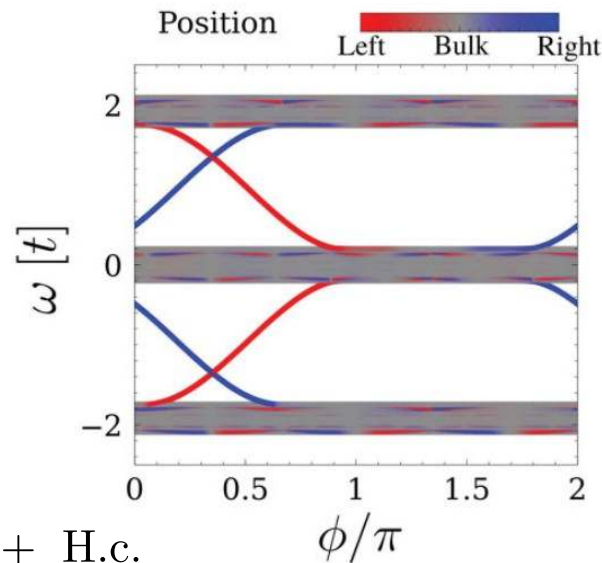
A minimal example of topological
modes in a many-body Hamiltonian

Quasiperiodic topology

Quantum Hall in a
two-dimensional lattice



Spectrum of the associated
quasiperiodic Hamiltonian



$\sim$

$$H = t \sum_N [1 + \lambda \cos(\alpha N + \phi)] c_N^\dagger c_{N+1} + \text{H.c.}$$

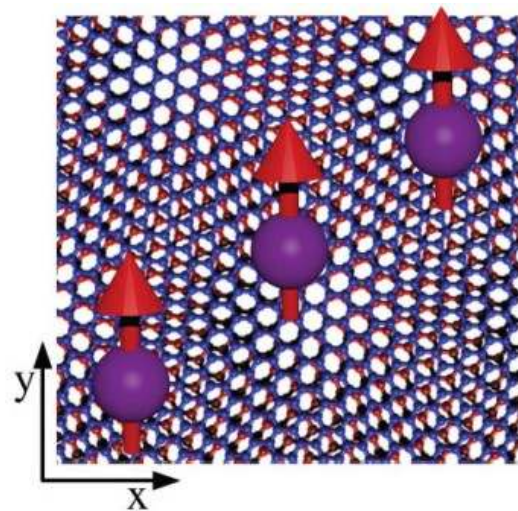
1D quasiperiodic models can be mapped to 2D quantum Hall states

This mapping is only valid in the non-interacting limit

Quasiperiodic spinon topology

Many-body spin Hamiltonians can be intrinsically quasiperiodic

For example, an atomic chain on top of a moire system

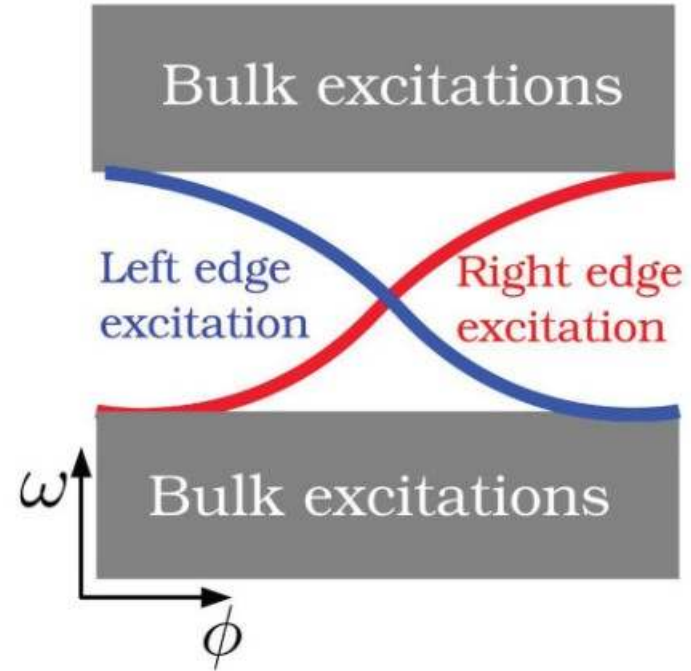
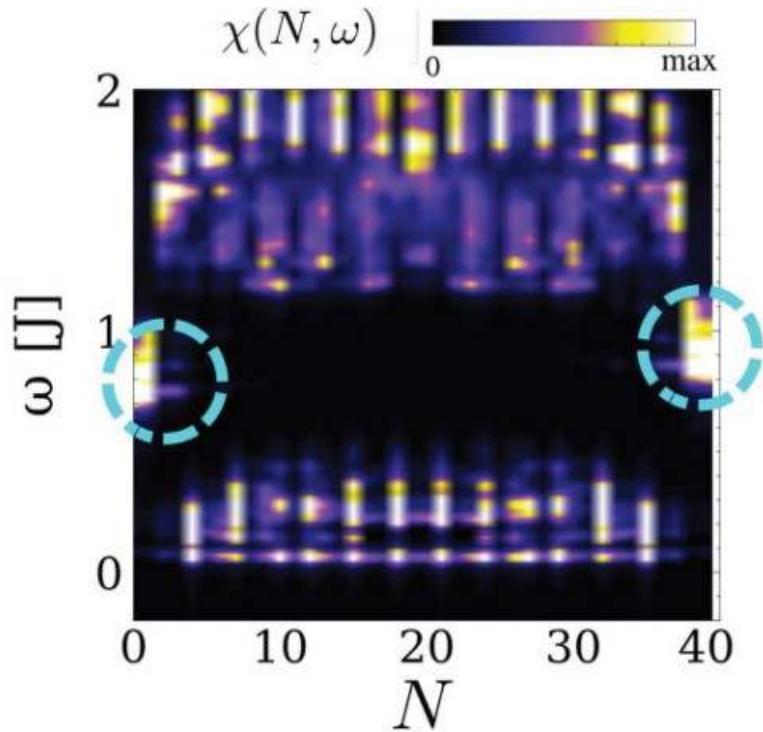


We will consider the generalized quasiperiodic Heisenberg model

$$H = J \sum_N [1 + \lambda \cos(\alpha N + \phi)] \vec{S}_N \cdot \vec{S}_{N+1}$$

with $S = 1/2, 1, 3/2, 2$

Topological quasiperiodic spinons for $S=1/2$

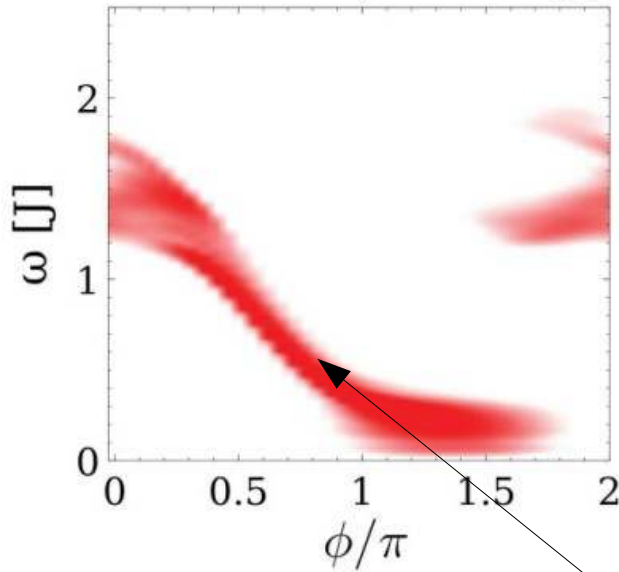


Quasiperiodic many-body interactions create topological spinon modes

Topological quasiperiodic spinons for $S=1/2$

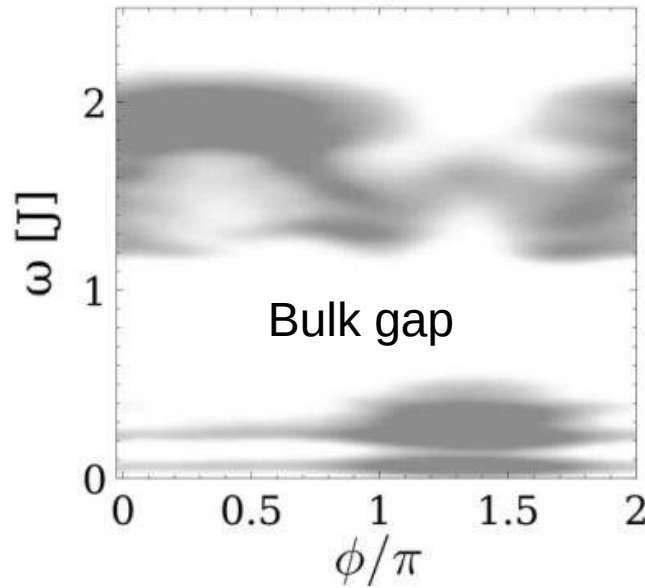
Left edge

$\chi(0, \omega)$ 0 max



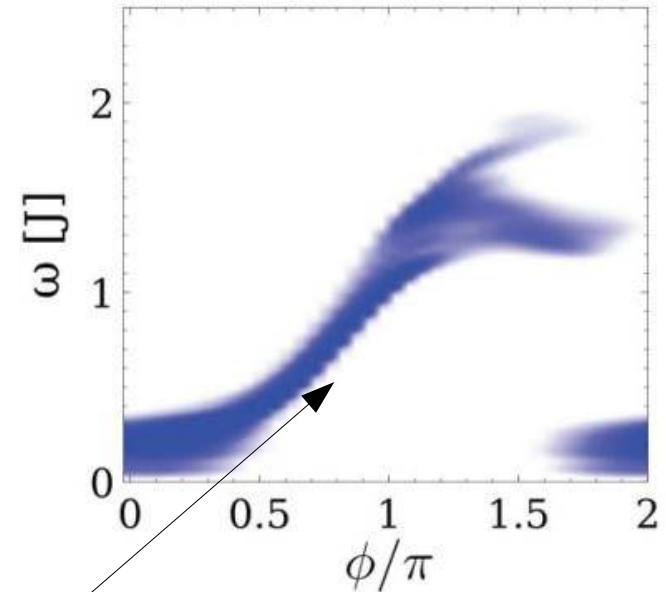
Bulk

$\chi(L/2, \omega)$ 0 max



Right edge

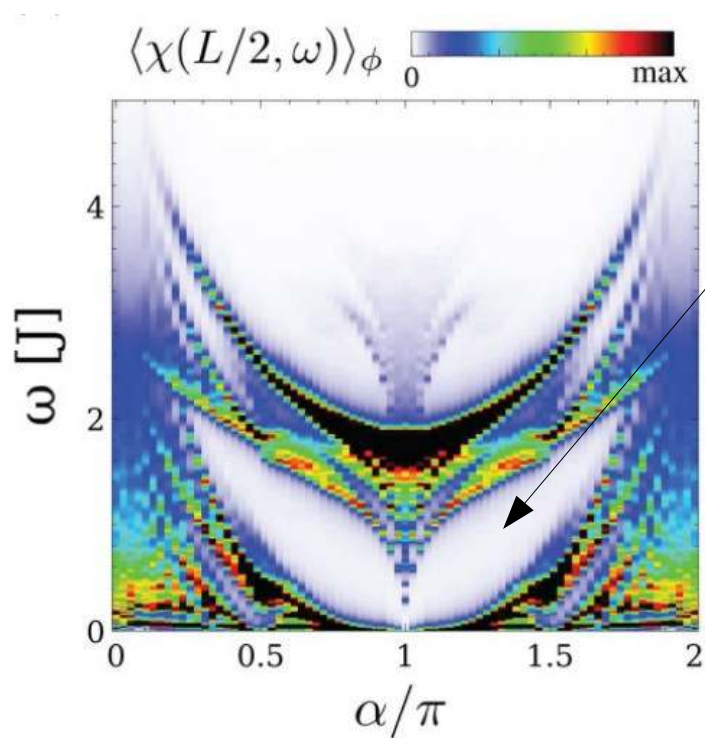
$\chi(L-1, \omega)$ 0 max



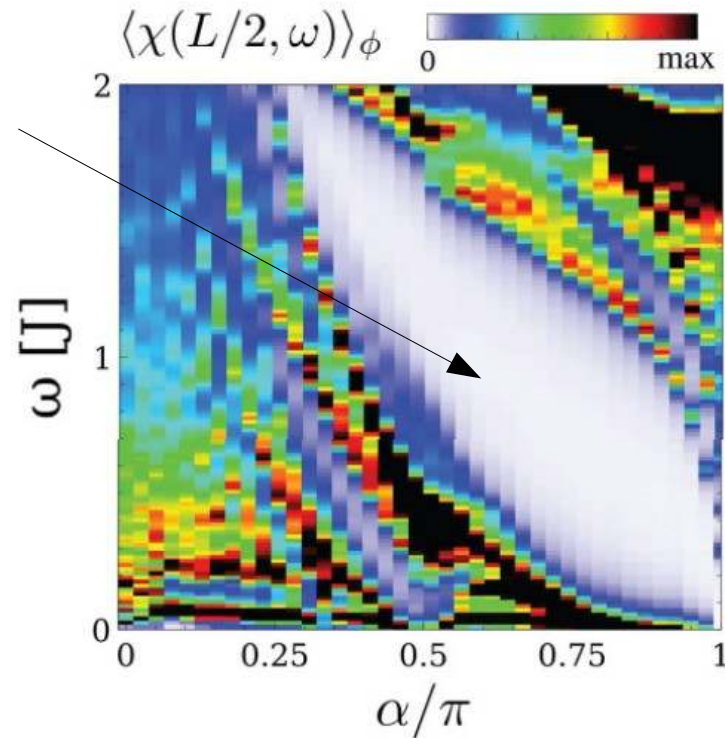
$$H = J \sum_N [1 + \lambda \cos(\alpha N + \phi)] \vec{S}_N \cdot \vec{S}_{N+1}$$

Topological spinon pumping modes

Spinon Hofstadter spectra $S=1/2$



Topological gap

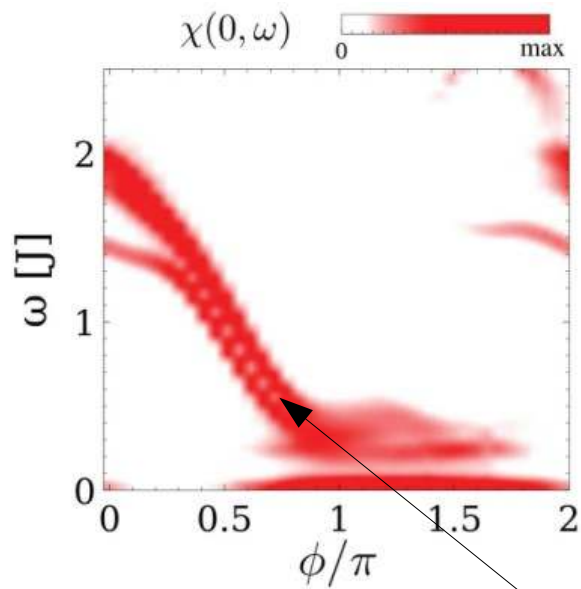


Topological gaps appear for arbitrary quasiperiodicity

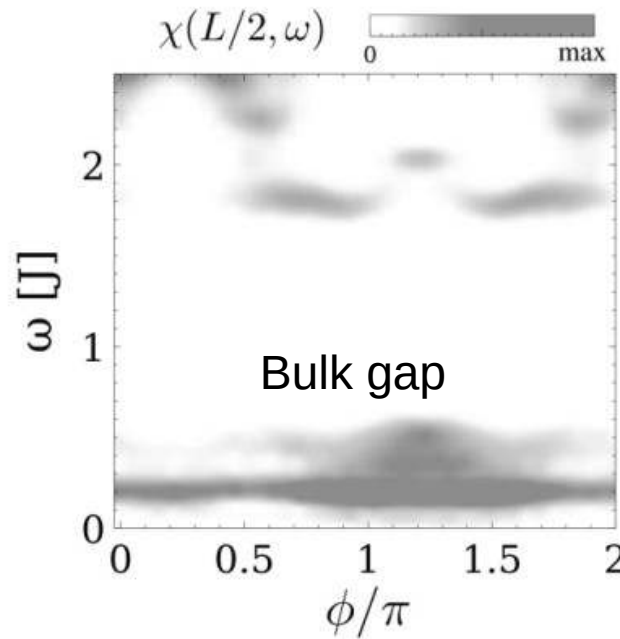
$$H = J \sum_N [1 + \lambda \cos(\alpha N + \phi)] \vec{S}_N \cdot \vec{S}_{N+1}$$

Topological quasiperiodic spinons for $S=1$

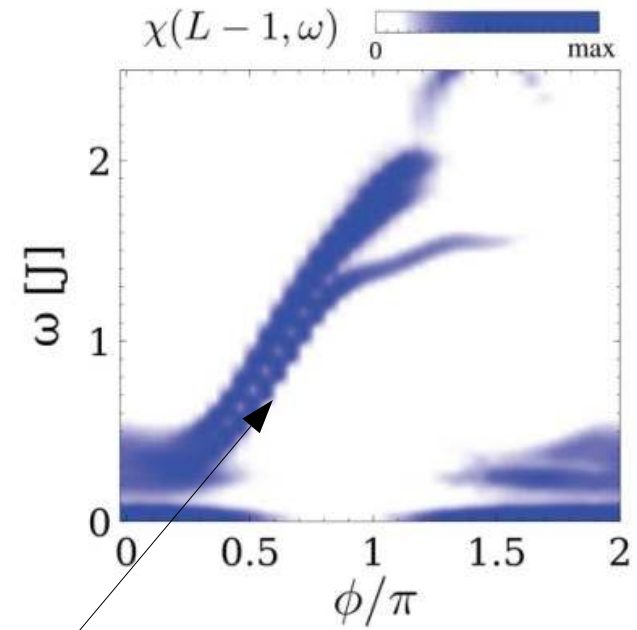
Left edge



Bulk



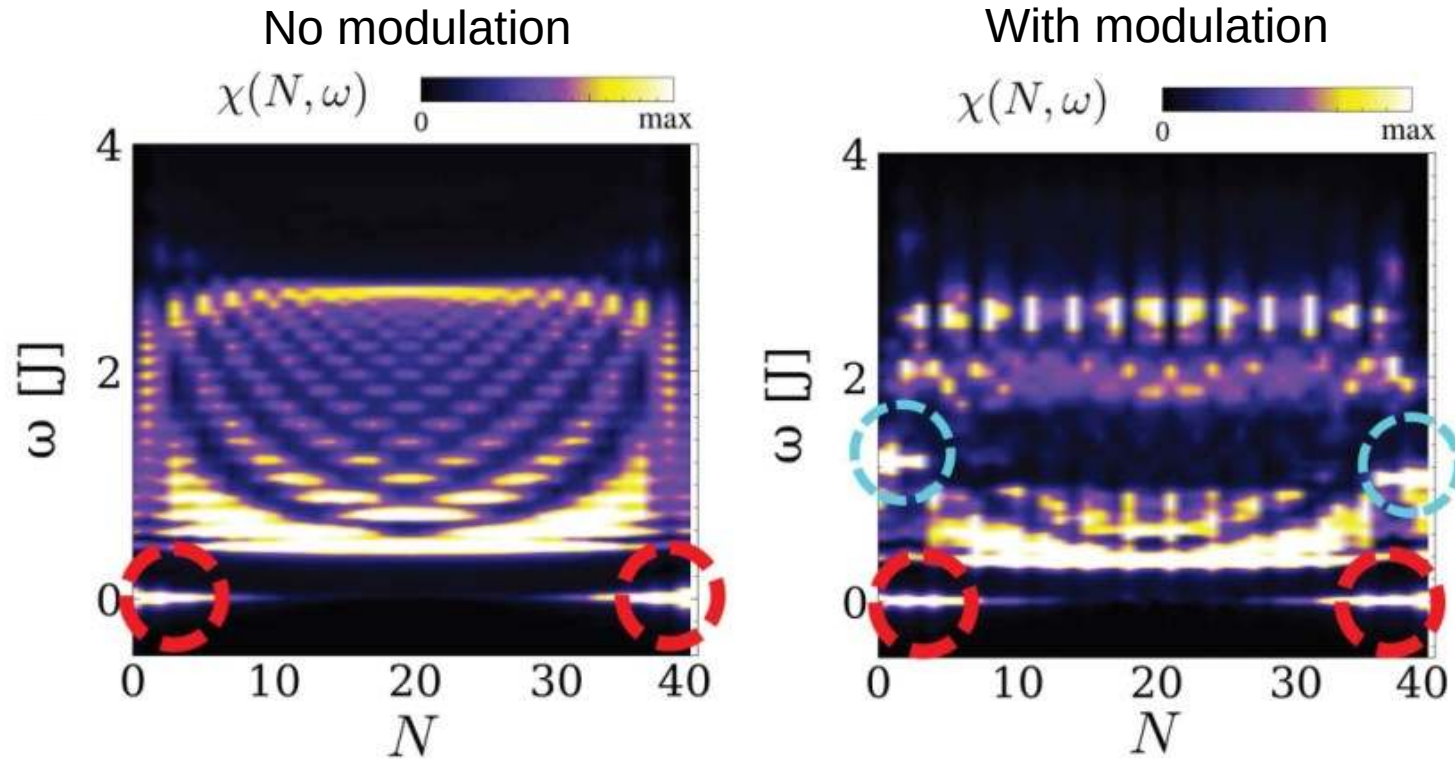
Right edge



Topological spinon pumping modes

$$H = J \sum_N [1 + \lambda \cos(\alpha N + \phi)] \vec{S}_N \cdot \vec{S}_{N+1}$$

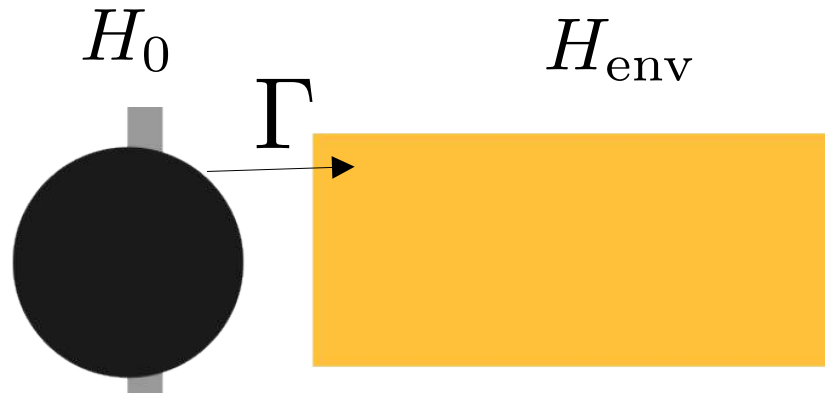
Topological quasiperiodic spinons for $S=1$



Quasiperiodicity creates additional topological modes away from $E=0$

Non-Hermitian topological many-body excitations

Coupling an electronic system to an environment



Full Hamiltonian of the system

$$H_{\text{full}} = \begin{pmatrix} H_0 & \Gamma \\ \Gamma^\dagger & H_{\text{env}} \end{pmatrix}$$

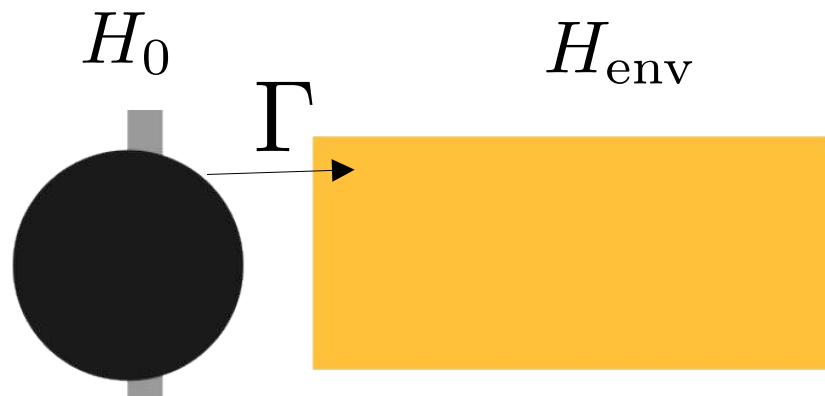
Selfenergy

$$\Sigma_{\text{env}} = \Gamma(\omega - H_{\text{env}})^{-1}\Gamma^\dagger$$

Green's function of the coupled system

$$G_0 = (\omega - H_0 - \Sigma_{\text{env}})^{-1}$$

Coupling an electronic system to an environment



Full Hamiltonian of the system

$$H_{\text{full}} = \begin{pmatrix} H_0 & \Gamma \\ \Gamma^\dagger & H_{\text{env}} \end{pmatrix}$$

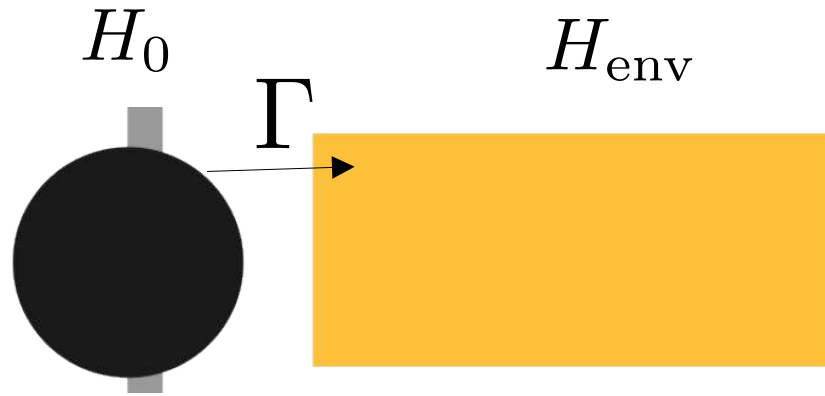
Effective Hamiltonian associated to the Green's function

$$H_0^{eff} = H_0 + \frac{1}{2}(\Sigma_{\text{env}} + \Sigma_{\text{env}}^\dagger) + \frac{1}{2}(\Sigma_{\text{env}} - \Sigma_{\text{env}}^\dagger)$$

Hermitian
renormalization

Non-Hermitian
environment induced
terms

Coupling an electronic system to an environment



Full Hamiltonian of the system

$$H_{\text{full}} = \begin{pmatrix} H_0 & \Gamma \\ \Gamma^\dagger & H_{\text{env}} \end{pmatrix}$$

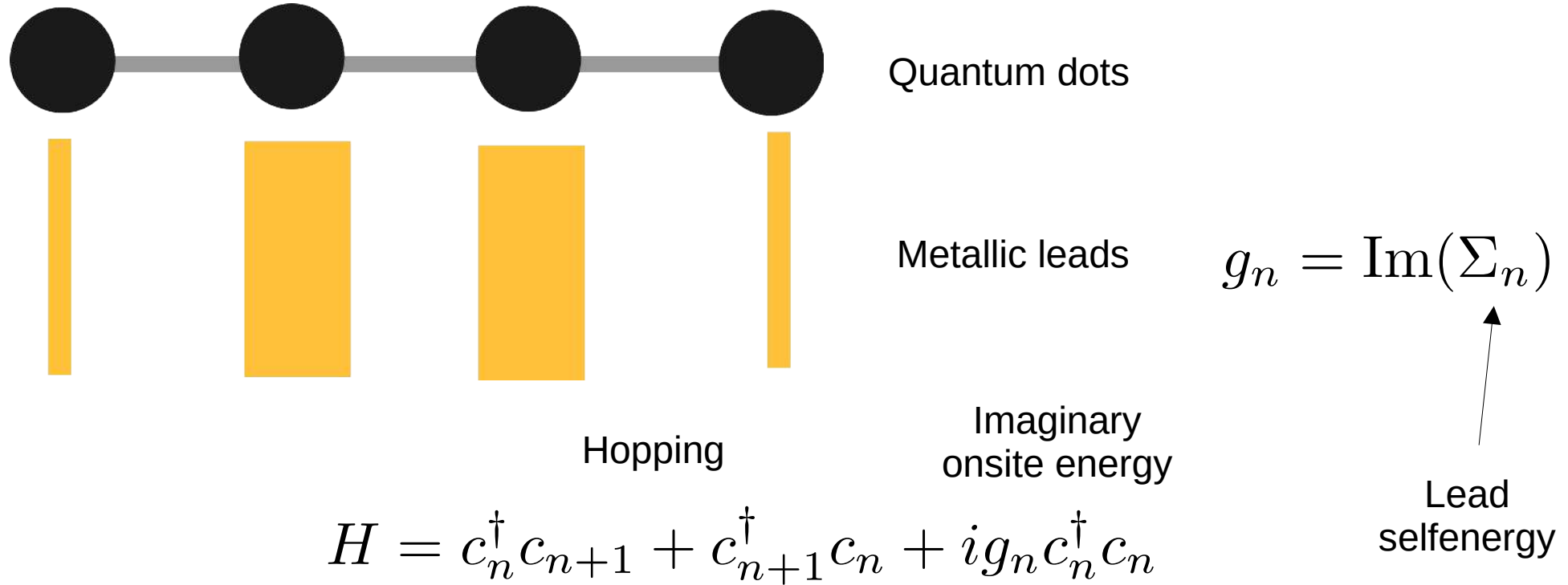
Effective Hamiltonian associated to the Green's function

$$H_0^{eff} = \bar{H}_0 + H_0^N H$$

Hermitian
terms

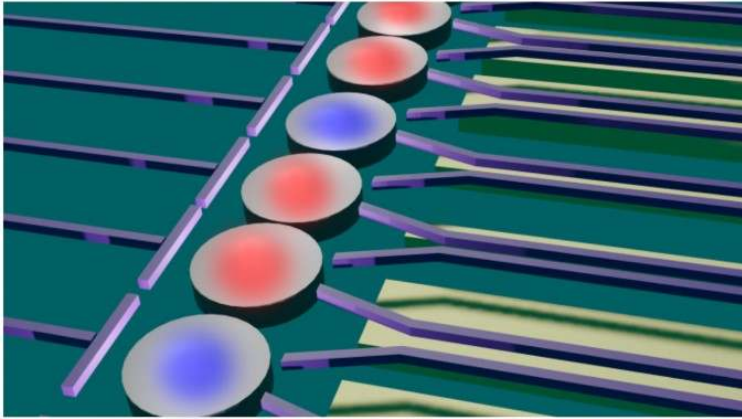
Non-Hermitian
terms

Non-Hermiticity in quantum dots



Coupling to metallic leads generates non-Hermitian terms in an effective Hamiltonian

Topological excitations driven by modulated losses

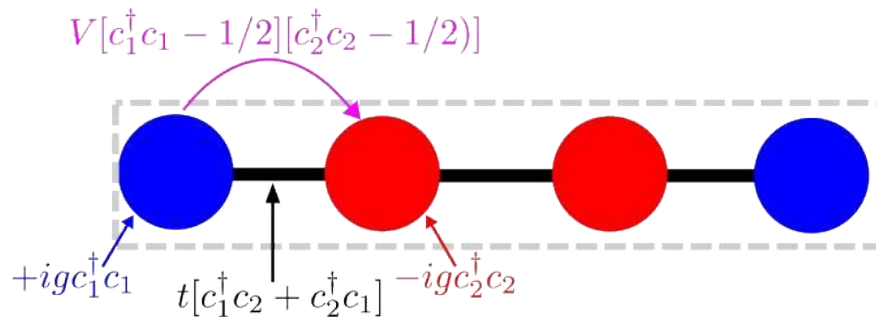


Bloch Hamiltonian

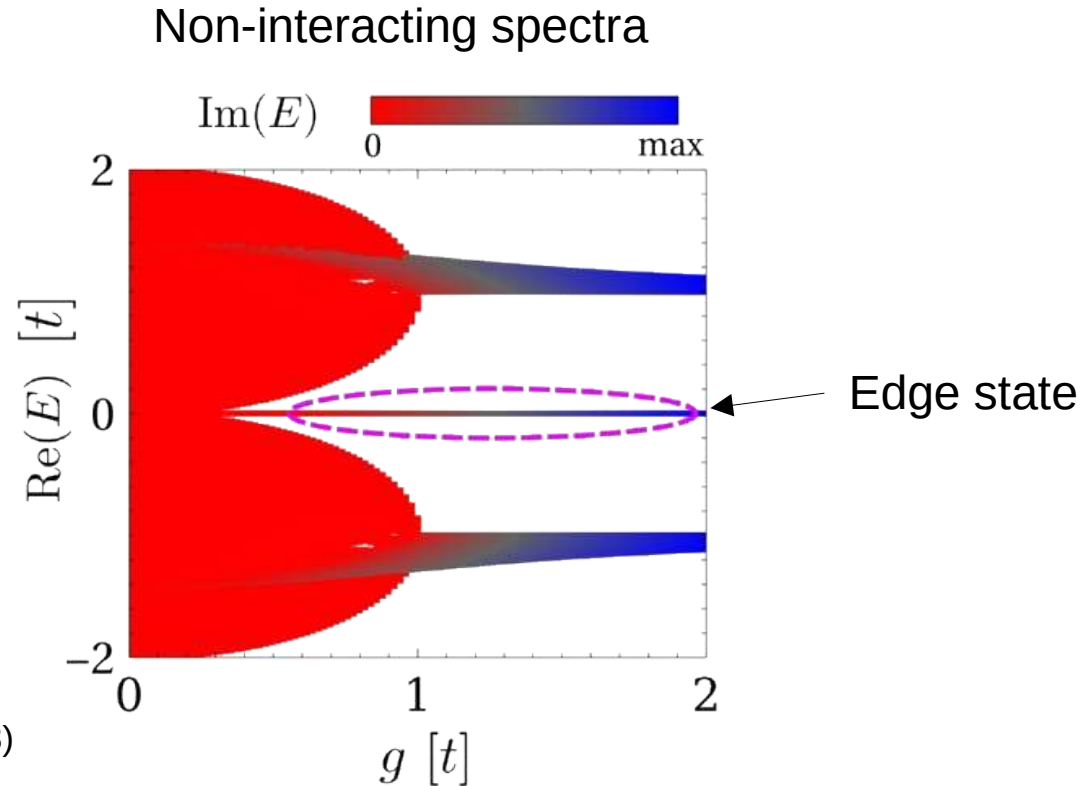
$$H_k = \begin{pmatrix} ig_1 & t & 0 & te^{-ik} \\ t & ig_2 & t & 0 \\ 0 & t & ig_3 & t \\ te^{ik} & 0 & t & ig_4 \end{pmatrix}$$

Losses coming from a self-energy

$$g_n = \text{Im}(\Sigma_n)$$



Edge mode driven by losses



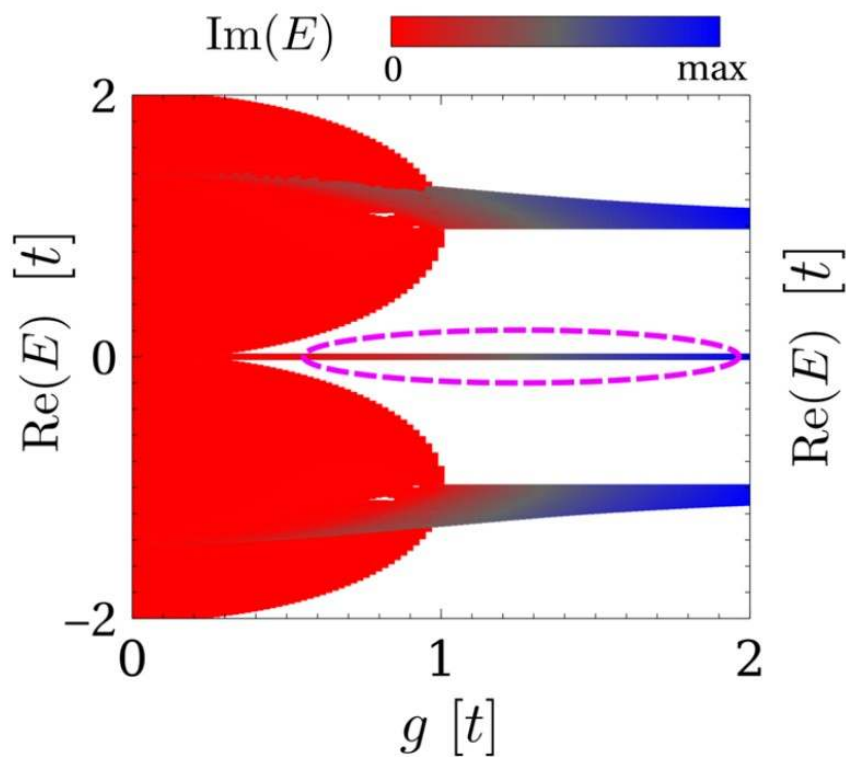
Phys. Rev. Lett. 121, 213902 (2018)

Phys. Rev. B 100, 161105® (2019)

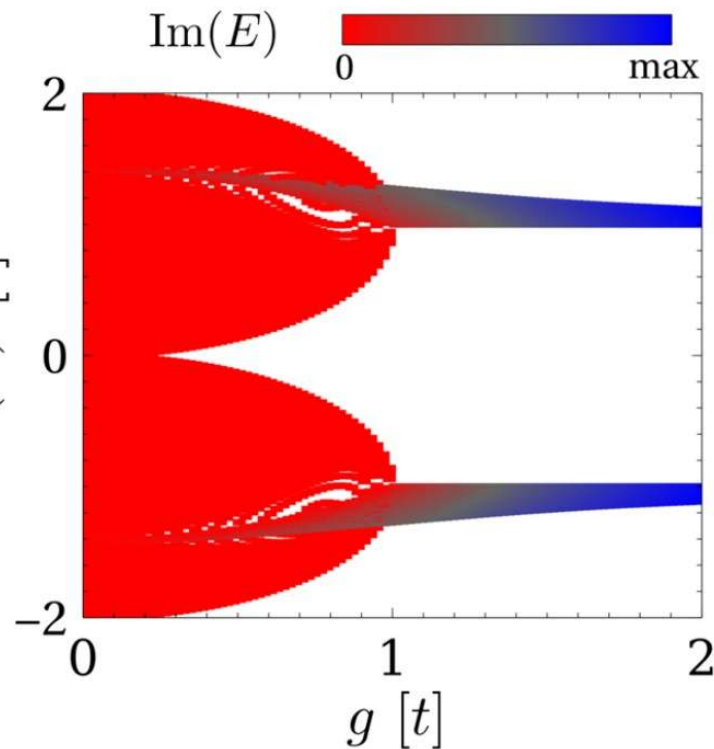
Does this topological mode survive the presence of many-body interactions?

Edge mode driven by losses

Open boundary conditions



Closed boundary conditions



Going to the many-body limit

Let us now add interacting to the non-Hermitian Hamiltonian

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_I$$

*Non-Hermitian single
particle term*

$$\mathcal{H}_0 = c_n^\dagger c_{n+1} + c_{n+1}^\dagger c_n + i g_n c_n^\dagger c_n$$

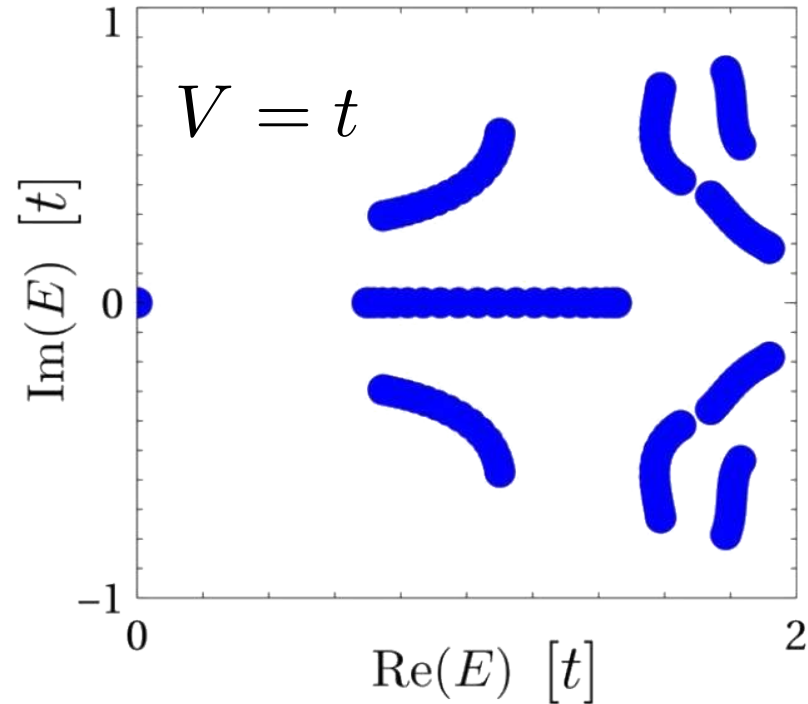
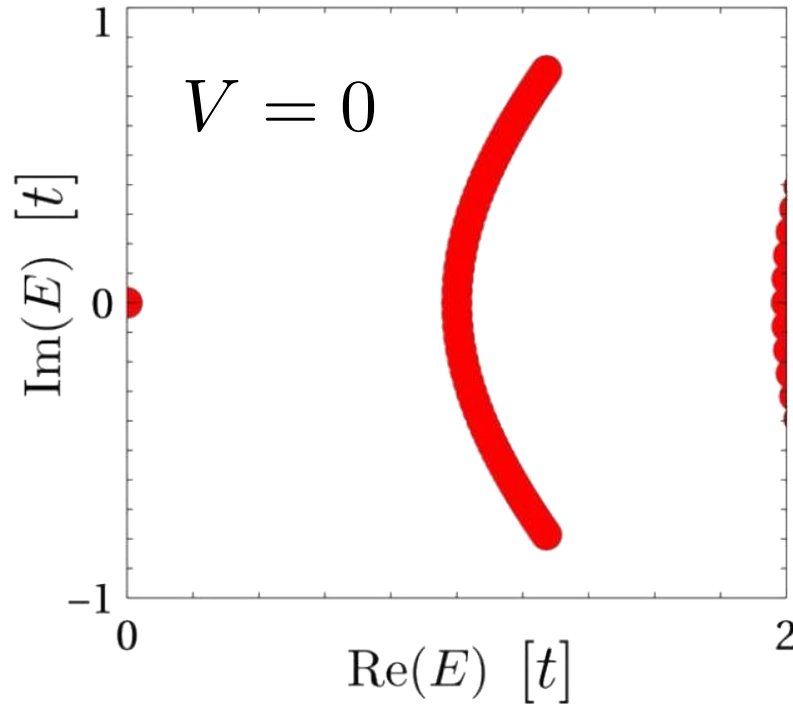
Interacting term

$$\mathcal{H}_I = V \sum_n \left(c_n^\dagger c_n - 1/2 \right) \left(c_{n+1}^\dagger c_{n+1} - 1/2 \right)$$

How do we assess the existence of edge modes in the interacting many-body case?

Adiabatic connection between non interacting and interacting limits

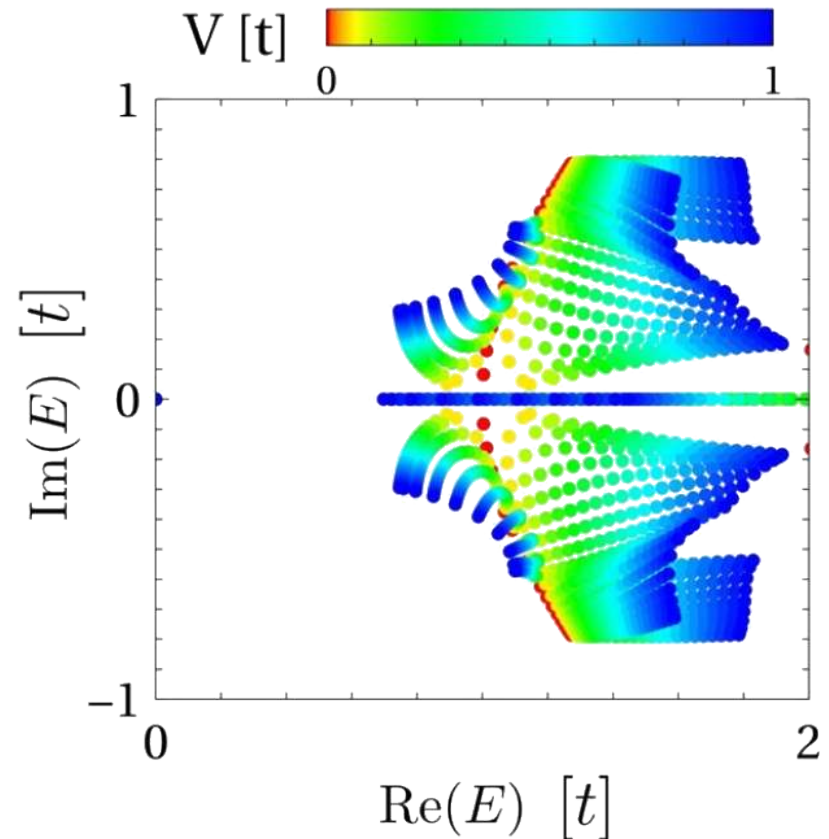
Many-body eigenenergies of the interacting non-Hermitian Hamiltonian



A many-body gap remains open from the non-interacting to the interacting limit

Adiabatic connection between non interacting and interacting limits

As interactions are ramped up, a many-body gap remains open with closed boundary conditions



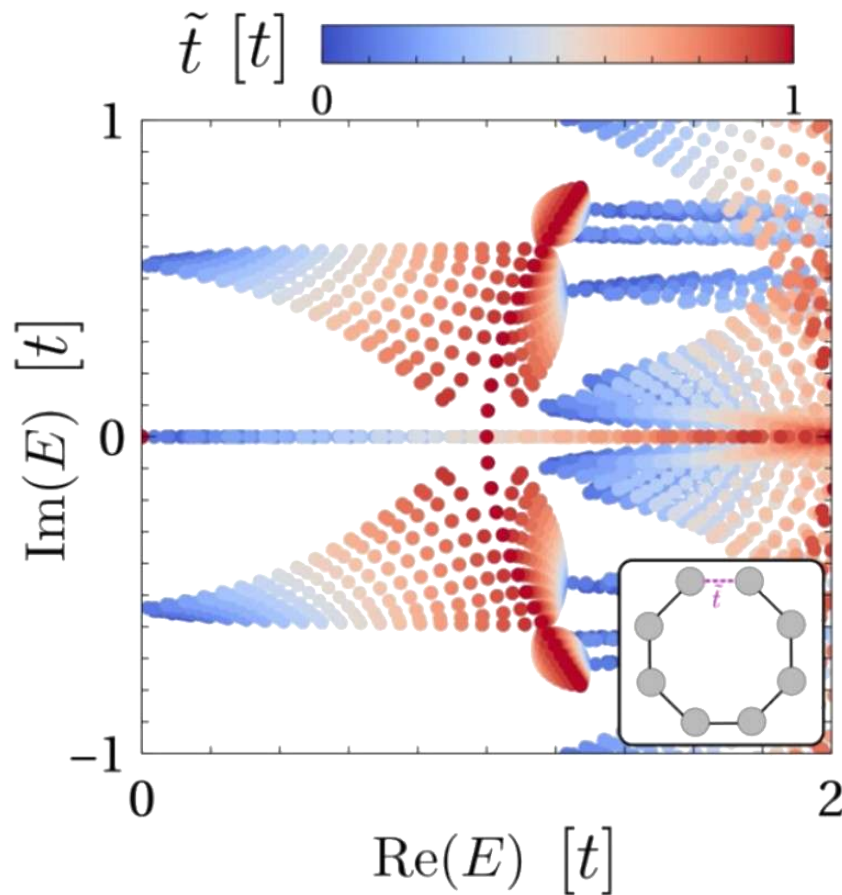
A many-body gap remains open from the non-interacting to the interacting limit

From closed to open boundary conditions

As a bond is disconnected, a topological degeneracy in the real part of the energy emerges

$\tilde{t} = 1$ Closed boundary

$\tilde{t} = 0$ Open boundary



Probing the edge modes with tunneling

The zero bias conductance becomes proportional to

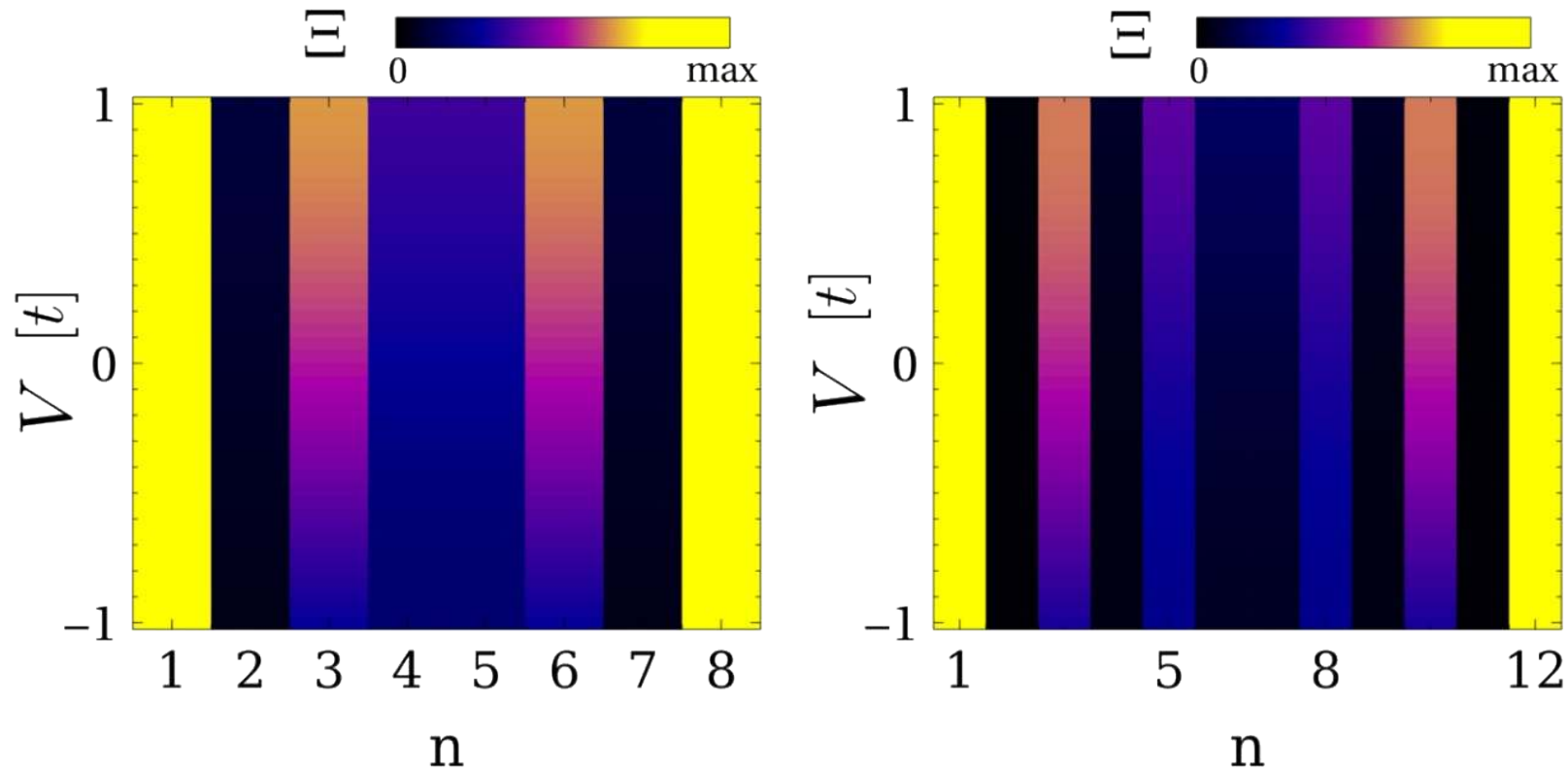
$$\Xi(n) = \sum_i |\langle \Psi_i | c_n | \Omega \rangle|^2$$

Degenerate ground state manifold

Ground state

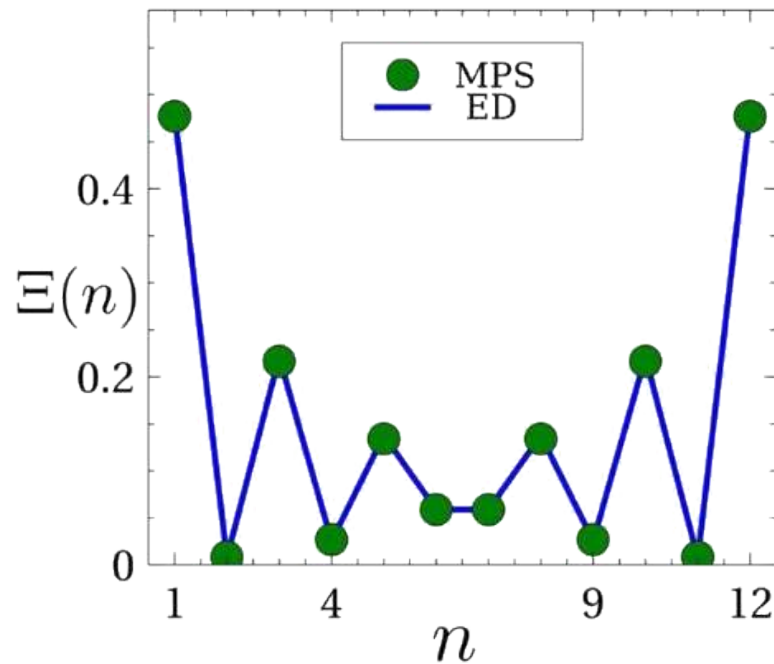
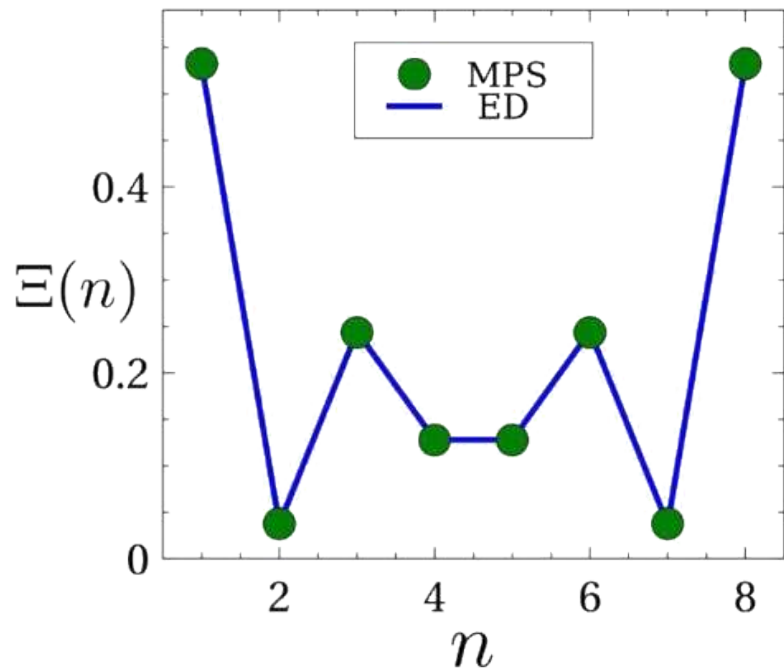
This quantity allow to visualize edge modes in the many-body limit

Edge modes with finite many-body interactions



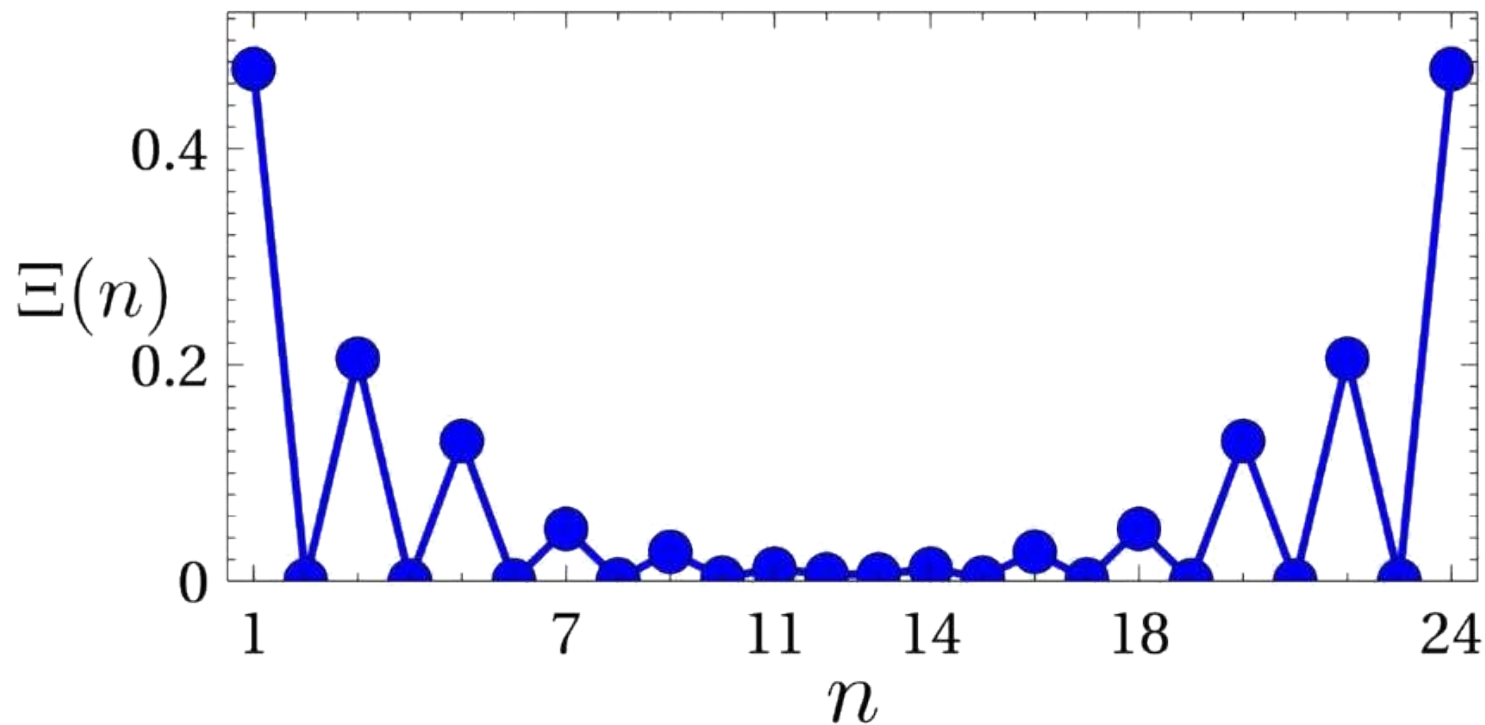
Edge modes remain in the presence of many-body interactions

Non-Hermitian edge modes



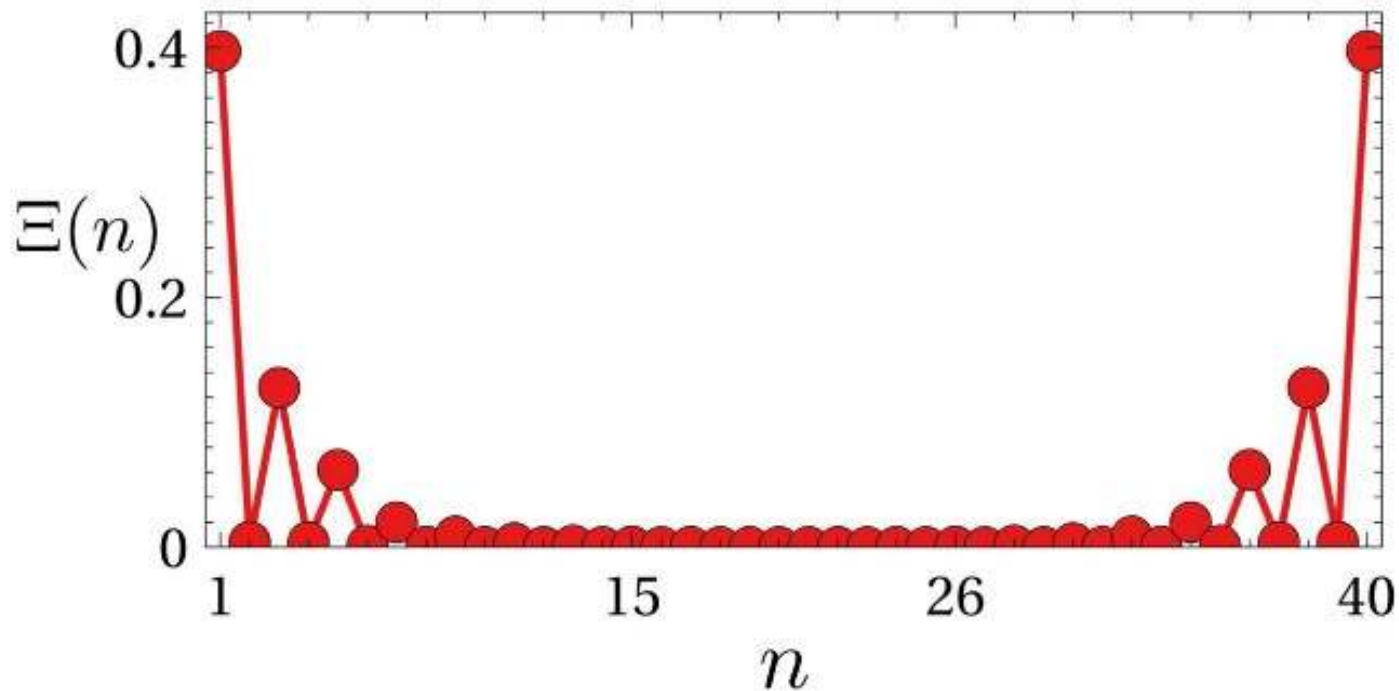
In short chains two overlapping edge modes appear in the correlator

Non-Hermitian edge modes



In long chains solved with MPS, the non-Hermitian mode becomes well localized at the edge

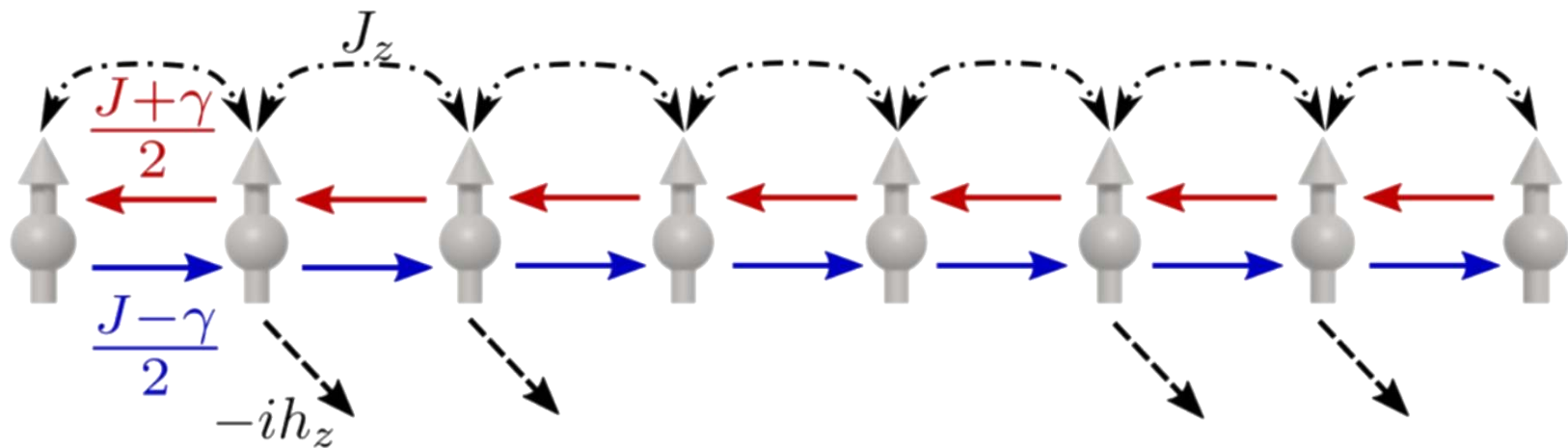
Non-Hermitian edge modes



In long chains solved with MPS, the non-Hermitian mode becomes well localized at the edge

Dynamical correlators in topological interacting non-Hermitian systems

A generic non-Hermitian quantum many-body model



$$\tilde{H} = \sum_{l=1}^{L-1} \left(\frac{J+\gamma}{2} c_l^\dagger c_{l+1} + \frac{J-\gamma}{2} c_{l+1}^\dagger c_l + J_z \left(c_l^\dagger c_l - \frac{1}{2} \right) \left(c_{l+1}^\dagger c_{l+1} - \frac{1}{2} \right) \right) + \sum_{l=1}^L i h_l^z \left(c_l^\dagger c_l - \frac{1}{2} \right)$$

Drives skin effect

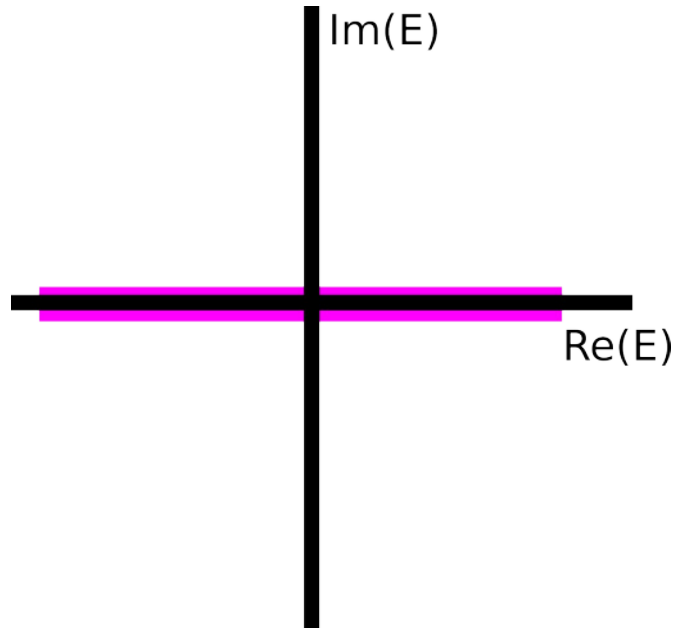
Many-body interaction

Local loss

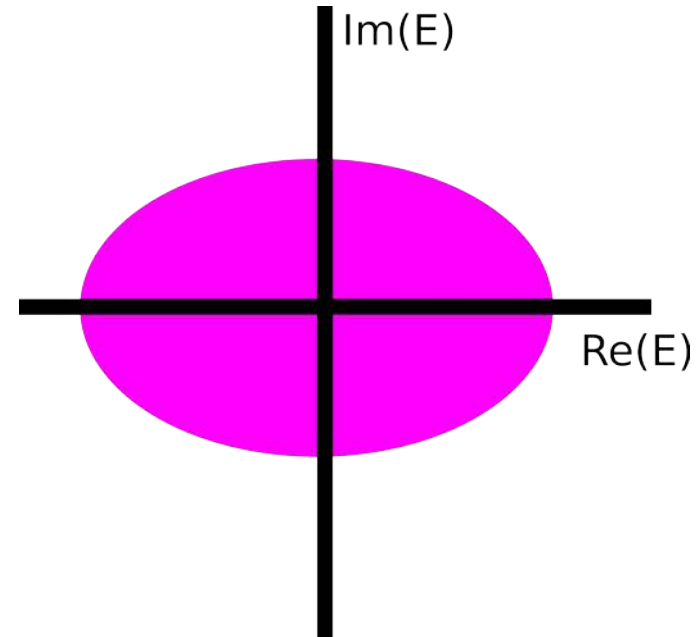
(in the fermionic Jordan Wigner representation)

Dynamical correlators in non-Hermitian models

In the Hermitian case



In the non-Hermitian case



In the non-Hermitian case, dynamical correlators are defined in the complex energy plane

Dynamical correlator of a many-body non-Hermitian model

In a non-Hermitian model, the dynamical correlator cannot be computed with a usual Kernel polynomial expansion

$$S(\omega, l) = \left\langle \text{GS}_L \left| c_l \delta^2 (\omega + E_{\text{GS}} - H) c_l^\dagger \right| \text{GS}_R \right\rangle + \left\langle \text{GS}_L \left| c_l^\dagger \delta^2 (\omega + E_{\text{GS}} - H) c_l \right| \text{GS}_R \right\rangle$$

$$\delta^2(\omega - H) = \sum_n \delta(\Re(\omega - E_n)) \delta(\Im(\omega - E_n)) |\Psi_{R,n}\rangle \langle \Psi_{L,n}|$$

We define a Hermitrized Hamiltonian

$$\mathcal{H} = \begin{pmatrix} & \omega - H \\ \omega^* - H^\dagger & \end{pmatrix}$$

And perform a KPM expansion of

$$f(\omega) = \langle \psi_L | \delta^2(\omega - H) | \psi_R \rangle$$

$$f(\omega) = \frac{1}{\pi} \partial_{\omega^*} G(E=0)$$

$$G(E) = \langle L | (E - \mathcal{H})^{-1} | R \rangle$$

Dynamical correlator of a topological non-Hermitian model

We take the following non-Hermitian model

$$\tilde{H} = \sum_{l=1}^{L-1} \left(\frac{J+\gamma}{2} c_l^\dagger c_{l+1} + \frac{J-\gamma}{2} c_{l+1}^\dagger c_l + J_z \left(c_l^\dagger c_l - \frac{1}{2} \right) \left(c_{l+1}^\dagger c_{l+1} - \frac{1}{2} \right) \right) + \sum_{l=1}^L i h_l^z \left(c_l^\dagger c_l - \frac{1}{2} \right)$$

Drives skin effect

Many-body interaction

Local loss

Full dynamical correlator

$$S(\omega, l) = \left\langle \text{GS}_L \left| c_l \delta^2 (\omega + E_{\text{GS}} - H) c_l^\dagger \right| \text{GS}_R \right\rangle + \left\langle \text{GS}_L \left| c_l^\dagger \delta^2 (\omega + E_{\text{GS}} - H) c_l \right| \text{GS}_R \right\rangle$$

Integrated in the imaginary axis
(equivalent to the Hermitian one)

$$\tilde{\rho}(E \in \mathbb{R}, l) = \left| \int S(E + iy, l) dy \right|$$

Single particle Hermitian VS many-body non-Hermitian dynamical correlators

Single particle Hermitian

Local density of states

$$\rho(\omega, l)$$

Total density of states

$$\rho_{tot}(\omega) = \sum_{l=1}^L \rho(\omega, l)$$

Many-body non-Hermitian

Local dynamical correlator

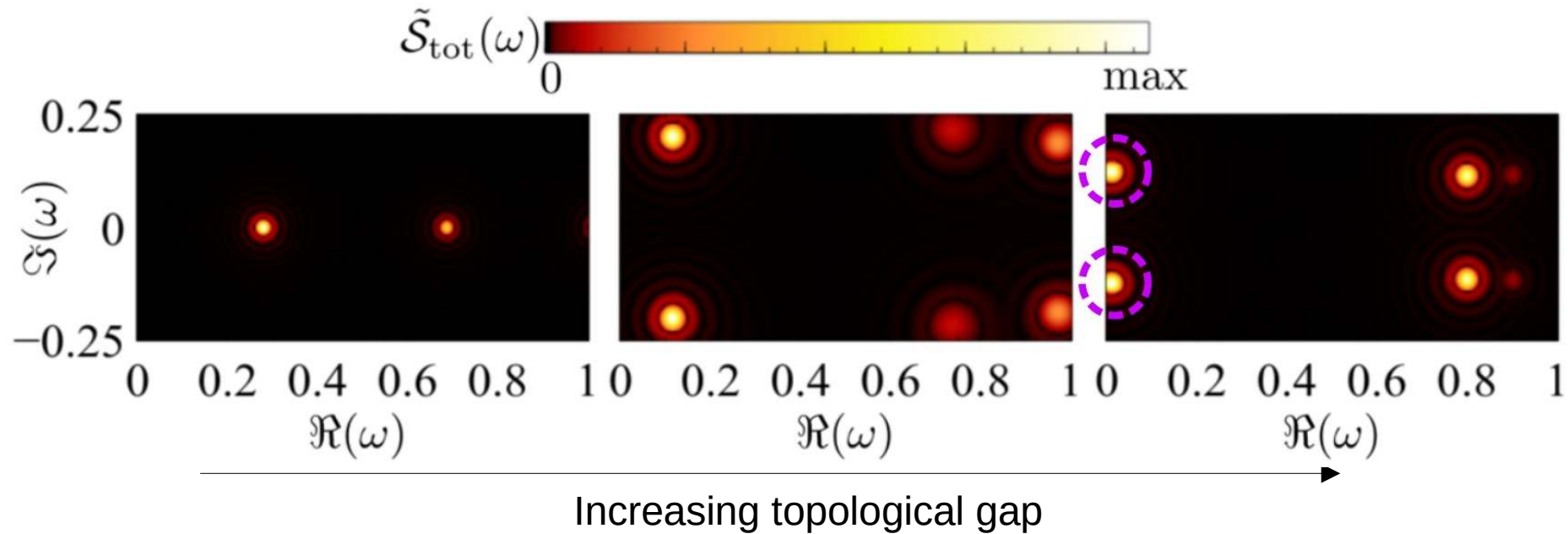
$$\mathcal{S}(\omega, l)$$

Total dynamical correlator

$$\tilde{\mathcal{S}}_{tot}(\omega) = \sum_{l=1}^L \mathcal{S}(\omega, l)$$

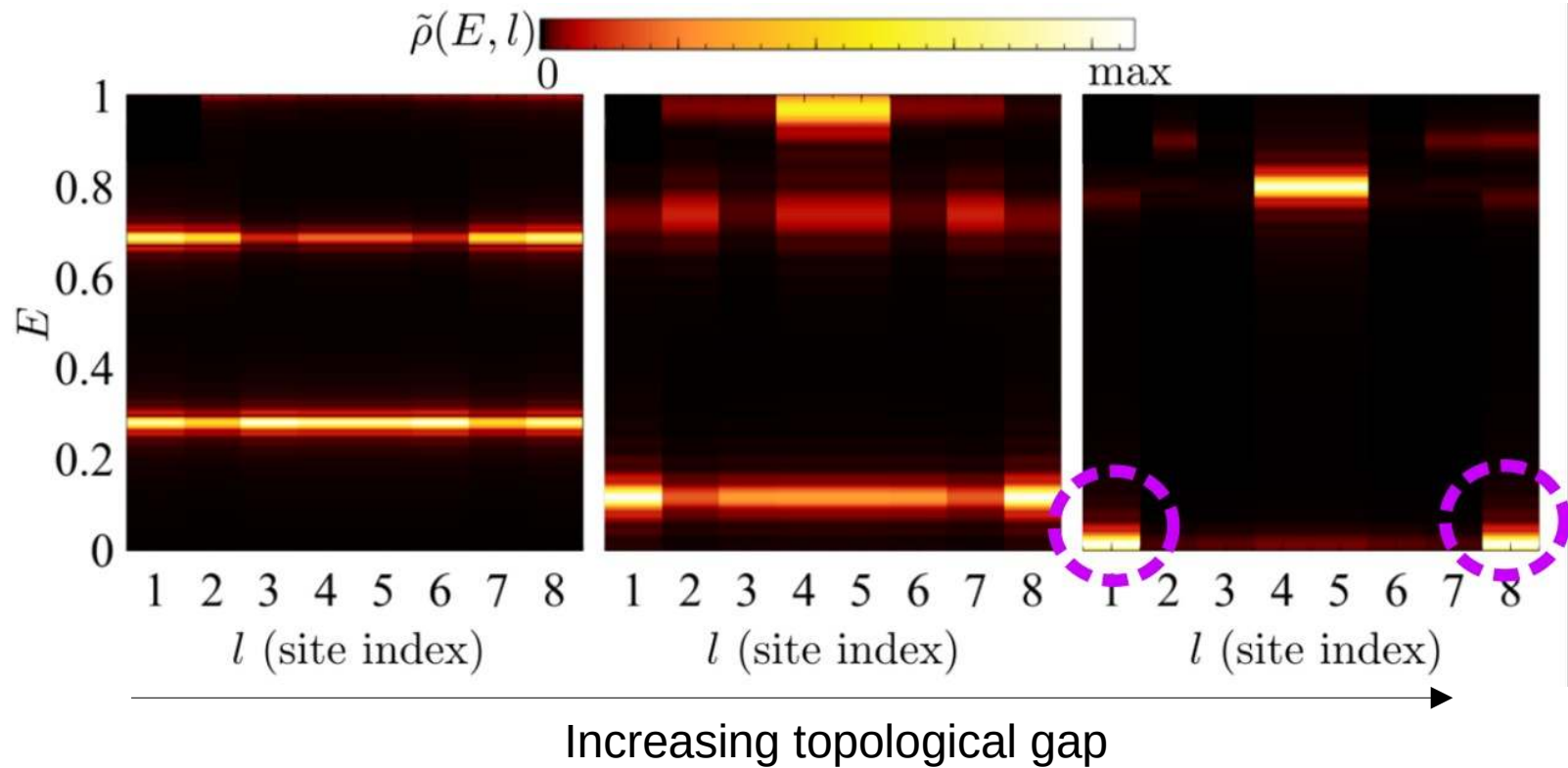
Topological degeneracies in a dynamical correlator

Total structure factor $\tilde{\mathcal{S}}_{\text{tot}}(\omega) = \left| \sum_{l=1}^L \mathcal{S}(\omega, l) \right|$



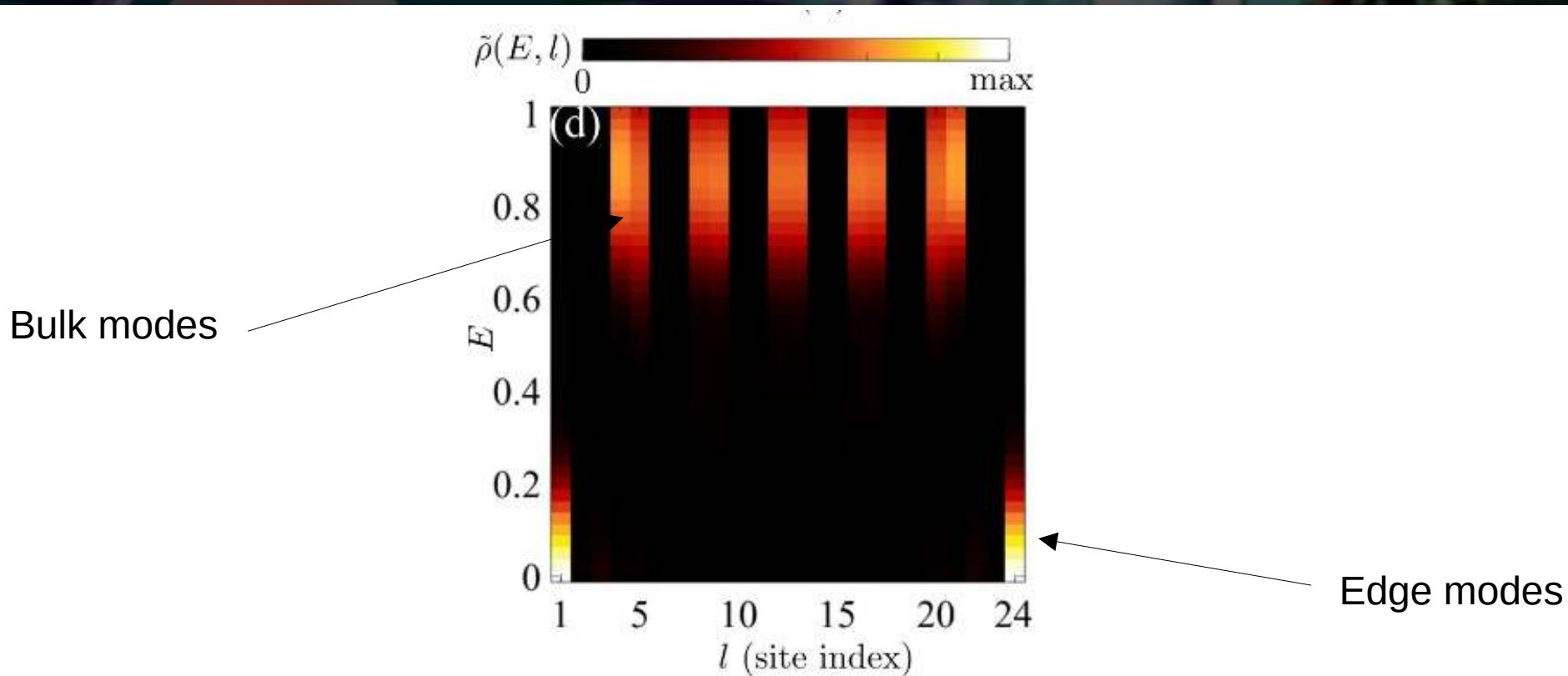
As the topological gap gets bigger, degeneracies at zero real frequency emerge

Topological modes in the local dynamical correlator



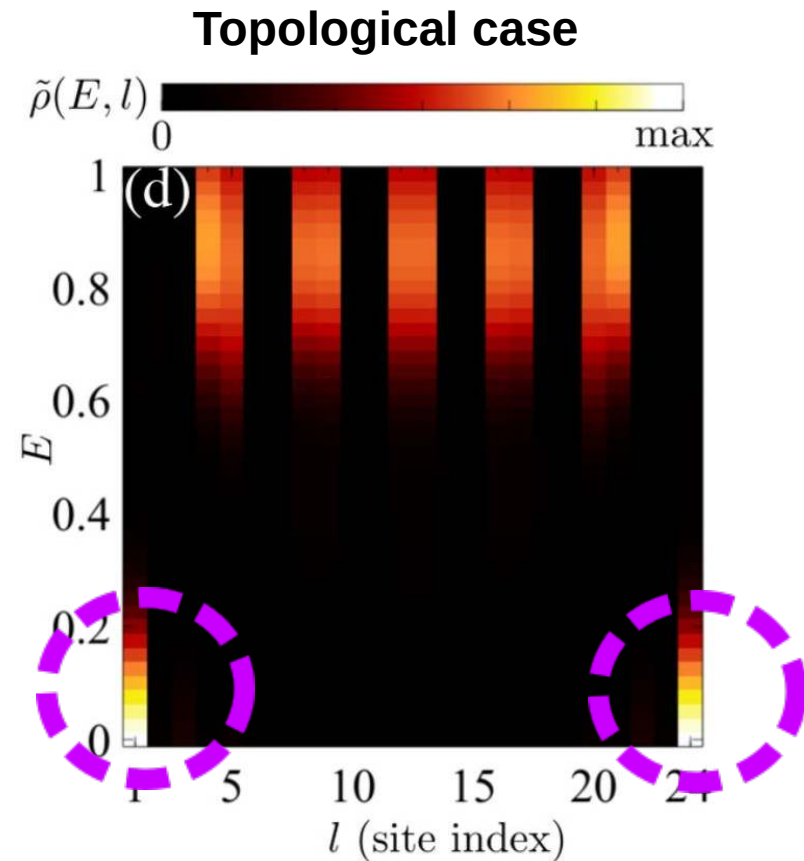
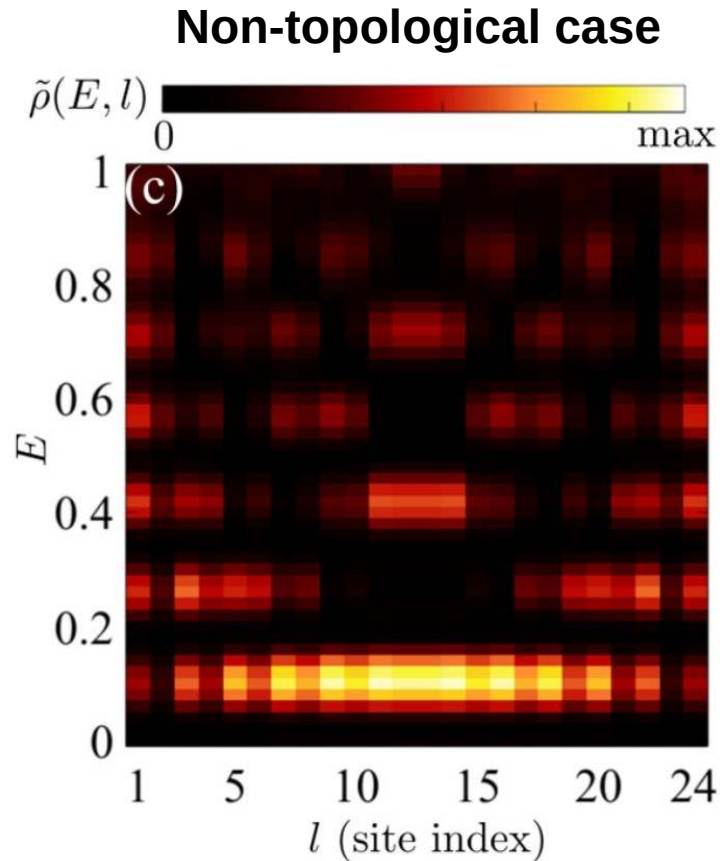
As the topological gap gets bigger, edge modes at zero real frequency emerge

Non-Hermitian topological modes



The non-Hermitian dynamical correlator allows visualizing the topological modes

Trivial VS topological spectral functions



Skin effect and topological modes

In the presence of skin effect, left/right eigenvectors get localized in one edge



With the definition of the dynamical correlator mirror symmetry is recovered

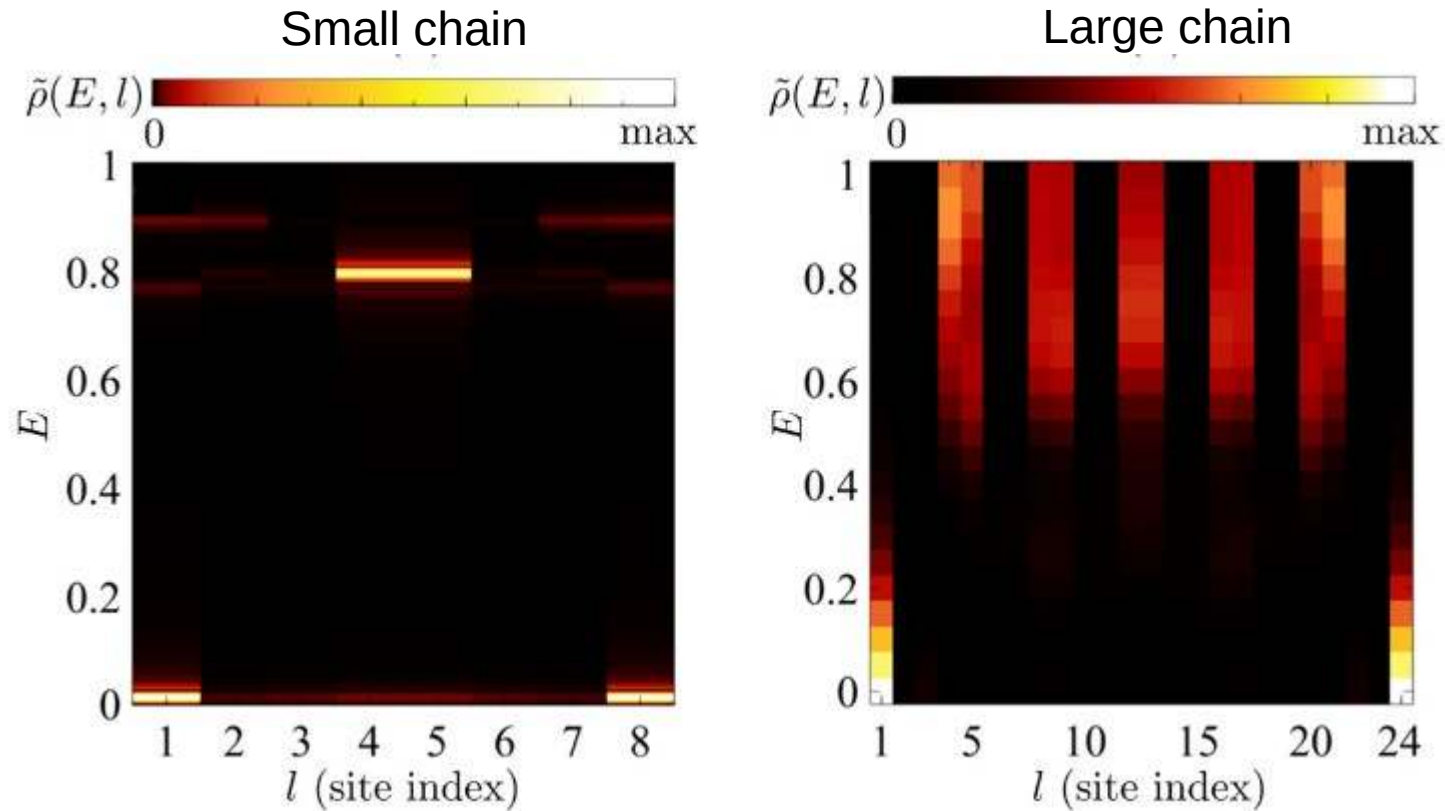
$$\mathcal{S}(\omega, l) \sim \left\langle GS_L \left| c_l \delta^2 (\omega + E_{GS} - H) c_l^\dagger \right| GS_R \right\rangle$$

Right
eigenvector

They localize in opposite edges

Right
eigenvector

Topological modes in the presence of skin effect



The non-Hermitian dynamical correlator allows visualizing the topological modes

Open-source software for
topological matter

Open-source software for tensor-network kernel polynomial algorithms

For general interacting models

dmrgpy

<https://github.com/joselado/dmrgpy>

```
from dmrgpy import spinchain
spins = ["S=1" for i in range(40)] # S=1 chain
sc = spinchain.Spin_Chain(spins) # create spin chain object
h = 0 # initialize Hamiltonian
for i in range(len(spins)-1):
    h = h + sc.Sx[i]*sc.Sx[i+1]
    h = h + sc.Sy[i]*sc.Sy[i+1]
    h = h + sc.Sz[i]*sc.Sz[i+1]
sc.set_hamiltonian(h)
sc.get_dynamical_correlator(name=(sc.Sz[0], sc.Sz[0]))
```

Generic Python library for tensor-network kernel polynomial algorithms for spins, fermions, parafermions, with static solvers for Hermitian and non-Hermitian modes

Specialized non-Hermitian kernel polynomial algorithm

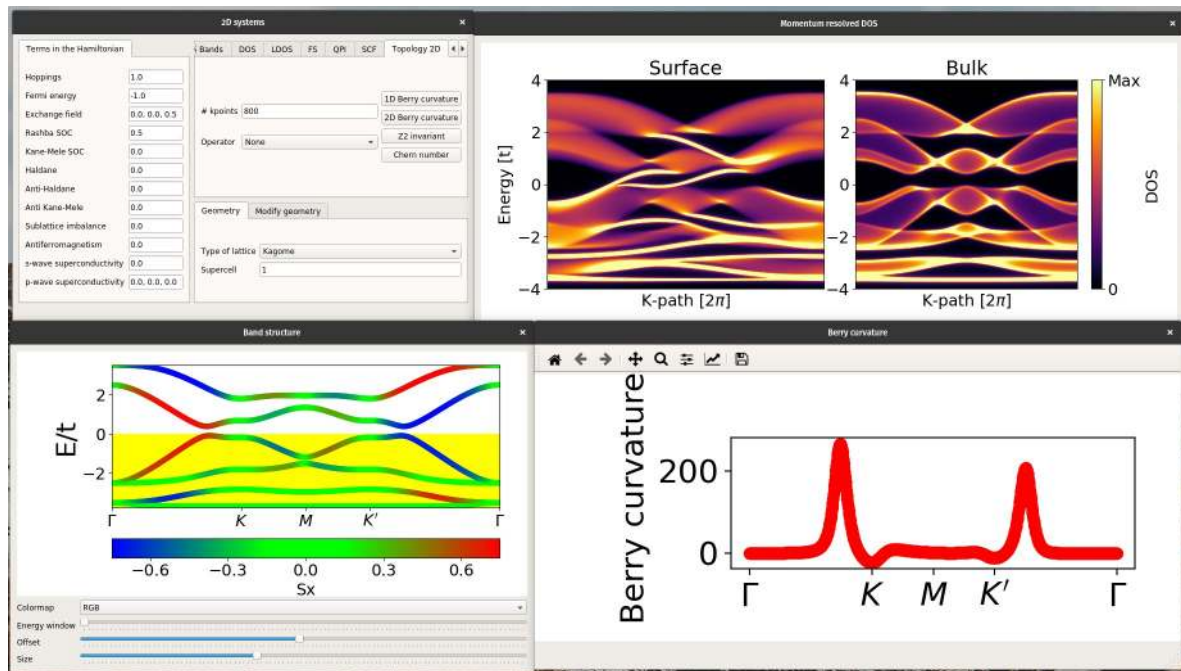
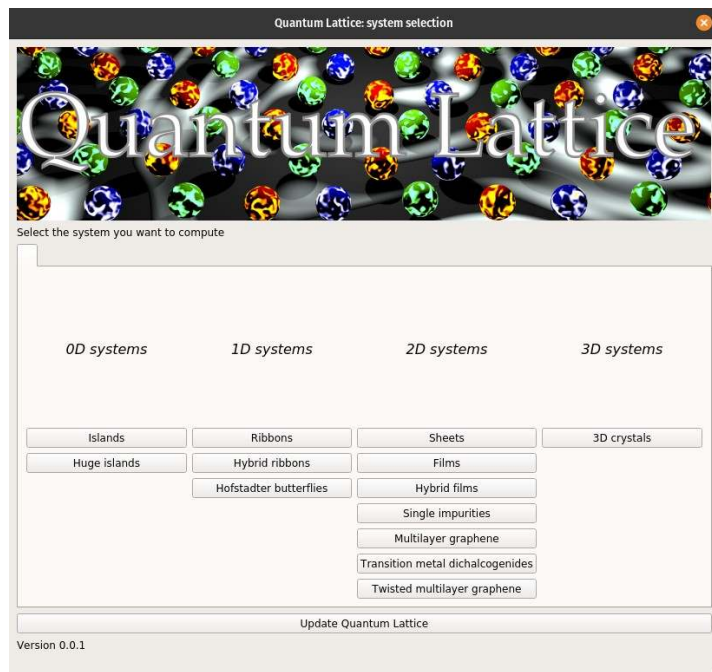
NHKPM.jl

<https://github.com/GUANGZECHEN/NHKPM.jl>

Based on



Quantum Lattice: A user interface to compute electronic properties

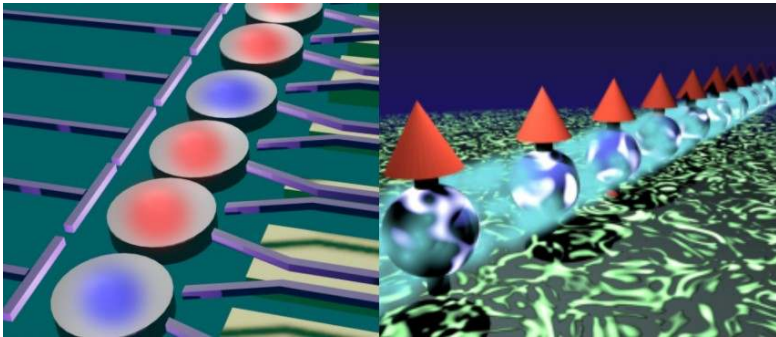


Quantum Lattice: open source interactive interface for tight binding modeling

<https://github.com/joselado/quantum-lattice>

Take home

Tensor-network kernel polynomial algorithms allow computing dynamical excitations in interacting non-Hermitian quantum many-body systems



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Phys. Rev. Lett. 130, 100401 (2023)

Thank you!

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