# Quantum computing topological matter on a superconducting quantum computer

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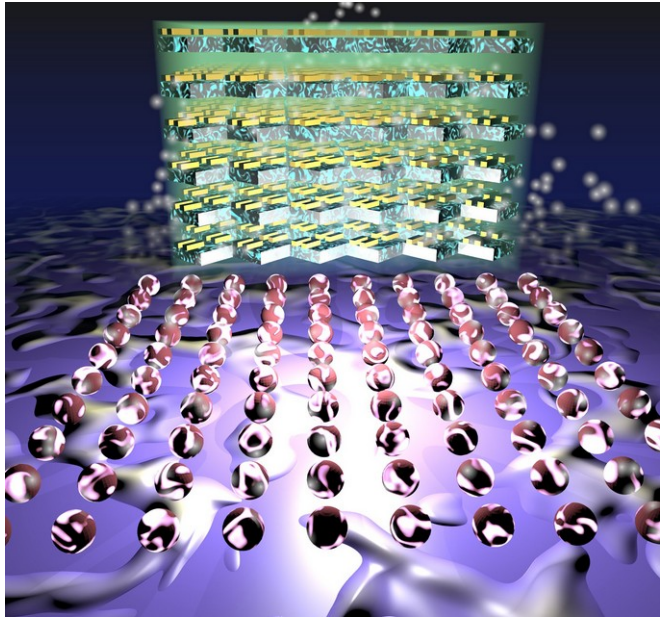
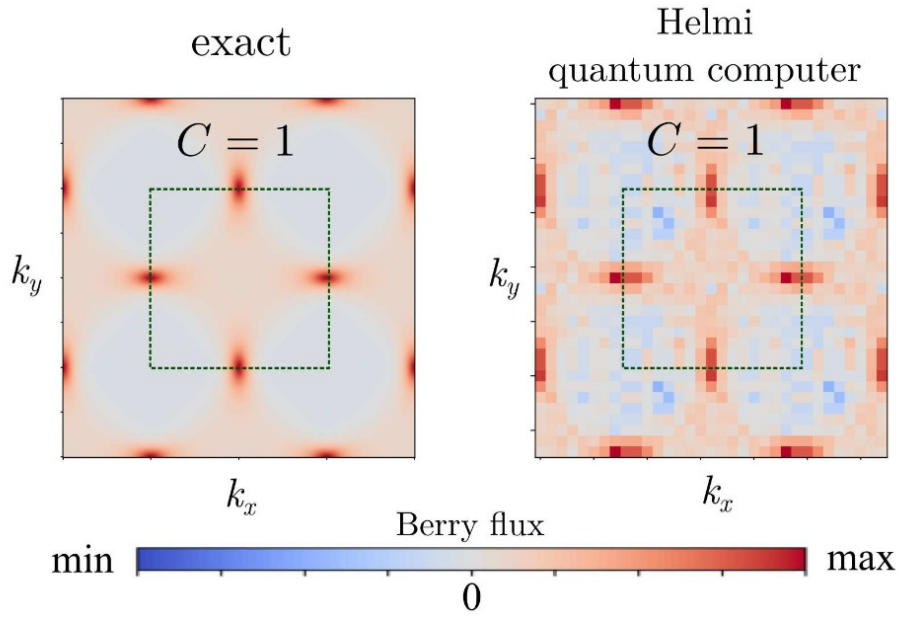


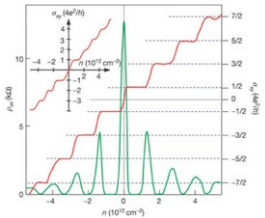



*Phys. Rev. Lett. 130, 100401 (2023), arXiv:2304.01751 (2023), arXiv:2404.06048 (2024)*

Theory seminar, University of Jyväskylä, May 27th 2024

# Topological quantum matter

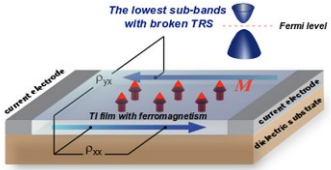
## Quantum Hall effect



graphene

Nature 438, 197-200 (2005)

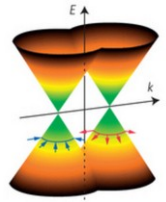
## Quantum Anomalous Hall effect




Cr-doped $(Bi,Sb)_2Te_3$

Science 340.6129 (2013): 167-170

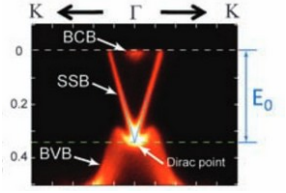
## Weyl semimetal



TaAs

Nature Physics 11, 724–727 (2015)

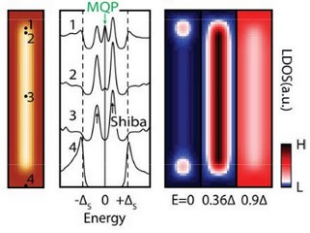
## Quantum Spin Hall effect




$Bi_2Se_3$

Science 325.5937 (2009): 178-181

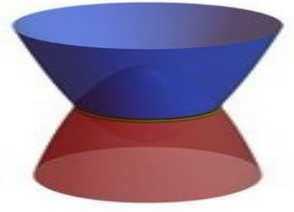
## Topological superconductor




Fe@Pb

Science 346.6209 (2014): 602-607

## Nodal line semimetals



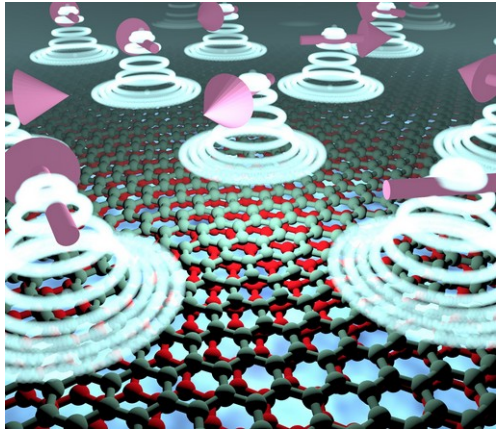
$Mg_3Bi_2$

Advanced Science 6.4 (2019): 1800897

***All these states are described by effective isolated single-particle Hamiltonians***

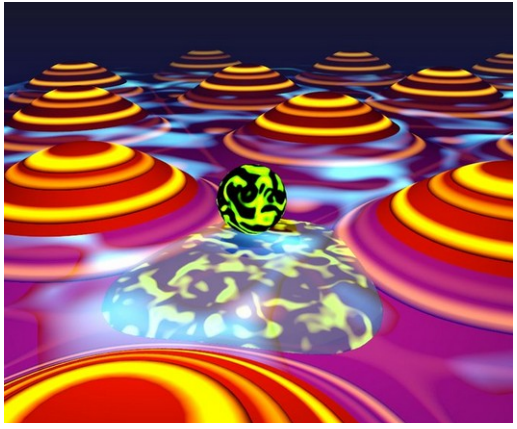
# Many-body topological quantum matter

**Quantum spin liquids**



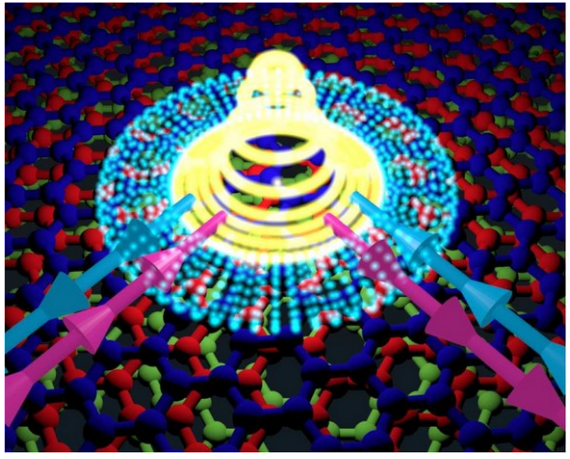
*Reports on Progress in Physics 80.1 (2016): 016502.*

**Fractional Chern insulators**



*Rev. Mod. Phys. 71, S298 (1999)*

**Parafermions**



*Annual Review of Condensed Matter Physics 7, 119-139 (2016)*

***Topological many-body quantum matter is an open challenge where quantum algorithms will potentially enable major breakthroughs***

# The computational challenge of quantum many-body systems

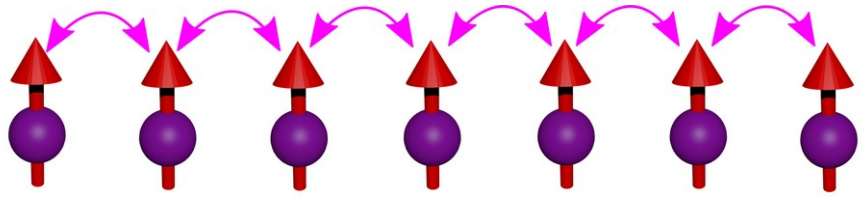
# The problem of dimensionality

In a many-body problem, the size of our vectors grows as

$$2^L$$

where L is the number of sites

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$



For a single-particle tight binding problem, we can reach up to $10^8$ sites in a laptop

For a many-problem, we cannot even store states for systems bigger than $L = 30$ sites

$$2^{30} \sim 10^9$$

# The quantum many-body problem

Let us go back to a simple many-body problem $\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$

A typical wavefunction is written as

$$|\Psi\rangle = \sum c_{s_1, s_2, \ldots, s_L} |s_1, s_2, \ldots s_L\rangle$$

We need to determine in total $2^L$ coefficients

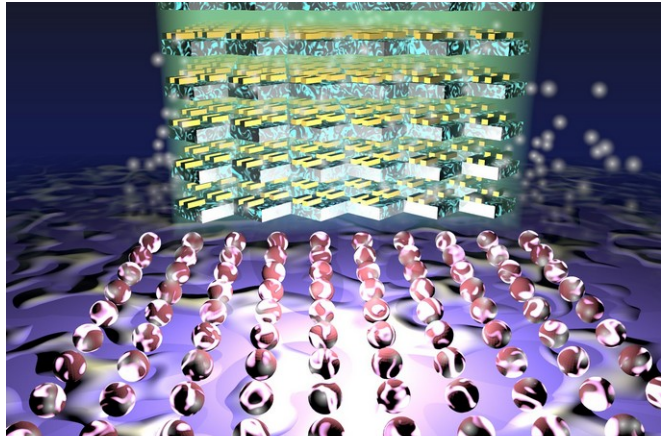
**Is there an efficient way of storing so many coefficients?**

# The fundamental idea of tensor-networks

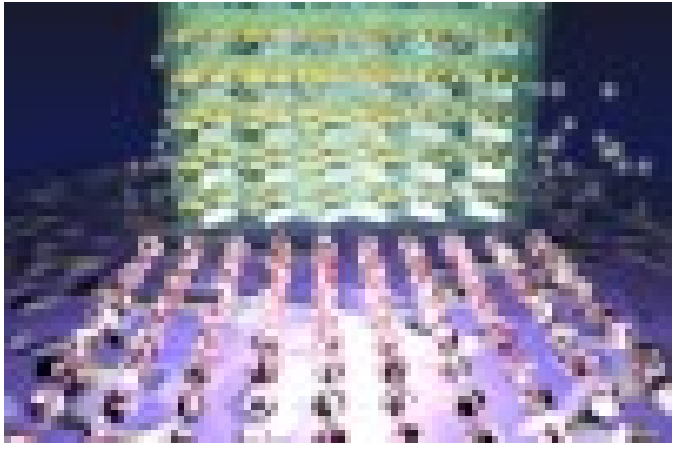
A many-body wavefunction a is a very high dimensional object

$$|\Psi\rangle = \sum c_{s_1,s_2,\ldots,s_L} |s_1, s_2, \ldots s_L\rangle$$

Tensor-networks allow “compressing” all that information in a very efficient way



“True wavefunction”



“Tensor-network wavefunction”

# The matrix-product state ansatz

For this wavefunction $|\Psi\rangle = \sum c_{s_1,s_2,\ldots,s_L} |s_1, s_2, \ldots s_L\rangle$

Let us imagine to propose a parametrization in this form

$$c_{s_1,s_2,\ldots,s_L} = M_1^{s_1} M_2^{s_2} \ldots . M_L^{s_3}$$

dimension $2^L$

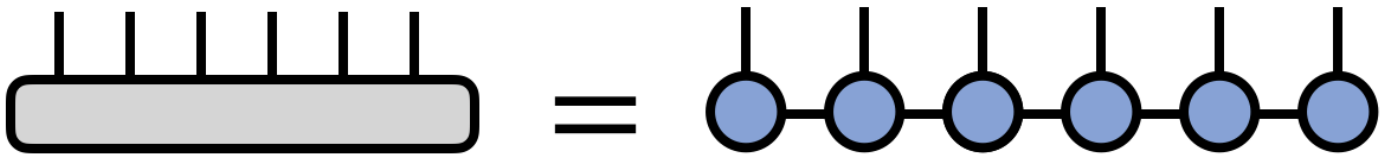
dimension $\sim Lm^2$

(m dimension of the matrix)

# State compression with tensor-networks

Given a many-body wavefunction, we can parametrice the components as

$$|\Psi\rangle = \sum_{\{s\}} \mathrm{Tr}\left[M_1^{(s_1)} M_2^{(s_2)} \cdots M_N^{(s_N)}\right] |s_1 s_2 \ldots s_N\rangle$$

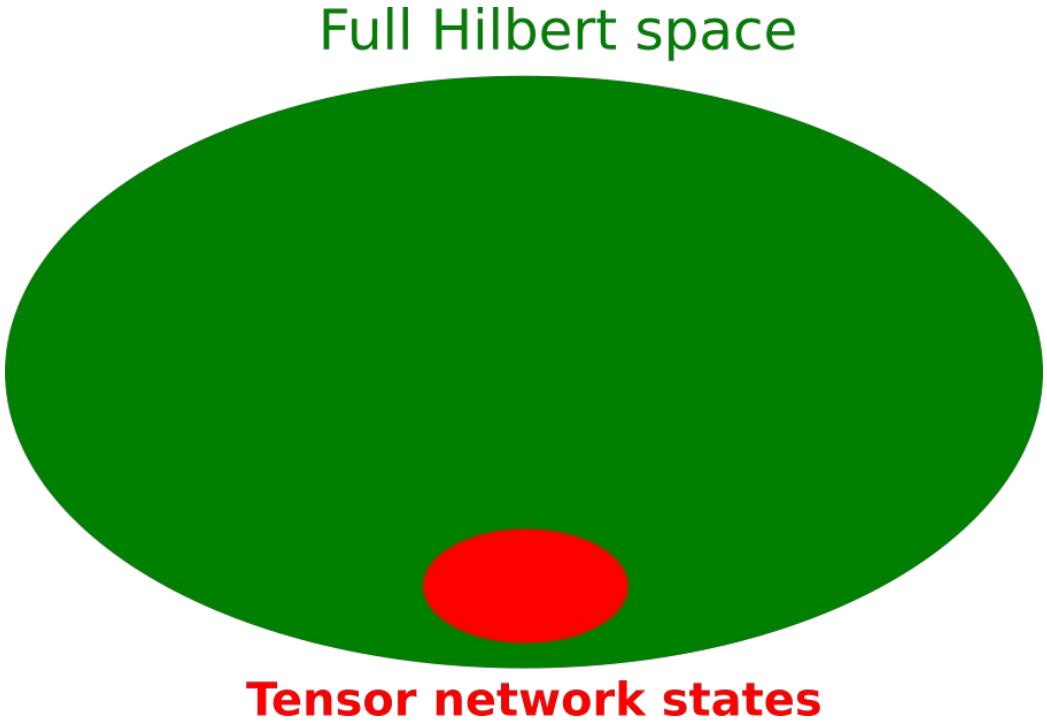


*Matrix product state*

The previous representation allows drastically reducing the memory required to store a state

*Annals of Physics 326, 96 (2011)*

# MPS as a parametrization of area law states



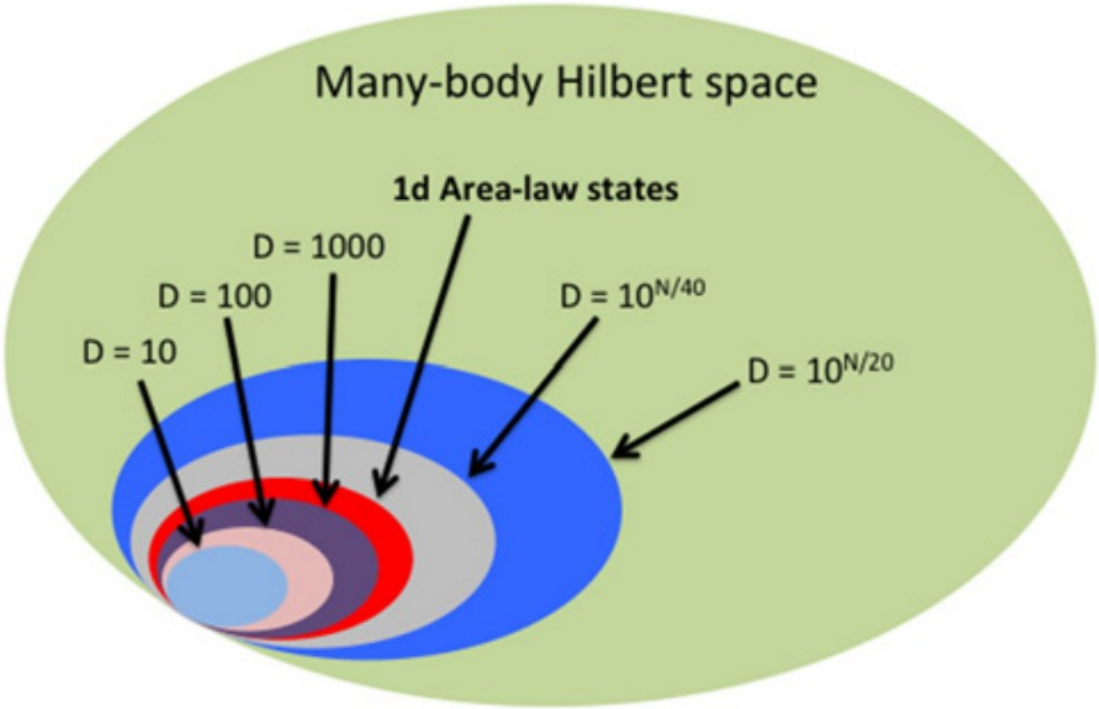

$$c_{s_1,s_2,\ldots,s_L} = M_1^{s_1} M_2^{s_2} \ldots M_L^{s_3}$$

MPS have an entanglement entropy bounded by the bond dimension

$$S \sim \log(m)$$

# A controlled way of parametrizing the Hilbert space

Sketch of the space parametrized with bond dimension D



# The matrix-product state ansatz

- This ansatz enforces a maximum amount of entanglement entropy in the state $S \sim \log m$
- One-dimensional problems have ground states that can be captured with this ansatz

$$c_{s_1,s_2,\ldots,s_L} = M_1^{s_1} M_2^{s_2} \ldots M_L^{s_3}$$

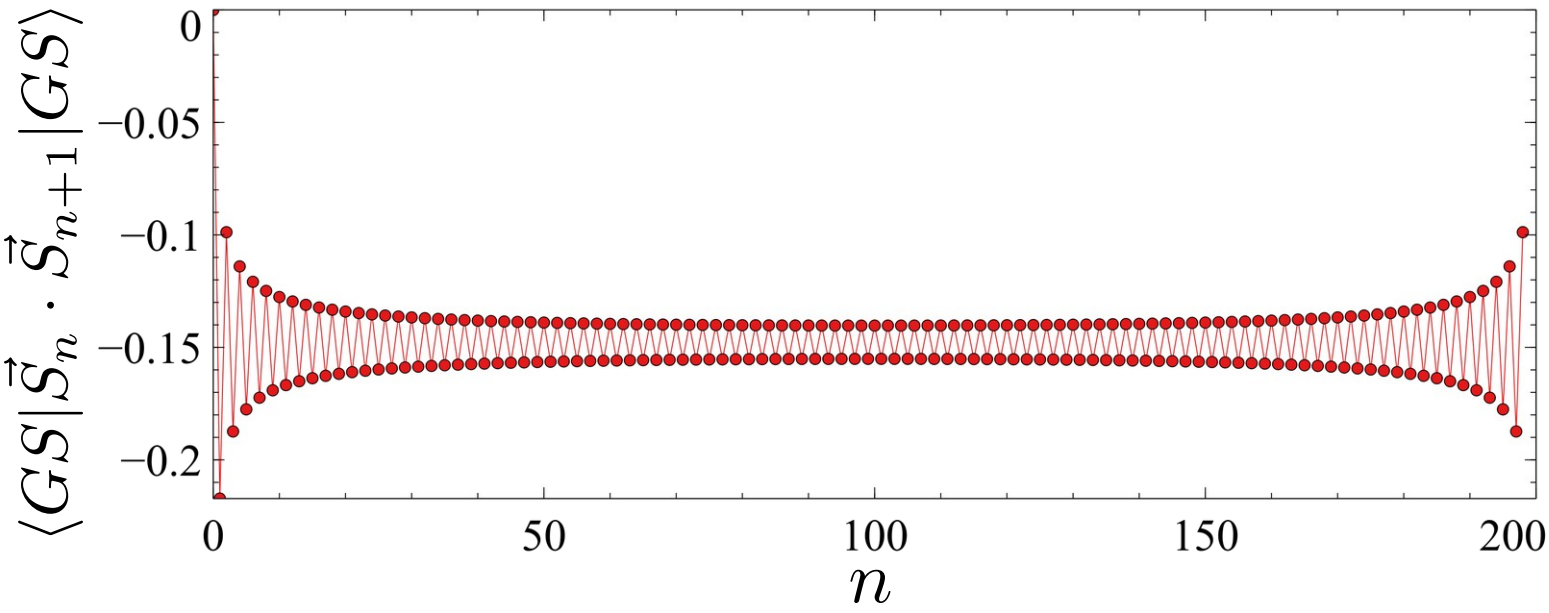
This ansatz can be generalized for time-evolution, excited states, or typical thermal states

# The Heisenberg model with tensor-networks

***Non-uniform Heisenberg model***

$$\mathcal{H} = \sum_n J(n)\vec{S}_n \cdot \vec{S}_{n+1}$$

$$J(n) = J_0 + \delta \cos \Omega n$$

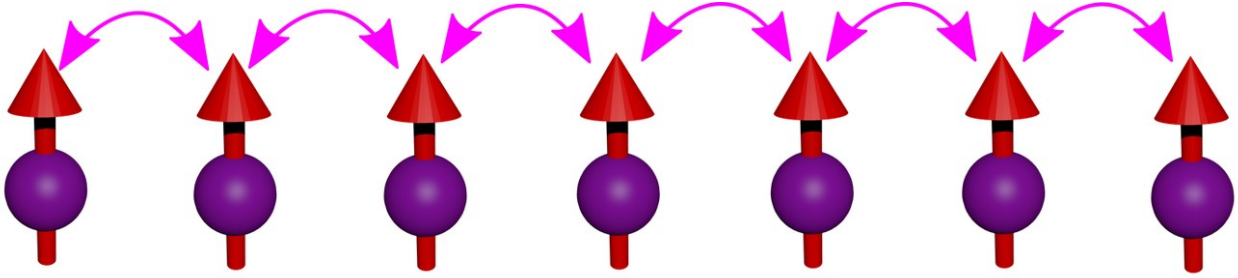



Tensor networks allow solving a 200 many-body spin model in a few seconds in a laptop

# Many-body dynamical correlators

***One dimensional Heisenberg Hamiltonian***

$$\mathcal{H} = \sum_{\langle ij \rangle} J \vec{S}_i \cdot \vec{S}_j$$

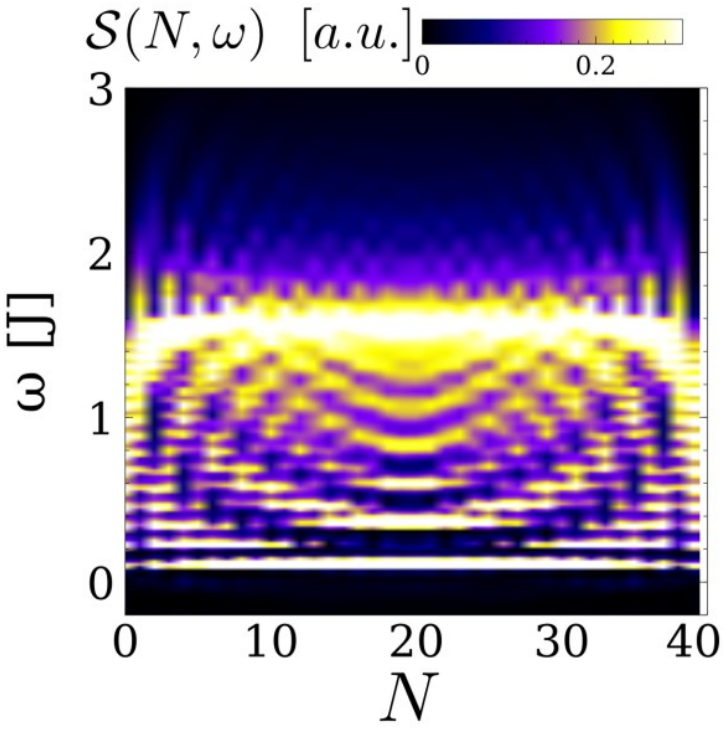


**Tensor networks allow computing dynamical correlators**

$$\mathcal{S}(N,\omega) = \langle GS | S^z_N \delta(\omega - \mathcal{H} + E_0) S^z_N | GS \rangle$$

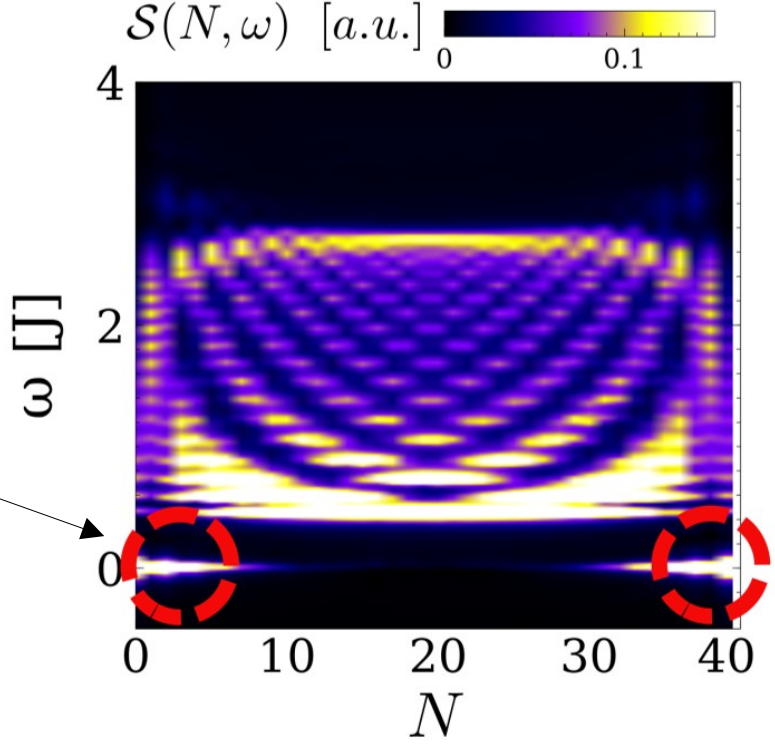
# Dynamical structure factor of a Heisenberg model

**S=1/2 chain**




Edge modes

**S=1 chain**



# Open-source software for tensor-network many-body algorithms

**dmrgpy**

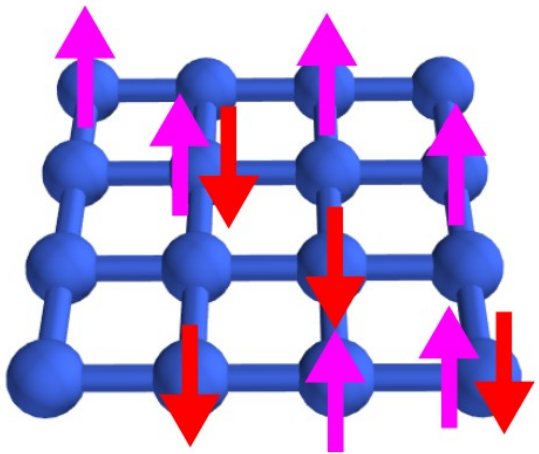
```
from dmrgpy import spinchain
spins = ["S=1" for i in range(40)] # S=1 chain
sc = spinchain.Spin_Chain(spins) # create spin chain object
h = 0 # initialize Hamiltonian
for i in range(len(spins)-1):
  h = h + sc.Sx[i]*sc.Sx[i+1]
  h = h + sc.Sy[i]*sc.Sy[i+1]
  h = h + sc.Sz[i]*sc.Sz[i+1]
sc.set_hamiltonian(h)
sc.get_dynamical_correlator(name=(sc.Sz[0],sc.Sz[0]))
```

*Generic Python library for tensor-network kernel polynomial algorithms for spins, fermions, parafermions, with static and dynamical solvers*

***https://github.com/joselado/dmrgpy***

# Some paradigmatic problems solved with matrix product states

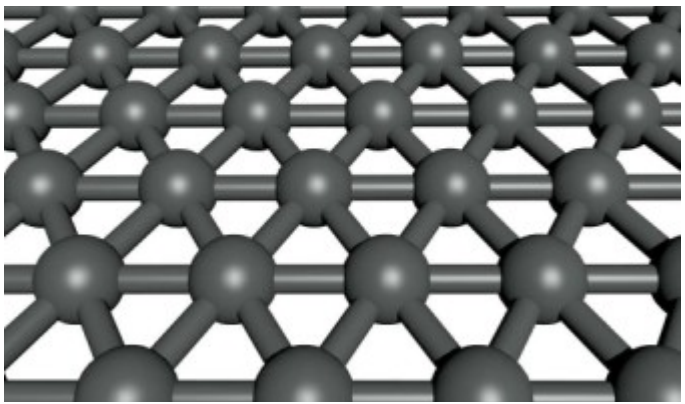
***Solving the 2D Hubbard model at finite doping***



$$H = \sum_{ij,s} t_{ij} c^{\dagger}_{is} c_{js} + \sum_{i} U c^{\dagger}_{i\uparrow} c_{i\uparrow} c^{\dagger}_{i\downarrow} c_{i\downarrow}$$

*Science, 365(6460), 1424-1428 (2019)*

***Solving the 2D Heisenberg model in frustrated lattices***

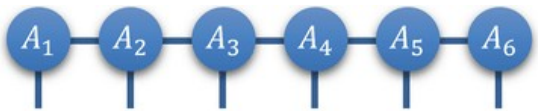


$$\mathcal{H} = \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

*Phys. Rev. Lett. 123, 207203 (2019)*

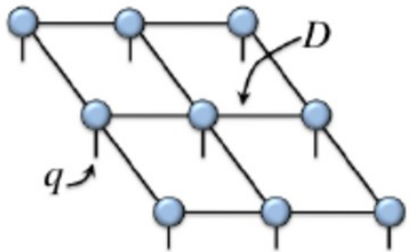
# Many-body state compression

**Matrix-product states**



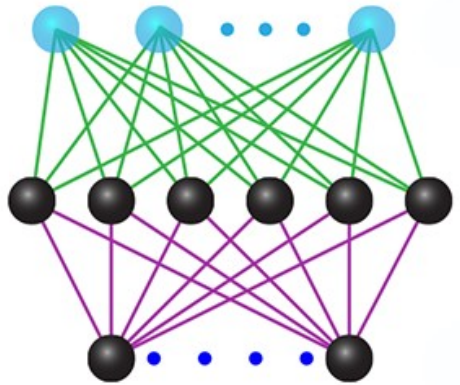

*Phys. Rev. Lett. 69, 2863 (1992)*

**Projected entangled pair-states**




*Annals of Physics 326, 96 (2011)*

**Neural-network quantum states**

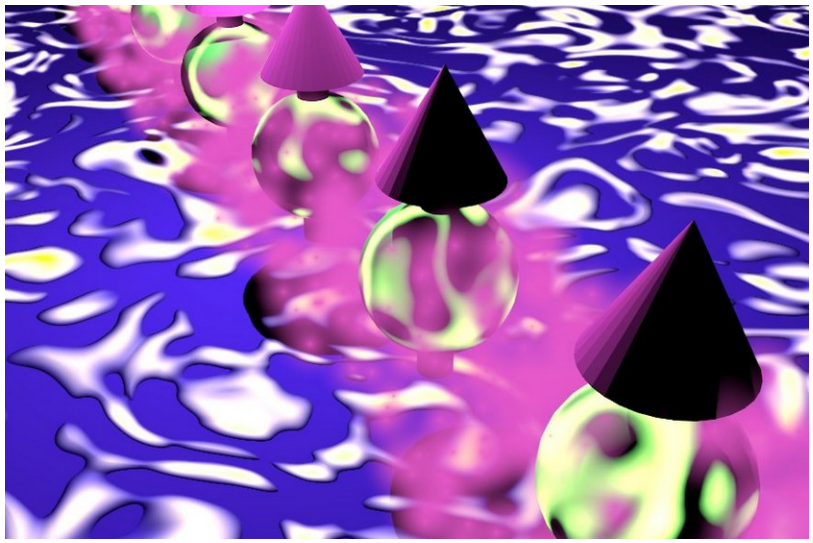


*Science 355.6325 (2017): 602-606.*

***Other compressed many-body states could be potentially used for noisy quantum circuit simulation***

# Two major challenges in quantum matter

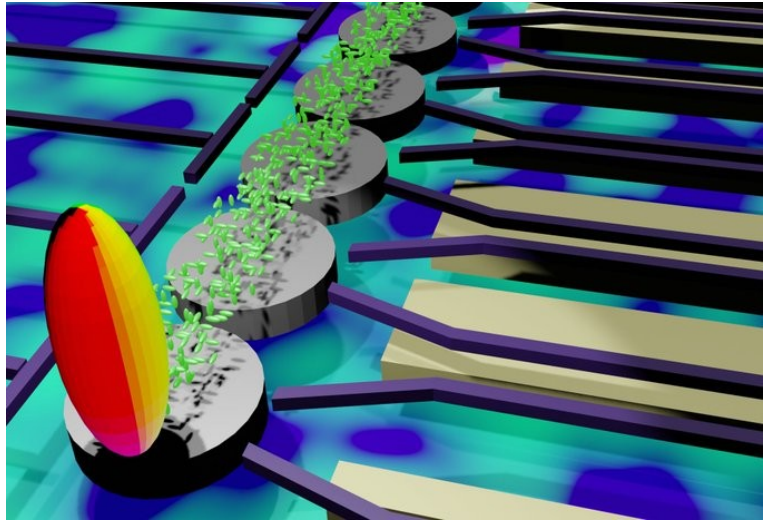
**Many-body interactions**



*Interacting quantum matter*

*Rep. Prog. Phys. 81 116501 (2018)*

**Coupling to the environment**



*Non-Hermitian quantum matter*

*Rev. Mod. Phys. 93, 015005 (2021)*

Currently available quantum computers bring together
quantum-many-body interactions and coupling to an environment

# Tensor-networks and entanglement

- How accurately can tensor-networks capture systems beyond Hermitian (closed) Hamiltonians?

- How accurately can tensor-networks capture systems with finite entanglement, as in currently available quantum computers?

# Plan for today

- Tensor network for (lossy) non-Hermitian quantum many-body Hamiltonians

  *Phys. Rev. Lett. 130, 100401 (2023)*

- Emulating quantum circuits with tensor-networks

  *arXiv:2304.01751 (2023)*

- Computing topological invariants in a superconducting quantum computer

  *arXiv:2404.06048 (2024)*

# Behind the scenes

***Non-Hermitian many-body Hamiltonians with tensor-networks***

Fei Song

Guangze Chen

Phys. Rev. Lett. 130, 100401 (2023)

***Quantum circuits for topological physics with tensor-networks and quantum computers***

Marcel Niedermeier

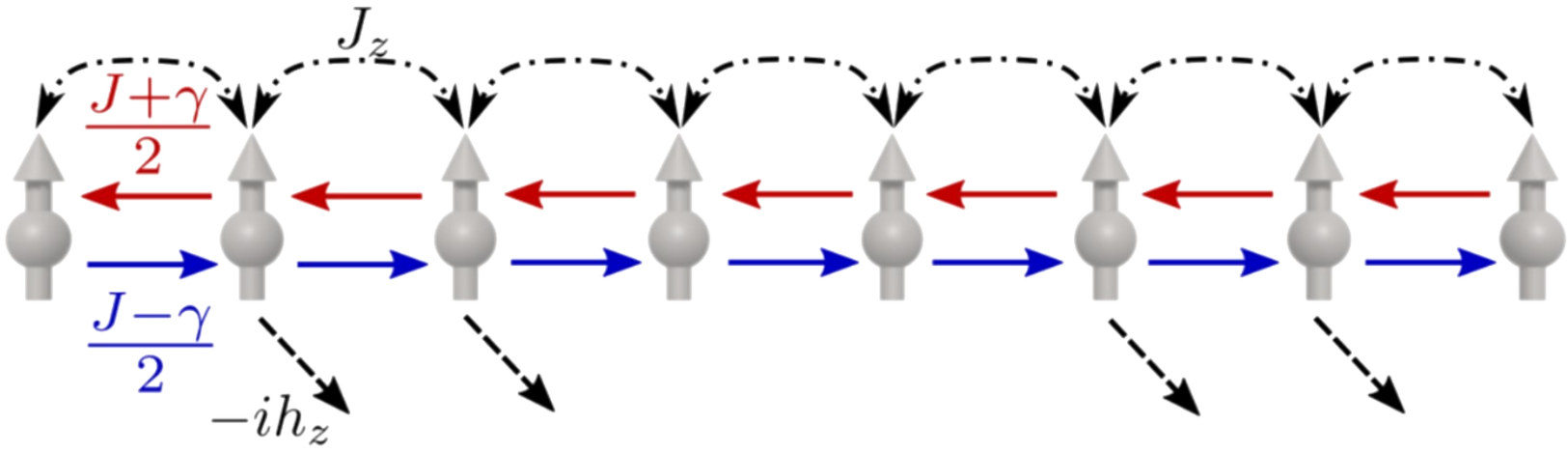
Marc Nairn

Christian Flindt

arXiv:2304.01751 (2023)
arXiv:2404.06048 (2024)

# Tensor-networks for many-body non-Hermitian models

# A generic non-Hermitian quantum many-body model




$$\tilde{H} = \sum_{l=1}^{L-1} \left( \frac{J+\gamma}{2} c_l^\dagger c_{l+1} + \frac{J-\gamma}{2} c_{l+1}^\dagger c_l + J_z \left( c_l^\dagger c_l - \frac{1}{2} \right) \left( c_{l+1}^\dagger c_{l+1} - \frac{1}{2} \right) \right) + \sum_{l=1}^{L} i h_l^z \left( c_l^\dagger c_l - \frac{1}{2} \right)$$

Drives skin effect

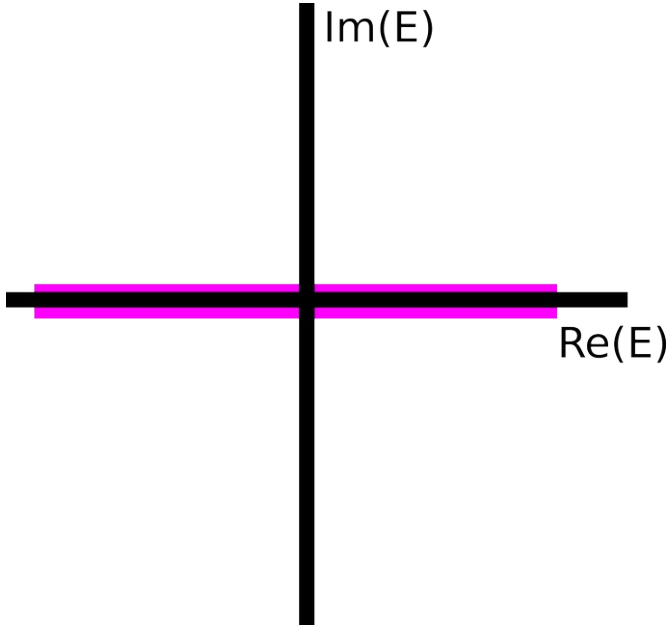
Many-body interaction

Local loss

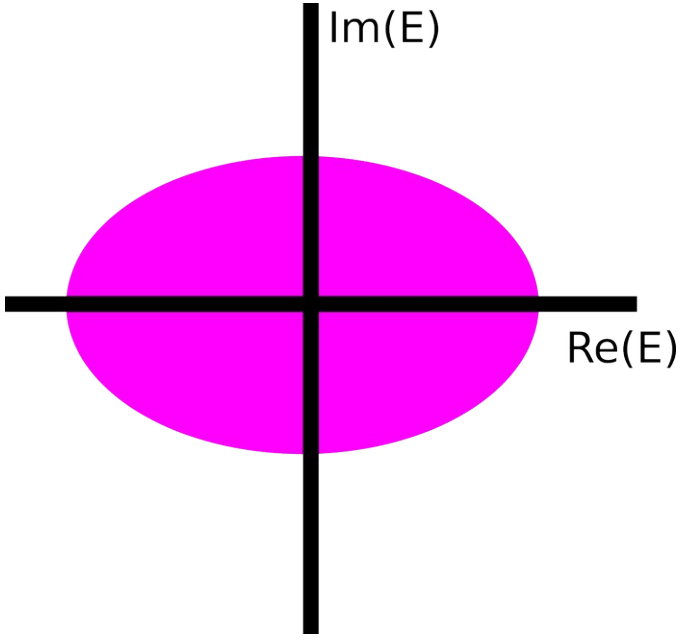
(in the fermionic Jordan Wigner representation)

# Dynamical correlators in non-Hermitian models

In the Hermitian case




In the non-Hermitian case




In the non-Hermitian case, dynamical correlators are defined in the complex energy plane

# Dynamical correlator of a many-body non-Hermitian model

Dynamical correlator of a non-Hermitian quantum many-body Hamiltonian

$$\mathrm{S}(\omega, l) = \left\langle \mathrm{GS_L} \left| c_l \delta^2 \left(\omega + E_{\mathrm{GS}} - H\right) c_l^\dagger \right| \mathrm{GS_R} \right\rangle + \left\langle \mathrm{GS_L} \left| c_l^\dagger \delta^2 \left(\omega + E_{\mathrm{GS}} - H\right) c_l \right| \mathrm{GS_R} \right\rangle$$

$$\delta^2(\omega - H) = \sum_n \delta(\Re(\omega - E_n))\, \delta\left(\Im\left(\omega - E_n\right)\right) |\Psi_{\mathrm{R},n}\rangle \langle \Psi_{\mathrm{L},n}|$$

We define a Hermitrized Hamiltonian

$$\mathcal{H} = \begin{pmatrix} & \omega - H \\ \omega^* - H^\dagger & \end{pmatrix}$$

And perform a KPM expansion of

$$f(\omega) = \langle \psi_{\mathrm{L}} | \delta^2(\omega - H) | \psi_{\mathrm{R}} \rangle$$

$$f(\omega) = \frac{1}{\pi} \partial_{\omega^*} G(E = 0)$$

$$G(E) = \langle \mathrm{L} | (E - \mathcal{H})^{-1} | \mathrm{R} \rangle$$

*Phys. Rev. Lett. 130, 100401 (2023)*

# Dynamical correlator of a topological non-Hermitian model

**We take the following non-Hermitian model**

$$\tilde{H} = \sum_{l=1}^{L-1} \left( \frac{J+\gamma}{2} c_l^\dagger c_{l+1} + \frac{J-\gamma}{2} c_{l+1}^\dagger c_l + J_z \left( c_l^\dagger c_l - \frac{1}{2} \right) \left( c_{l+1}^\dagger c_{l+1} - \frac{1}{2} \right) \right) + \sum_{l=1}^{L} i h_l^z \left( c_l^\dagger c_l - \frac{1}{2} \right)$$

Drives skin effect

Many-body interaction

Local loss

**Full dynamical correlator**

$$\mathrm{S}(\omega, l) = \left\langle \mathrm{GS_L} \left| c_l \delta^2 \left( \omega + E_{\mathrm{GS}} - H \right) c_l^\dagger \right| \mathrm{GS_R} \right\rangle + \left\langle \mathrm{GS_L} \left| c_l^\dagger \delta^2 \left( \omega + E_{\mathrm{GS}} - H \right) c_l \right| \mathrm{GS_R} \right\rangle$$

Integrated in the imaginary axis

*(equivalent to the Hermitian one)*

$$\tilde{\rho}(E \in \mathbb{R}, l) = \left| \int \mathcal{S}(E + iy, l) dy \right|$$

# Single particle Hermitian VS many-body non-Hermitian dynamical correlators

**Single particle Hermitian**

*Local density of states*

$$\rho(\omega, l)$$

*Total density of states*

$$\rho_{tot}(\omega) = \sum_{l=1}^{L} \rho(\omega, l)$$

**Many-body non-Hermitian**

*Local dynamical correlator*

$$\mathcal{S}(\omega, l)$$

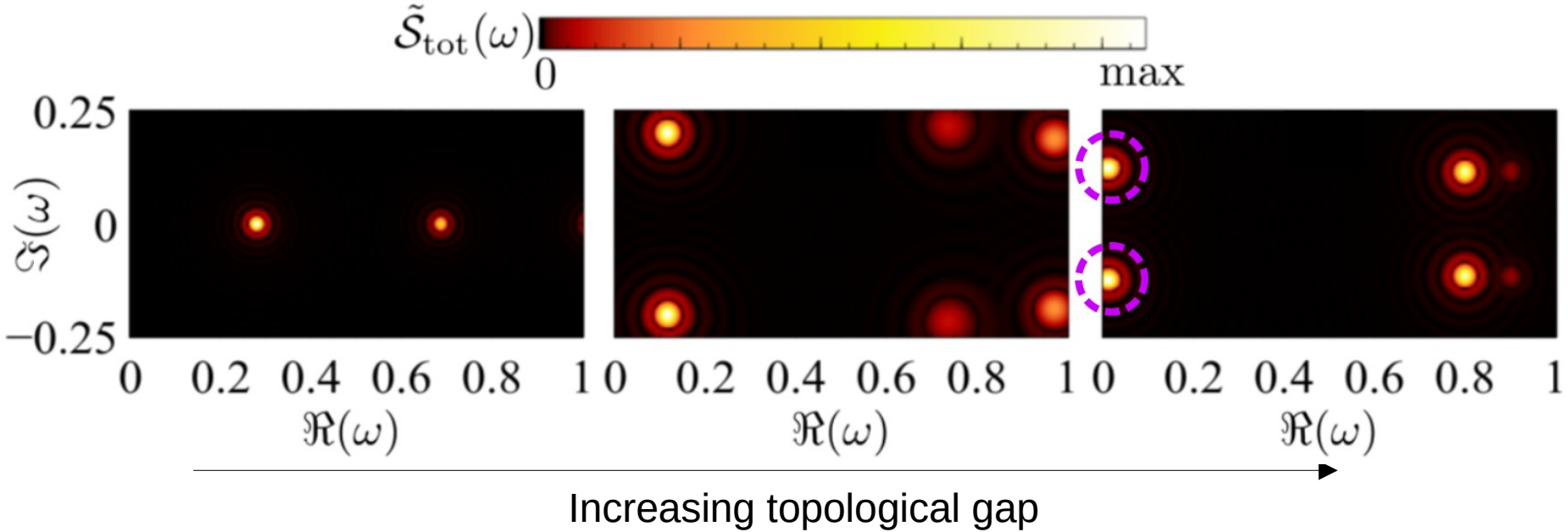
*Total dynamical correlator*

$$\tilde{\mathcal{S}}_{\mathrm{tot}}(\omega) = \sum_{l=1}^{L} \mathcal{S}(\omega, l)$$

*Phys. Rev. Lett. 130, 100401 (2023)*

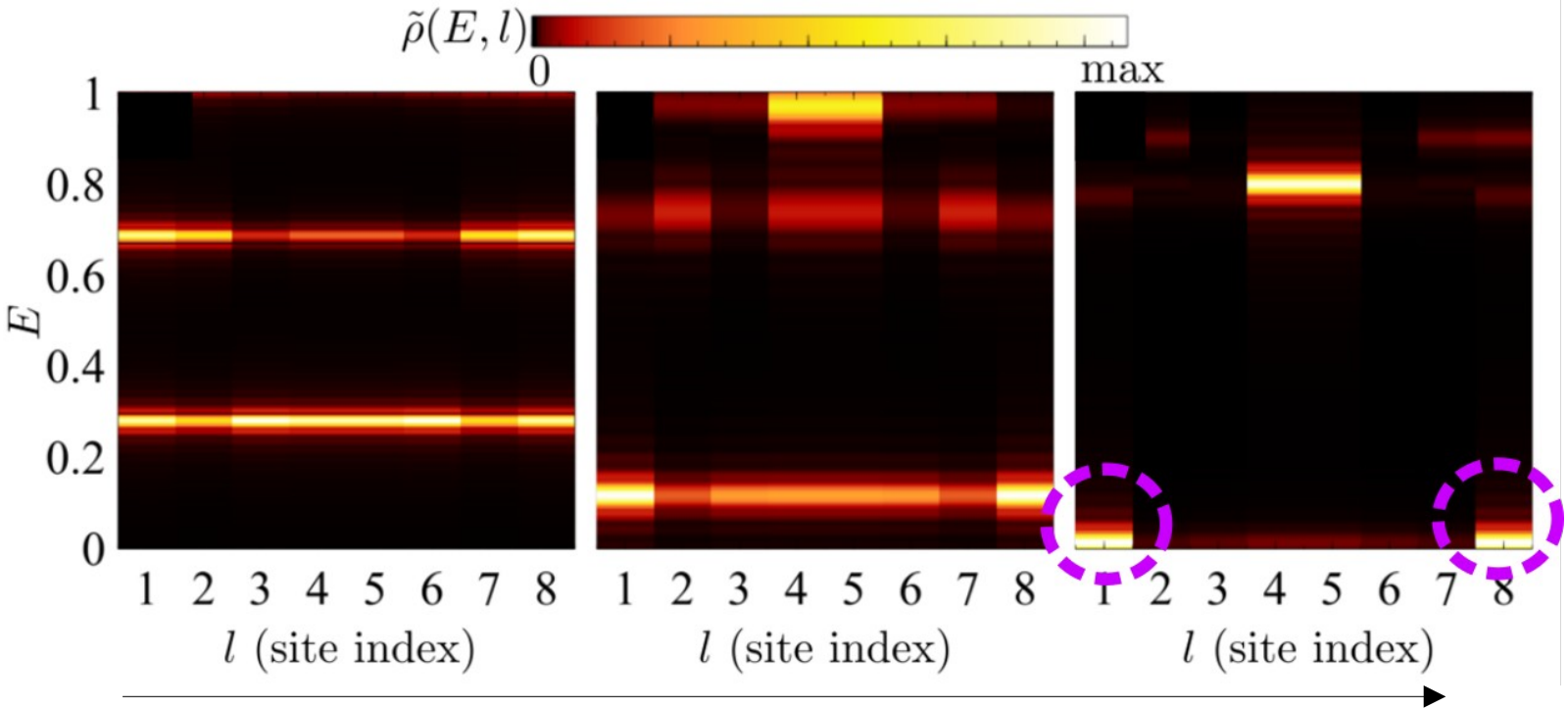
# Topological degeneracies in a dynamical correlator

Total structure factor $\tilde{\mathcal{S}}_{\mathrm{tot}}(\omega) = \left| \sum_{l=1}^{L} \mathcal{S}(\omega, l) \right|$



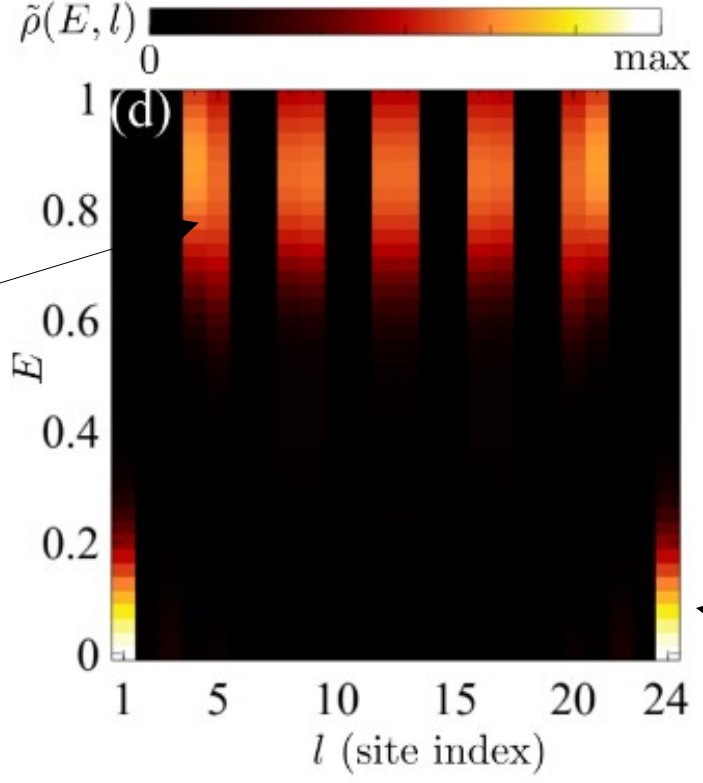

**As the topological gap gets bigger, degeneracies at zero real frequency emerge**

# Topological modes in the local dynamical correlator




**As the topological gap gets bigger, edge modes at zero real frequency emerge**

# Non-Hermitian topological modes



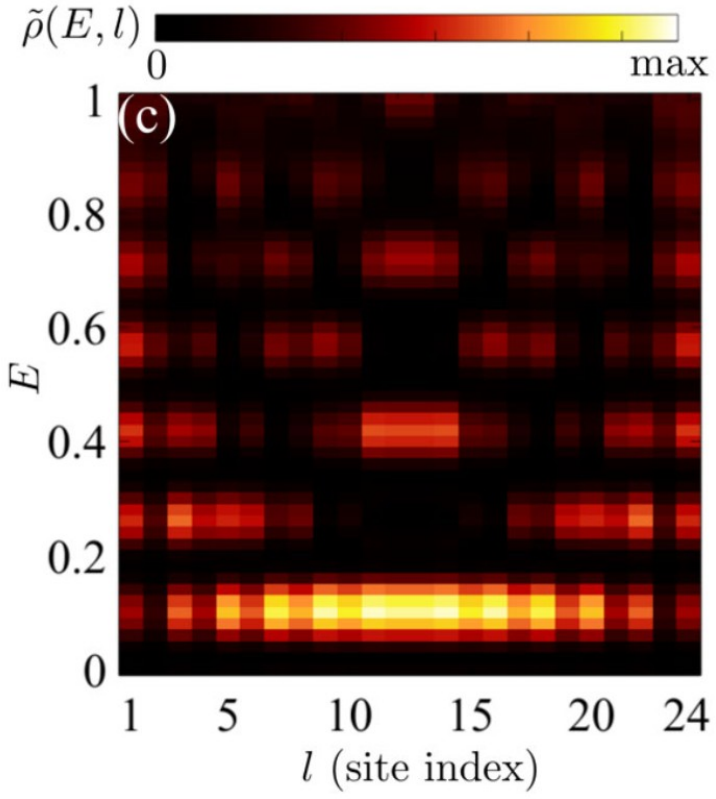

Bulk modes

Edge modes

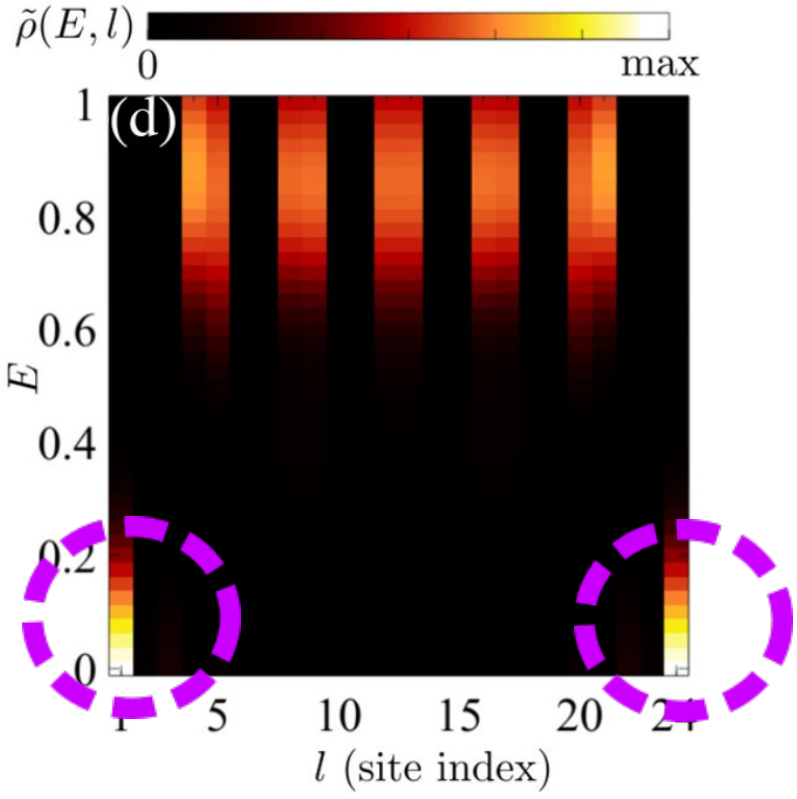
The non-Hermitian dynamical correlator allows visualizing the topological modes

# Trivial VS topological spectral functions

**Non-topological case**




**Topological case**

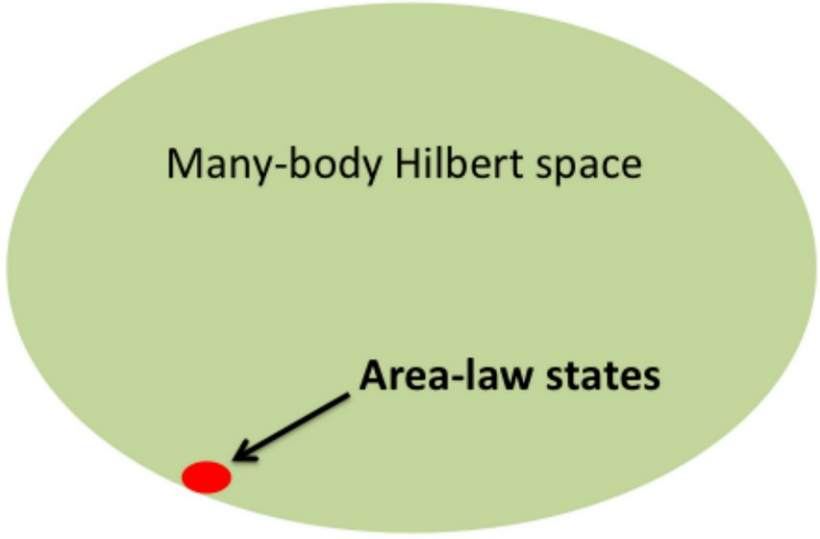


# Tensor-networks for noisy quantum circuits

# How much entanglement do we need

***Qubit fidelity limits which states we can create in a quantum circuit***

***Tensor-network bond dimension limits which states we parametrize***



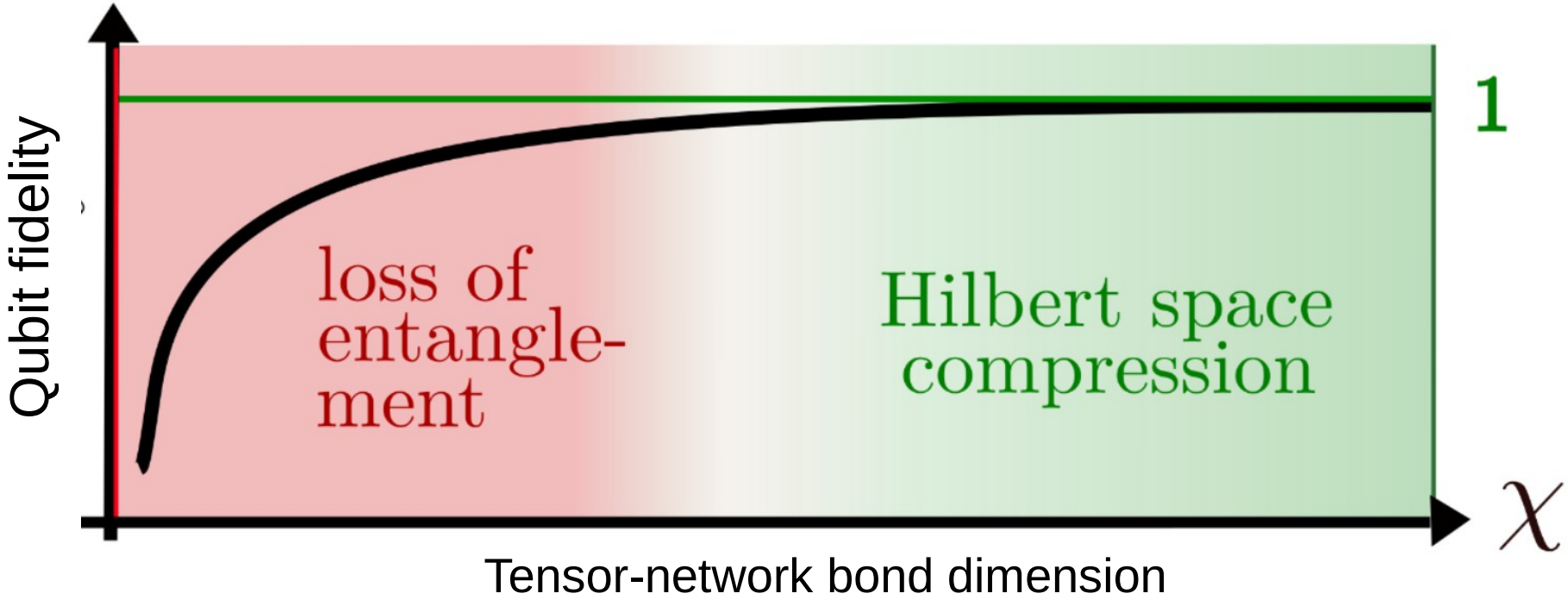

Both constrains put bounds to the entanglement of accessible states

*Phys. Rev. X 10, 041038 (2020)*

**Do we need the whole Hilbert space to successfully run a quantum algorithm?**

**Could we get meaningful results even if we can only access part of the Hilbert space?**

# What is the relation between a noisy qubit and a tensor network?

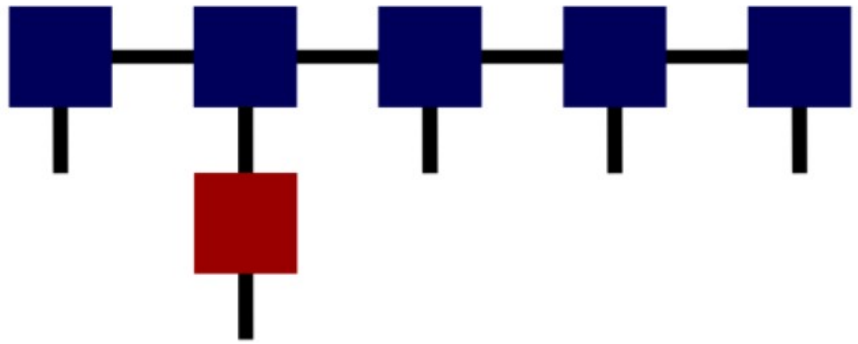



***The finite tensor-network bond dimension can be understood as a finite qubit fidelity***

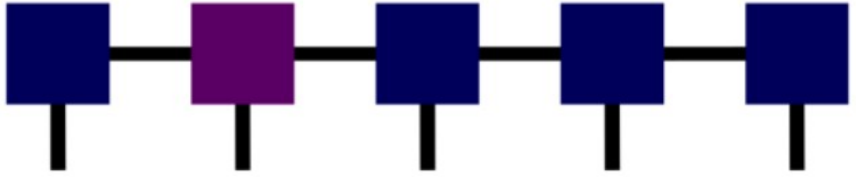
# Applying a single qubit gate to a tensor network

Original MPS

Single qubit gate



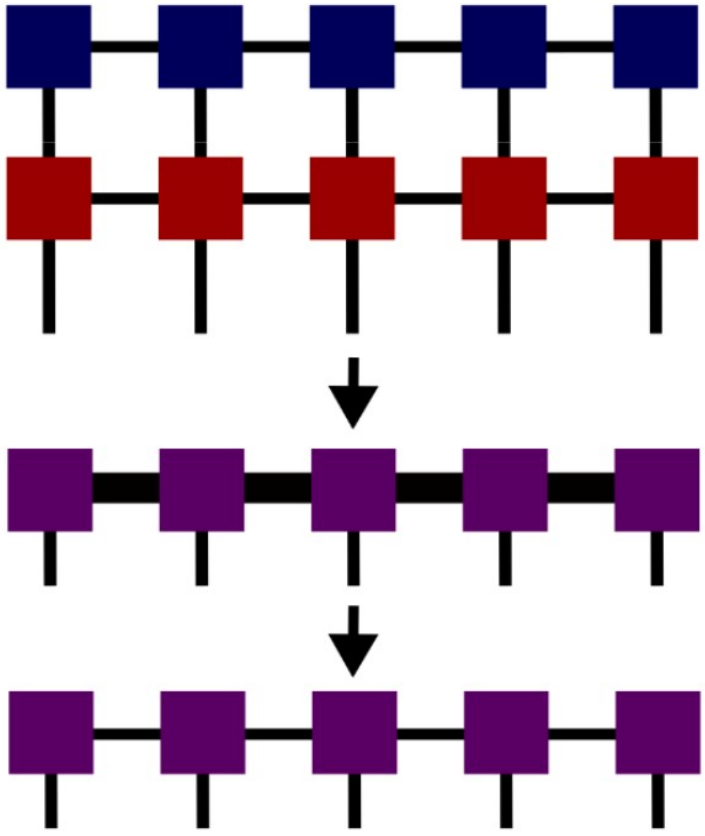
Transform MPS



# Applying a multi-qubit gate to a tensor network

Original MPS

Multi-qubit gate



Higher dimension updated MPS

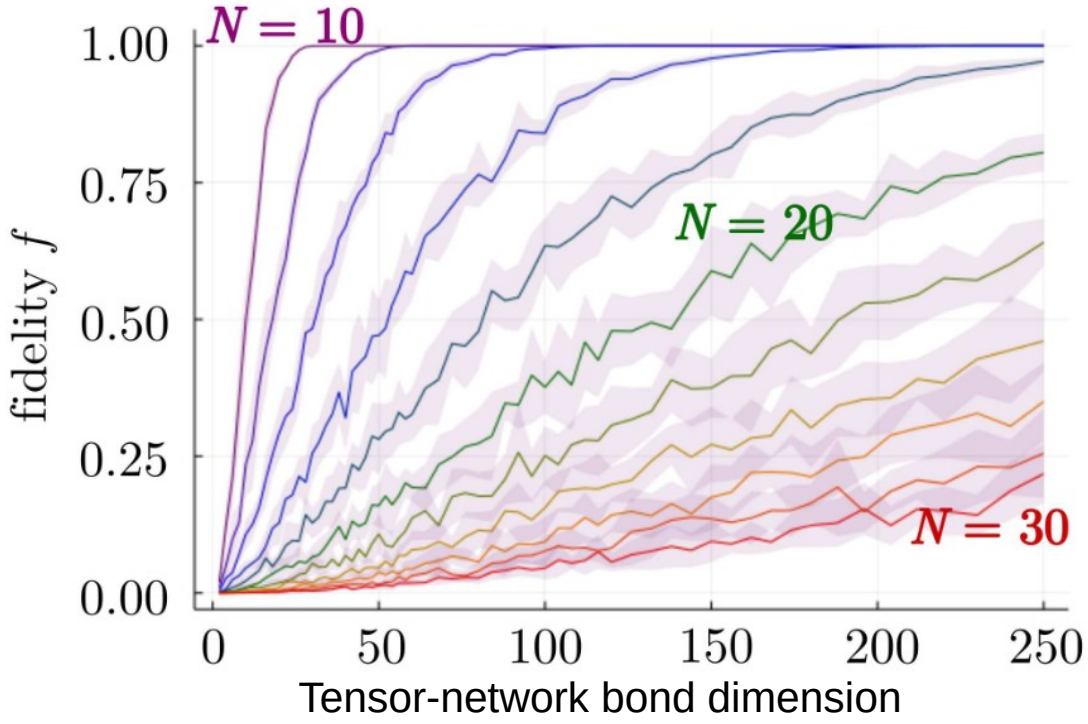
Compressed updated MPS

# Three quantum algorithms with tensor-network quantum circuits

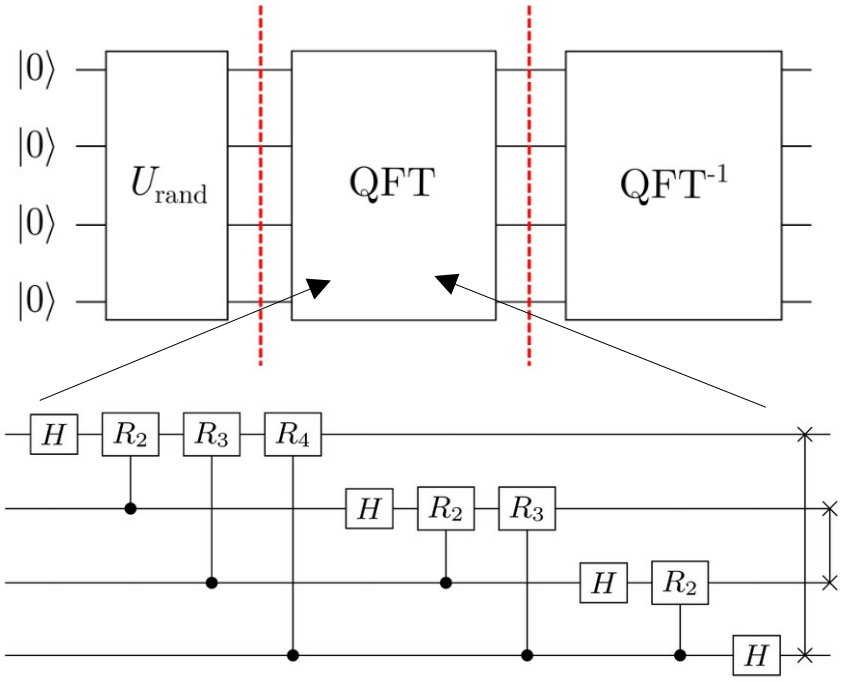
- ***Quantum Fourier transform:*** quantum analogue of the discrete Fourier transform
- ***Grover's algorithm:*** find elements in a database
- ***Quantum counting algorithm:*** counting the number of solutions for a given search problem

# The quantum Fourier transform

***N is the number of qubits***




***Circuit architecture***



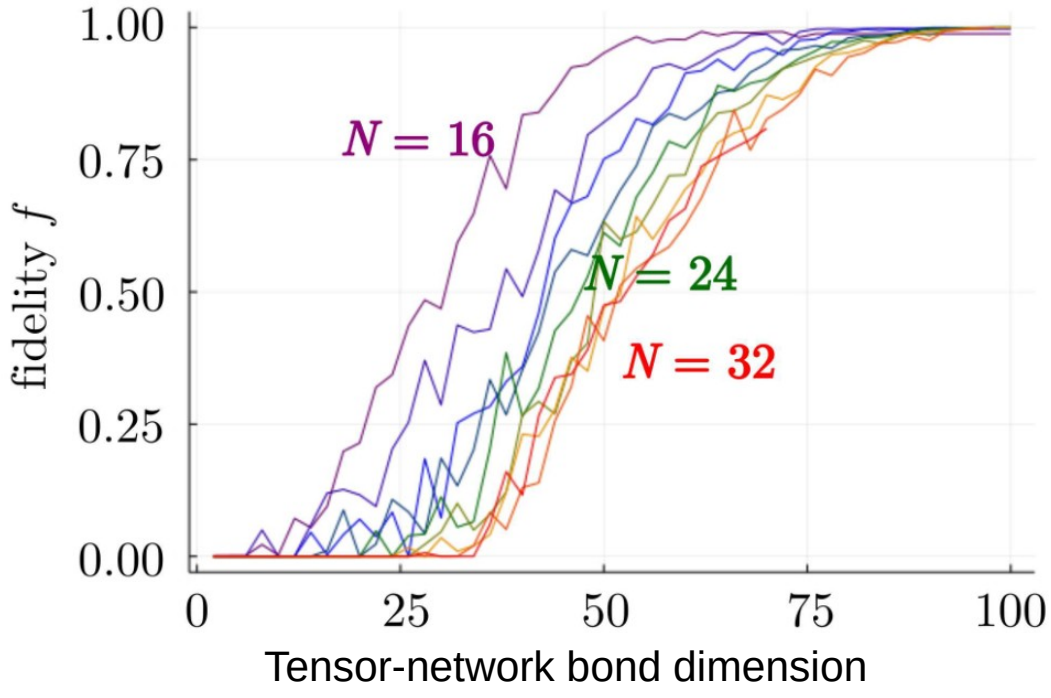

**QFT Fidelity**

$$f = |\langle \Psi_0' | \Psi_0 \rangle|^2 \qquad |\Psi_0'\rangle = \mathrm{QFT}^{-1}\mathrm{QFT}\,|\Psi_0\rangle$$

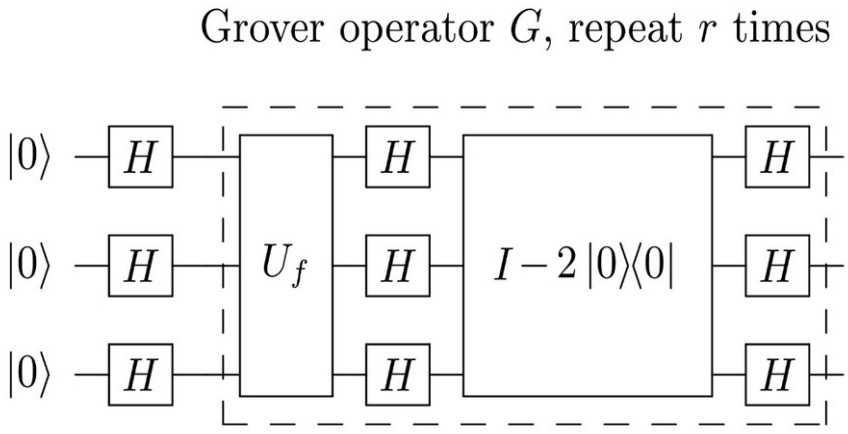
*arXiv:2304.01751 (2023)*

# Grover's search algorithm

***N is the number of qubits***




**Circuit architecture**




**Grover fidelity (m marked elements)**

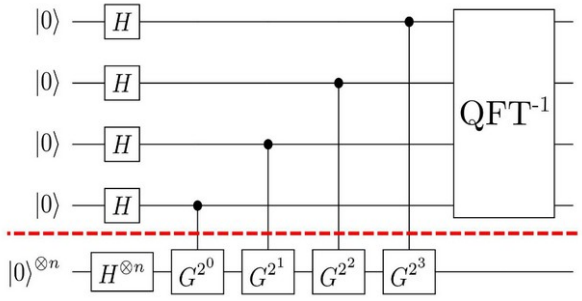
$$f_r = \frac{1}{m} \sum_{i}^{m} |\, \langle \omega_i | \Psi_r \rangle \,|^2$$

*arXiv:2304.01751 (2023)*

# Quantum counting algorithm

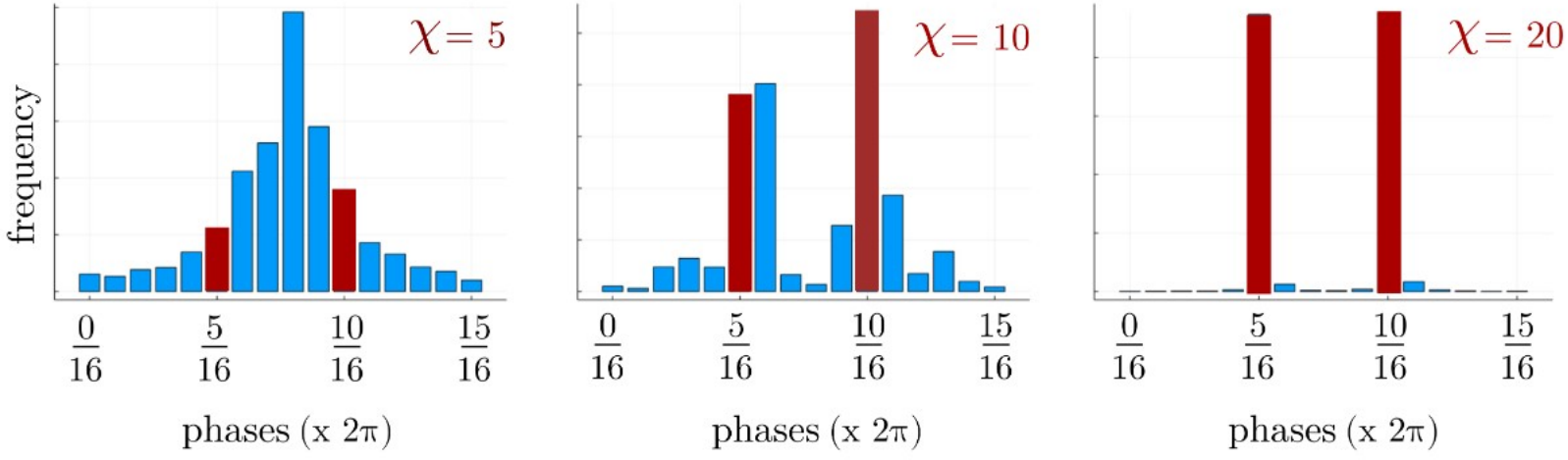
## Circuit architecture

*arXiv:2304.01751 (2023)*




## Phases of marked elements (correct in red)

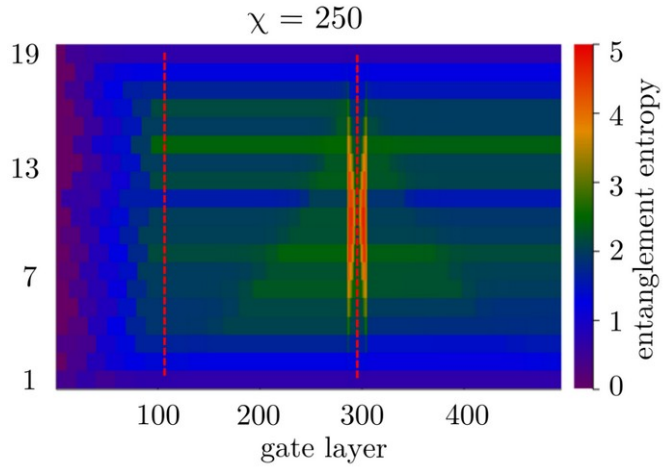
*low-quality circuit*



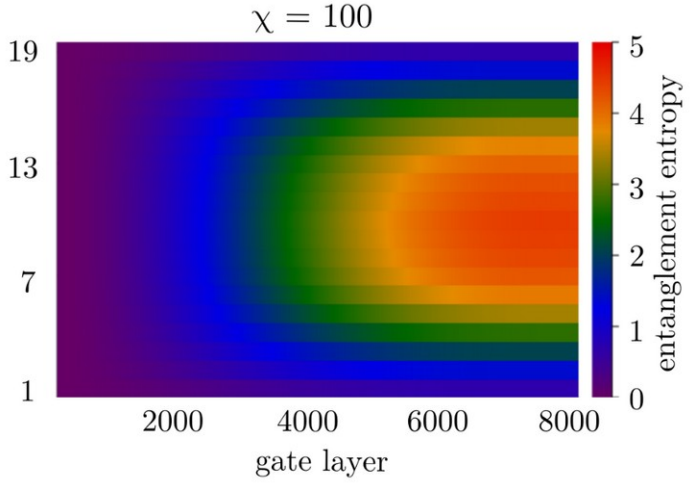

*high-quality circuit*

# Entanglement propagation in the quantum-circuit

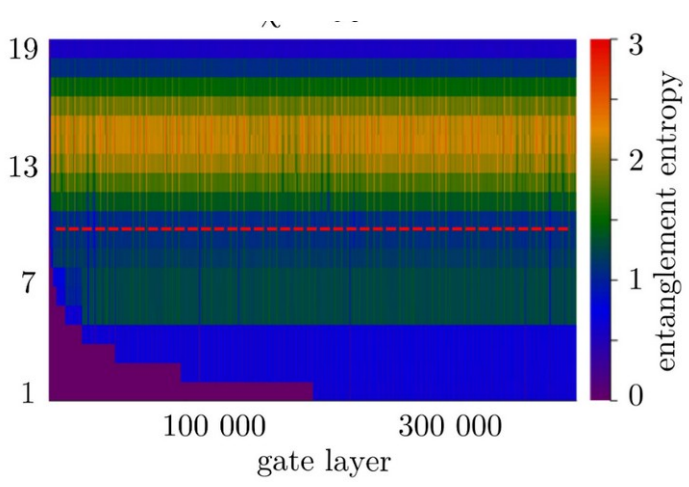
**Quantum Fourier transform**




**Grover's algorithm**




**Quantum counting algorithm**



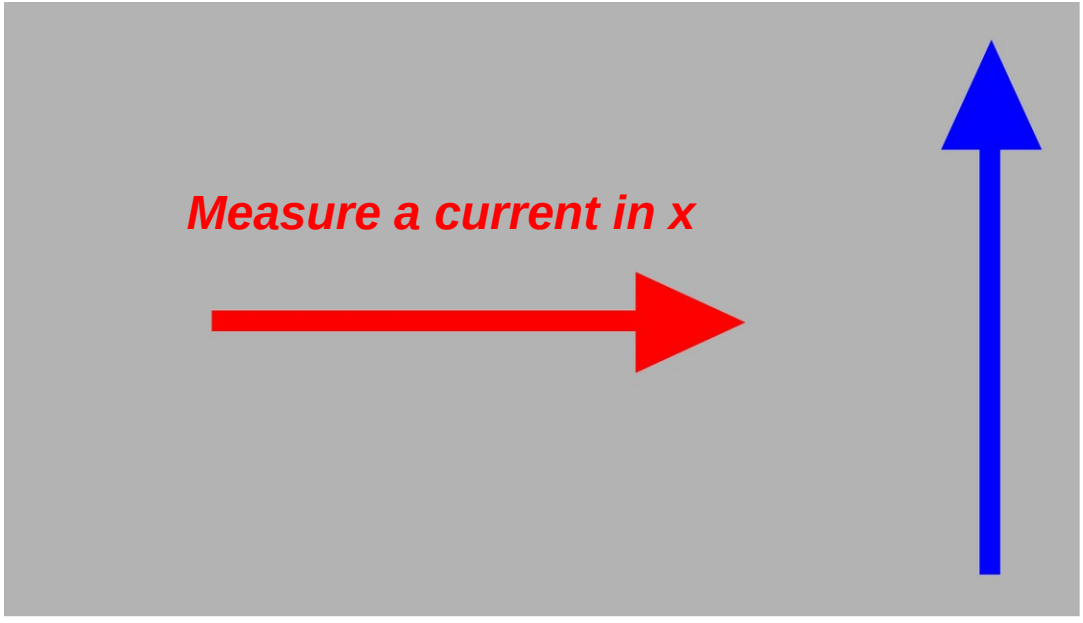

***Entanglement builds up in concrete locations and steps of the quantum circuit***

*arXiv:2304.01751 (2023)*

# Quantum circuits for topological quantum materials

# Chern number and the transverse conductivity

**Take a two-dimensional material**




$$J_x = \sigma_{xy} V_y$$

***Full Hamiltonian***

$$H = H_0 + \lambda V$$

***Perturbation***

$$V \sim y \sim i\partial_{k_y}$$

***Measure***

$$J_x \sim \langle \partial H / \partial k_x \rangle$$

# Linear response for transverse current

$$J_x = \sigma_{xy} V_y$$

**The Hall conductivity is obtained as** $\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_\alpha d^2\mathbf{k}$

with $\Omega_\alpha(\mathbf{k}) = i \sum_{\beta \neq \alpha} \frac{\langle \Psi_\alpha | \partial H/\partial k_x | \Psi_\beta \rangle \langle \Psi_\beta | \partial H/\partial k_y | \Psi_\alpha \rangle}{(\epsilon_\alpha - \epsilon_\beta)^2} - \alpha \leftrightarrow \beta$

***Berry curvature of a band***

***Expression coming from perturbation theory***

# The Hall conductivity

**The Hall conductivity is obtained as** $\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_\alpha d^2\mathbf{k}$

Using $\langle \Psi_\alpha | \partial H / \partial k_\mu | \Psi_\beta \rangle = \langle \partial_{k_\mu} \Psi_\alpha | \Psi_\beta \rangle (\epsilon_\alpha - \epsilon_\beta)$

***the Hall conductivity can be expressed in terms of***

**Berry curvature**

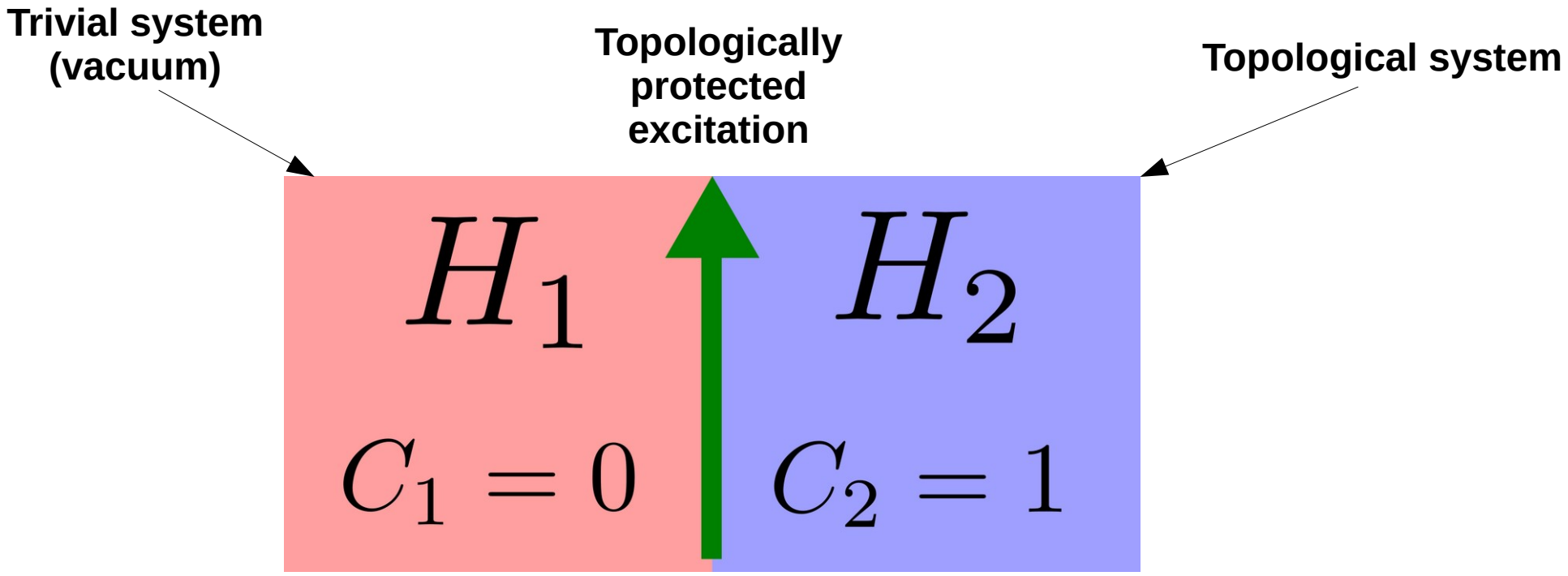
$$\Omega_\alpha = \partial_{k_x} A^\alpha_y - \partial_{k_y} A^\alpha_x$$

**Berry connection**

$$A^\alpha_\mu = i \langle \partial_{k_\mu} \Psi_\alpha | \Psi_\alpha \rangle$$

$$\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_\alpha d^2\mathbf{k} = \sum_\alpha C_\alpha = C$$ ← Chern number

# The consequence of different topological invariants




***Topological excitations appear between topologically different systems***

***<u>Topological invariants are integers, and thus potentially robust in noisy quantum circuits</u>***

# Topological invariant in a Hamiltonian

**We can classify Hamiltonians according to topological invariants**

**Hamiltonian**

$$H(\mathbf{k})$$

**Wavefunction**

$$|\Psi(\mathbf{k})\rangle$$

$$\Omega = \nabla \times \mathbf{A}|_z$$

**Metric (Berry curvature)**

$$C = \int_{BZ} \Omega d^2\mathbf{k}$$

**Topological invariant (Chern number)**

# Berry phase with a quantum circuit

Take the eigenstates of the Hamiltonian

$$H(\boldsymbol{k})\,|n(\boldsymbol{k})\rangle = E_n(\boldsymbol{k})\,|n(\boldsymbol{k})\rangle$$

Evolve them in reciprocal space

$$U(T,0)\,|n(\boldsymbol{k})\rangle = e^{i(\Theta_B - \Theta_D)}\,|n(\boldsymbol{k})\rangle$$

***Geometric phase***

$$\Theta_B = i\int_0^T dt\langle n(\boldsymbol{k}(t))|\partial_t|n(\boldsymbol{k}(t))\rangle$$

***Dynamical phase***

$$\Theta_D = \int_0^T dt E_n(\boldsymbol{k}(t))$$

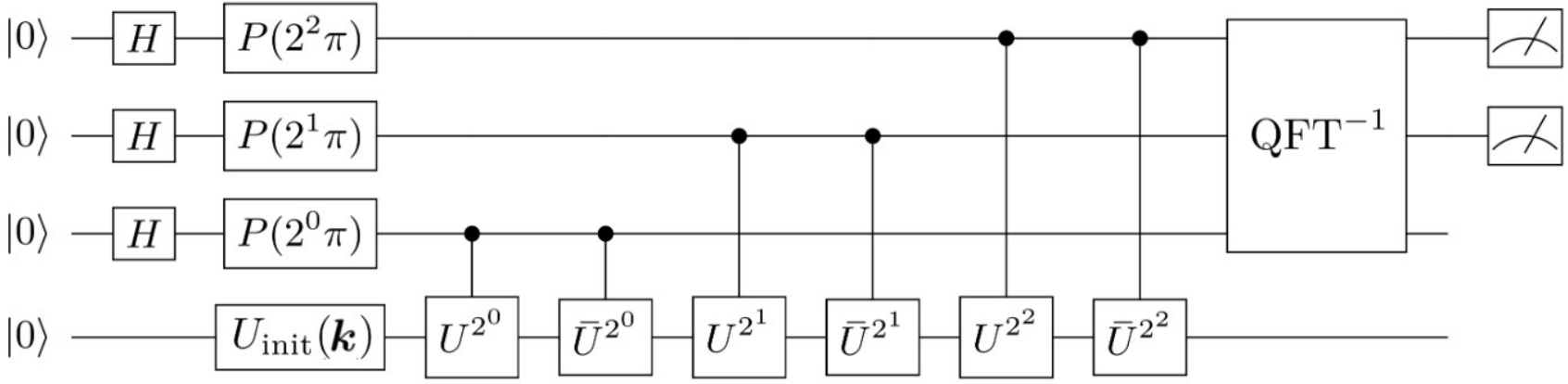
We want to cancel out the dynamical phase, so we can evolve first forward and then backwards in time

$$\bar{U}(T,0)U(T,0)\,|n(\boldsymbol{k})\rangle = e^{i2\Theta_B}\,|n(\boldsymbol{k})\rangle$$

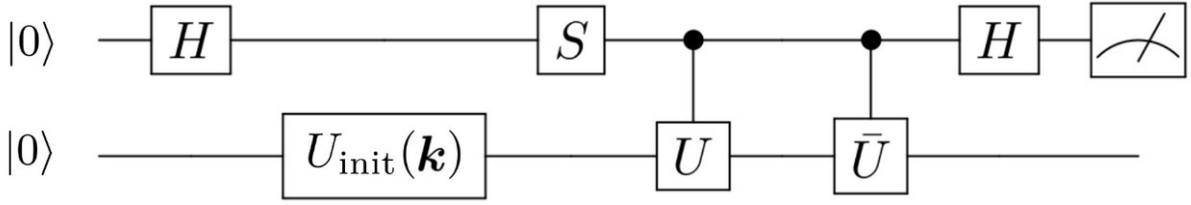
*arXiv:2404.06048 (2024)*

# Quantum circuits to compute topological invariants

**Berry phase with quantum phase estimation (based on QFT)**




**Berry flux (based on Hadamard test)**




*arXiv:2404.06048 (2024)*

# Two ways of computing a Chern number

**Method 1**: Integrate the Berry Curvature

$$C = \frac{1}{2\pi} \int_{BZ} d^2\boldsymbol{k} \Omega(\boldsymbol{k})$$

**Method 2**: Pumping of the Wannier charge centers

$$X_W(k_y) = i \int_{-\pi}^{\pi} dk_x \langle n(\boldsymbol{k}) | \partial_{k_x} | n(\boldsymbol{k}) \rangle / 2\pi$$

(and count how many times they wind)

*arXiv:2404.06048 (2024)*

# Five implementations of the topological invariant

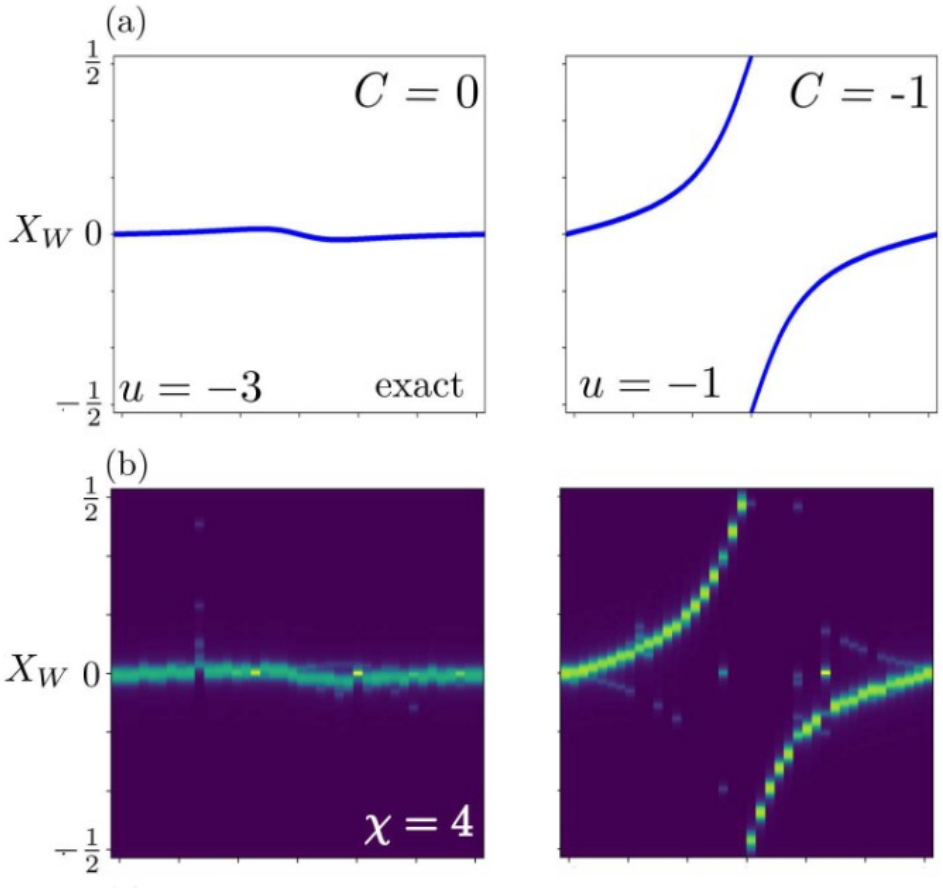
- Helmi quantum computer
- Exact quantum circuit emulator
- Noisy quantum circuit emulator
- Tensor-network quantum circuit emulator
- Exact classical algorithm

# Helmi quantum computer


*“Helmi, the first Finnish quantum computer, co-developed by VTT and IQM Quantum Computers, is operated by VTT in Espoo, Finland. Helmi is based on superconducting technology, and presently provides five qubits. Upgrades to 20, then 50 qubits is planned for the near future.”*

https://fiqci.fi/

# Chern number from Wannier winding

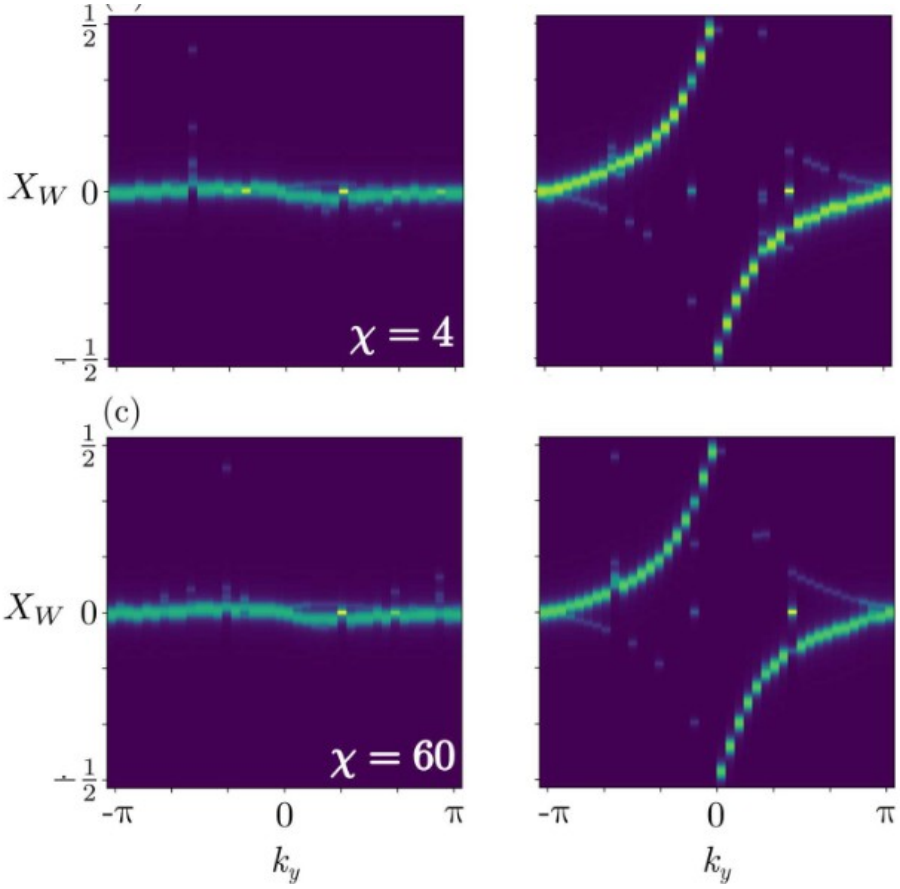



**Exact classical simulation**

**Tensor network simulator**

*arXiv:2404.06048 (2024)*

# Chern number from Wannier winding




**Low bond dimension**

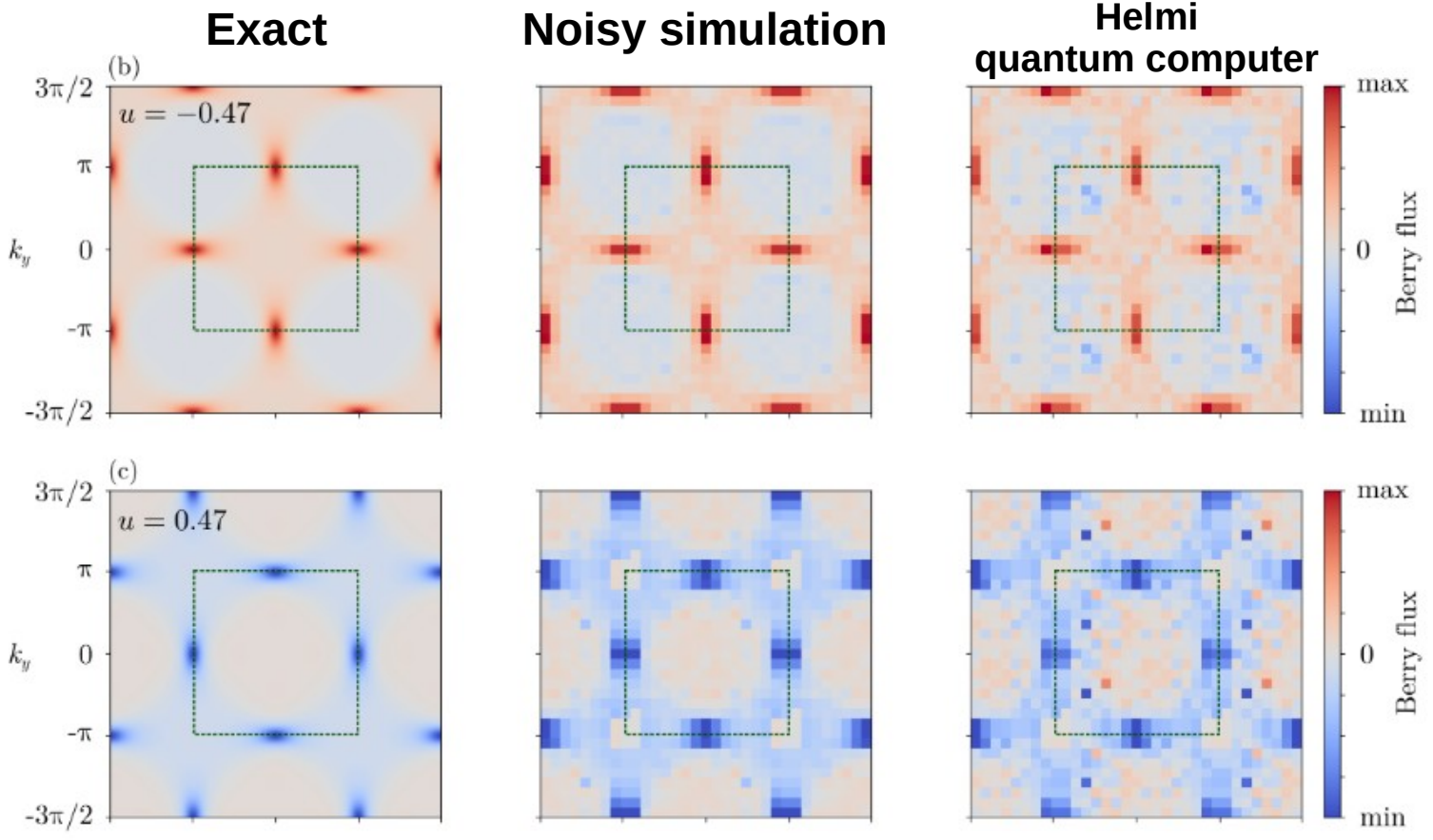
*Tiny part of the Hilbert space*

**High bond dimension**

*Most of the Hilbert space*

*arXiv:2404.06048 (2024)*

# Chern number from Berry curvature integration



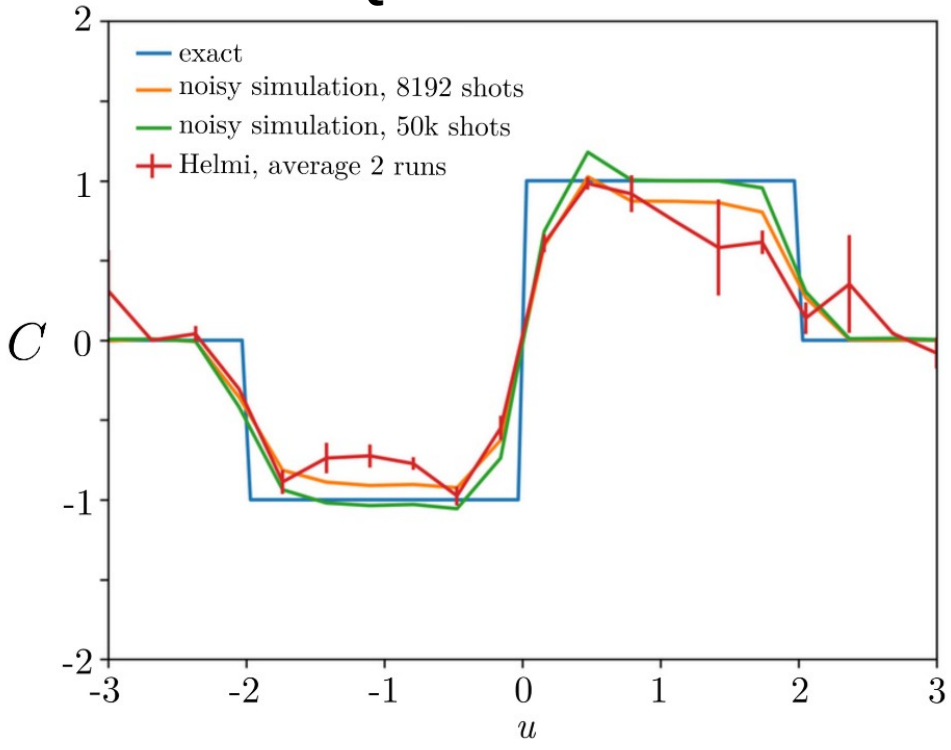

$$C = 1$$

$$C = -1$$

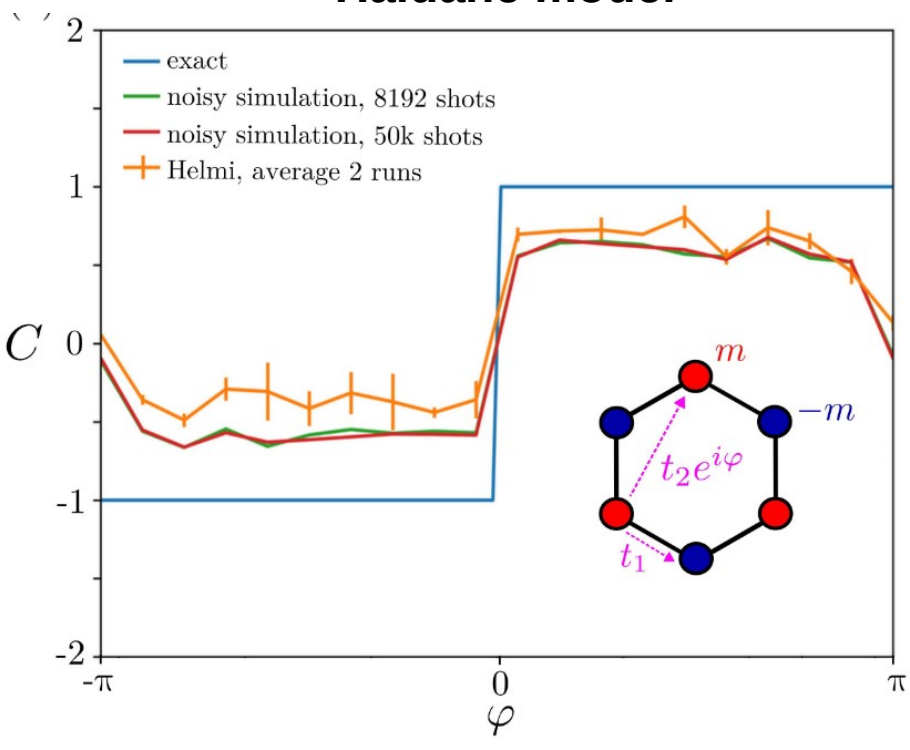
*arXiv:2404.06048 (2024)*

# Topological phase diagram

**QWZ model**

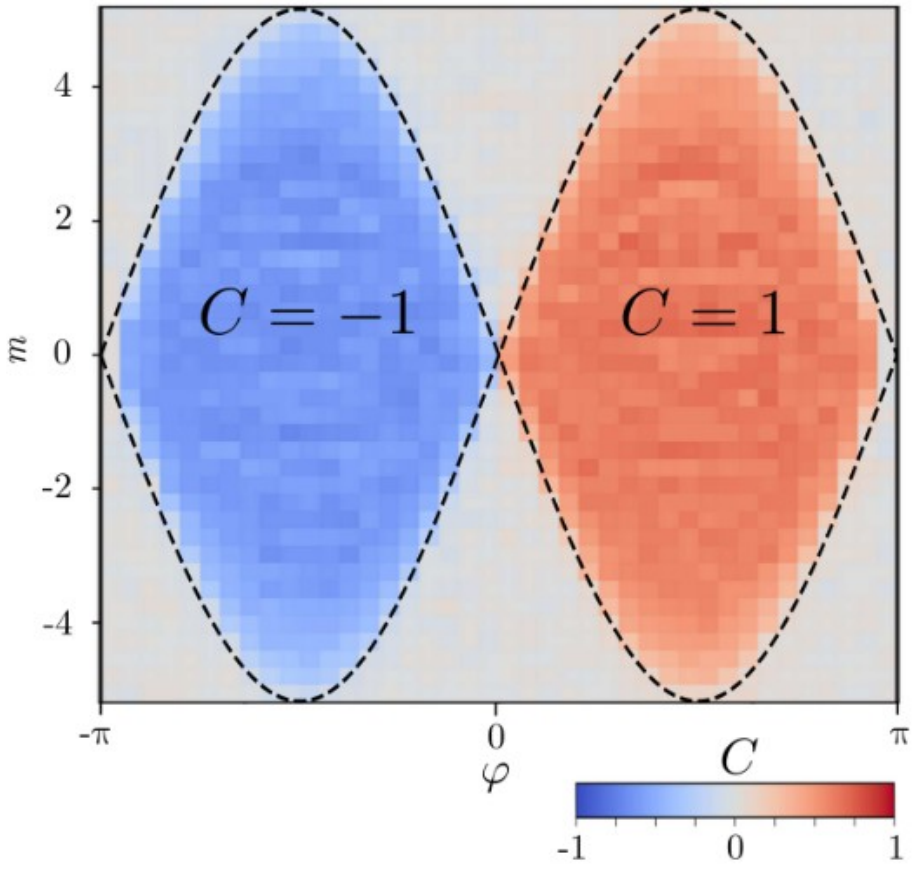



**Haldane model**




*arXiv:2404.06048 (2024)*

# Topological phase diagram




**Haldane model**

Noisy calculations provide qualitatively correct topological phase diagrams

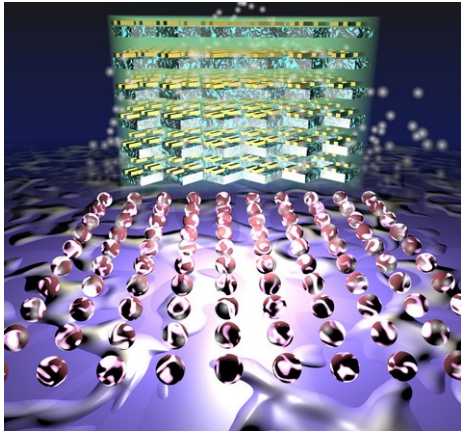
*arXiv:2404.06048 (2024)*

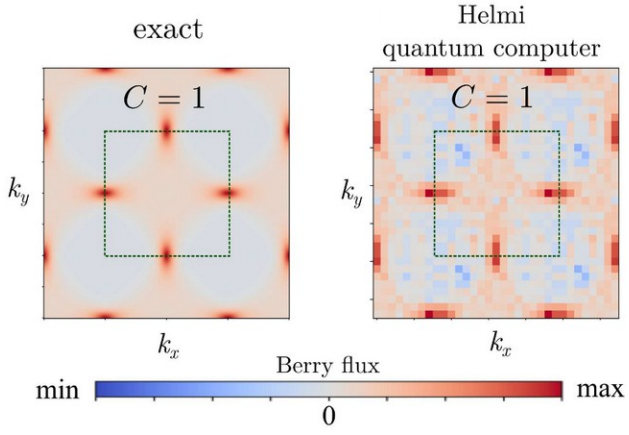
# Future steps in topology on quantum computers

- Next FiQCI quantum computers (20 and 50 qubits) would allow tackling many-body topological models
  - One dimensional topological spin liquids (S=1 Haldane)
  - Two dimensional topological spin liquids (S=1/2)

# Take home

Currently available quantum computers allow computing topological invariants, and can be effectively benchmarked with tensor-networks






*Phys. Rev. Lett. 130, 100401 (2023)*
*arXiv:2304.01751 (2023)*
*arXiv:2404.06048 (2024)*

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J&AE JANE AND AATOS ERKKO FOUNDATION

Institute Q

**Thank you**