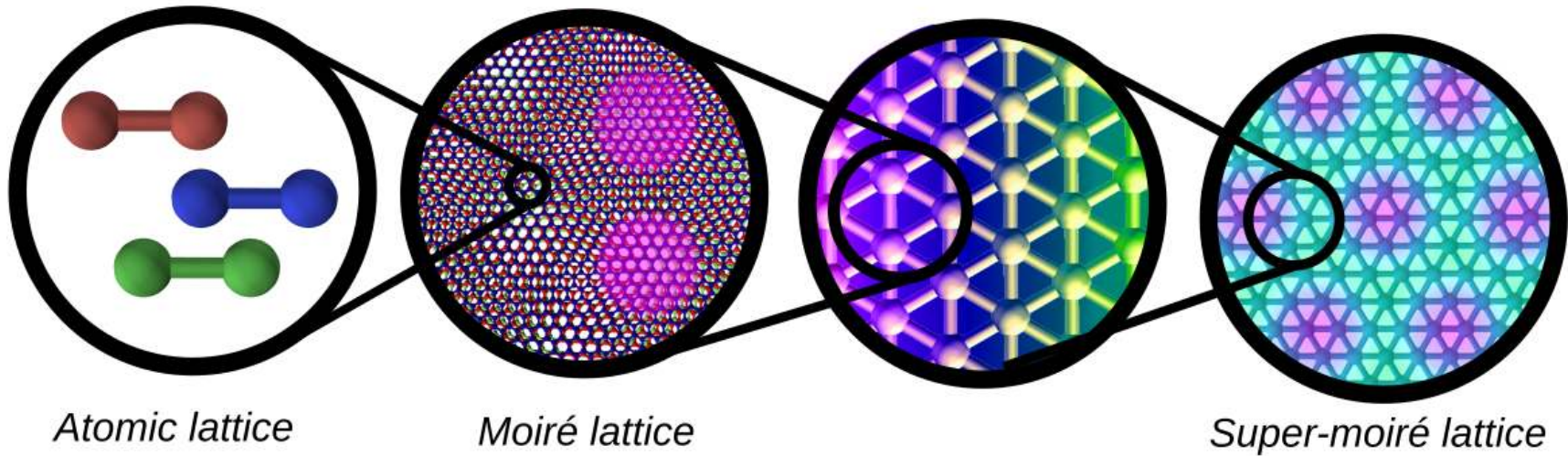


# Solving topological and correlated super-moire materials with tensor networks and tensor cross interpolation

Jose Lado

*Department of Applied Physics, Aalto University, Finland*



*2D Materials 12 (1), 015018 (2025)*

*arXiv:2503.04373 (2025)*

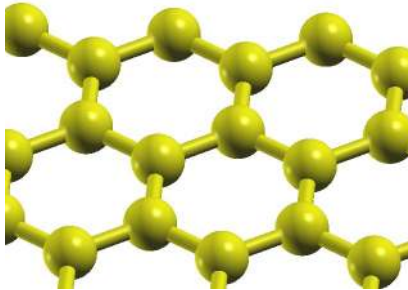
*arXiv:2506.05230 (2025)*

International workshop on tensor cross interpolation and other algorithms to learn tensor networks  
Grenoble, France, October 6<sup>th</sup> 2025

# The two-dimensional materials world

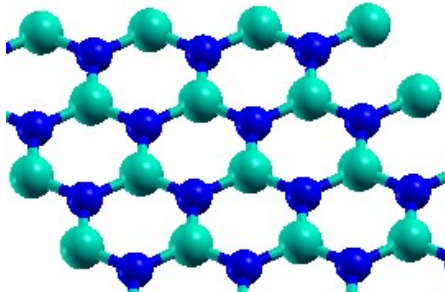
**Semimetal**

Graphene



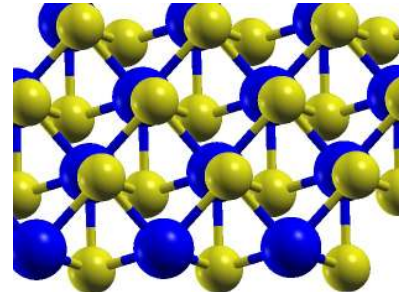
**Insulator**

BN



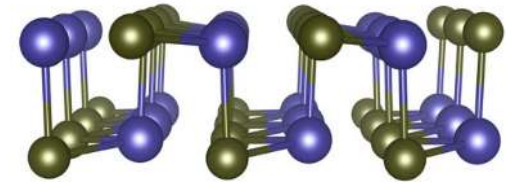
**Superconductor**

NbSe<sub>2</sub>



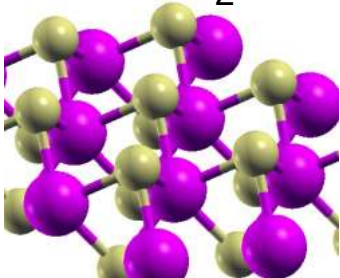
**Ferroelectric**

SnTe



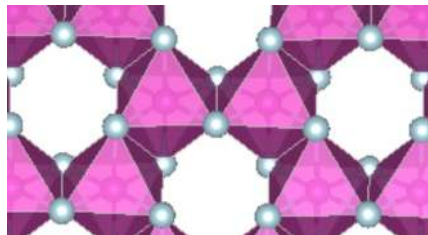
**Semiconductor**

WS<sub>2</sub>



**Ferromagnet**

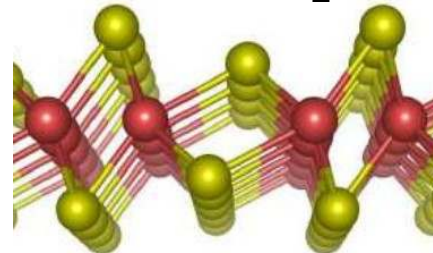
CrI<sub>3</sub>



**Quantum spin**

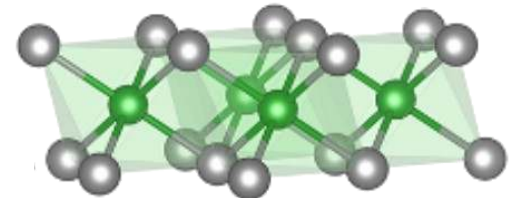
**Hall insulator**

WTe<sub>2</sub>



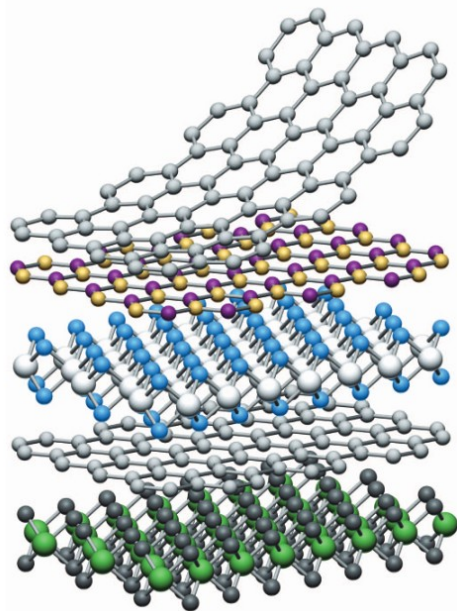
**Multiferroic**

NiI<sub>2</sub>



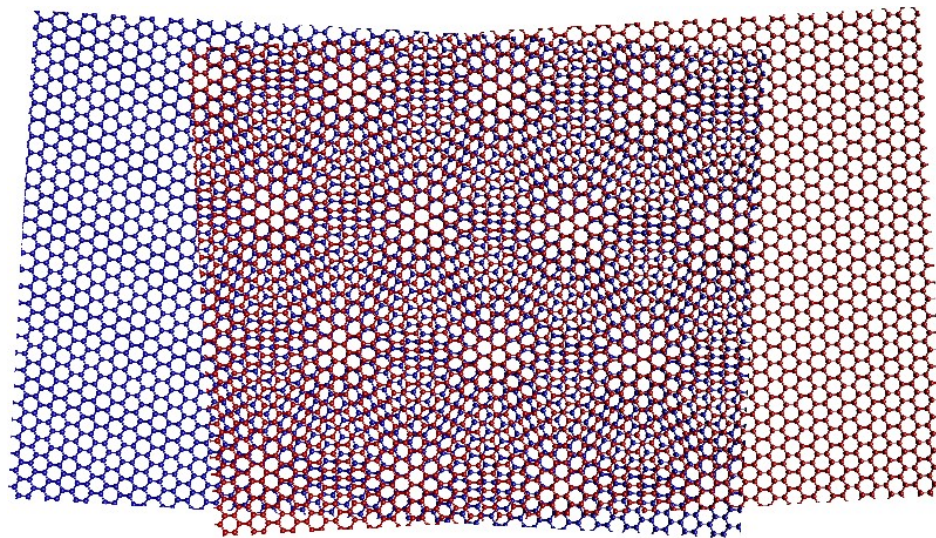
# The flexibility of two-dimensional materials

**They can be stacked**



*Nature* 499, 419–425 (2013)

**They can be rotated**

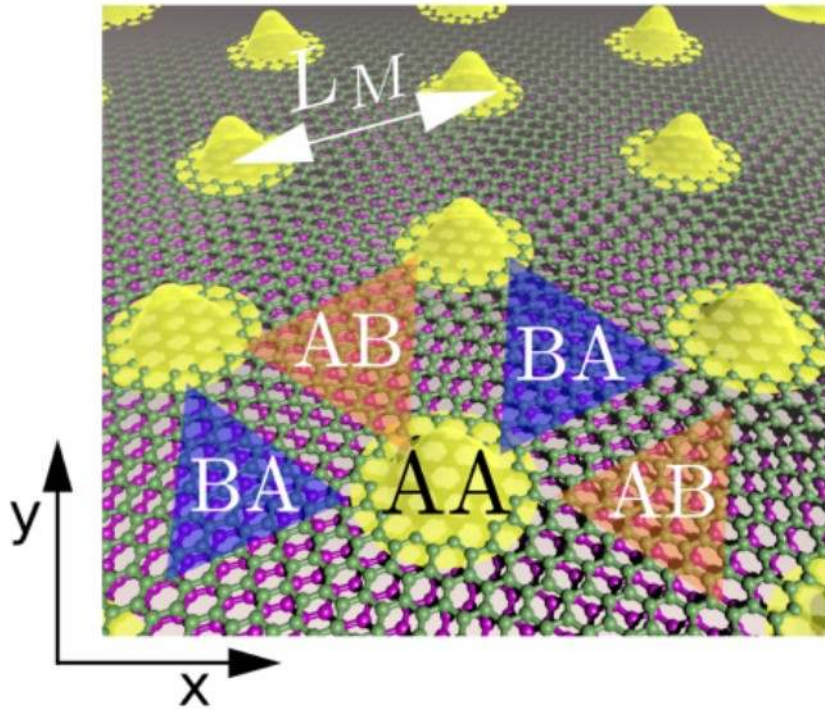


*Science* 361, 6403, 690-693 (2018)

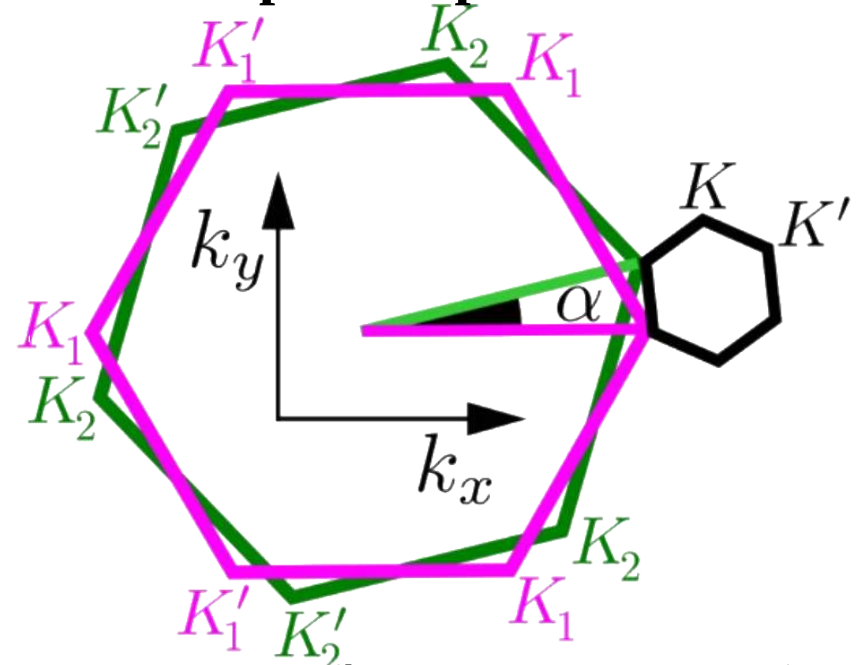
**These are unique features of two-dimensional materials**

# Twisted graphene bilayers

Structure



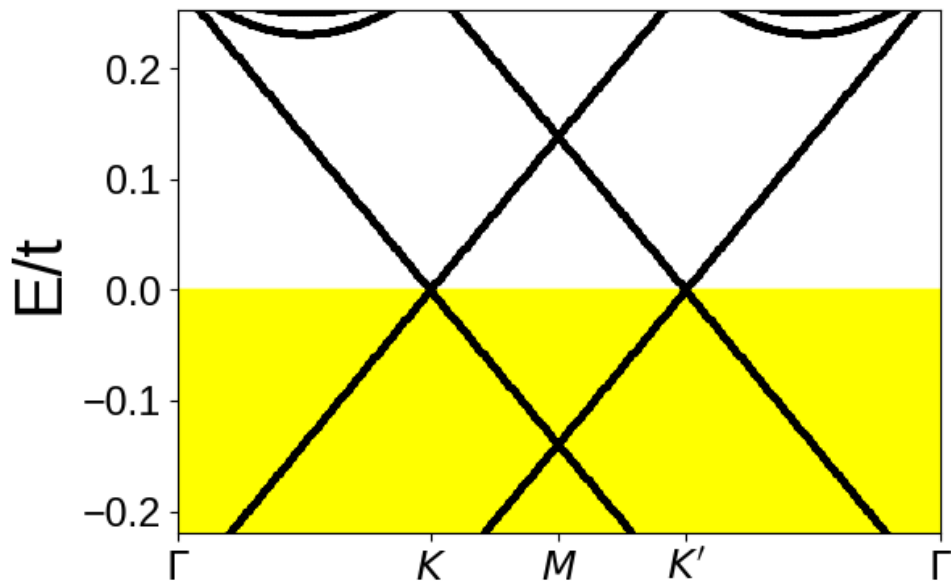
Reciprocal space



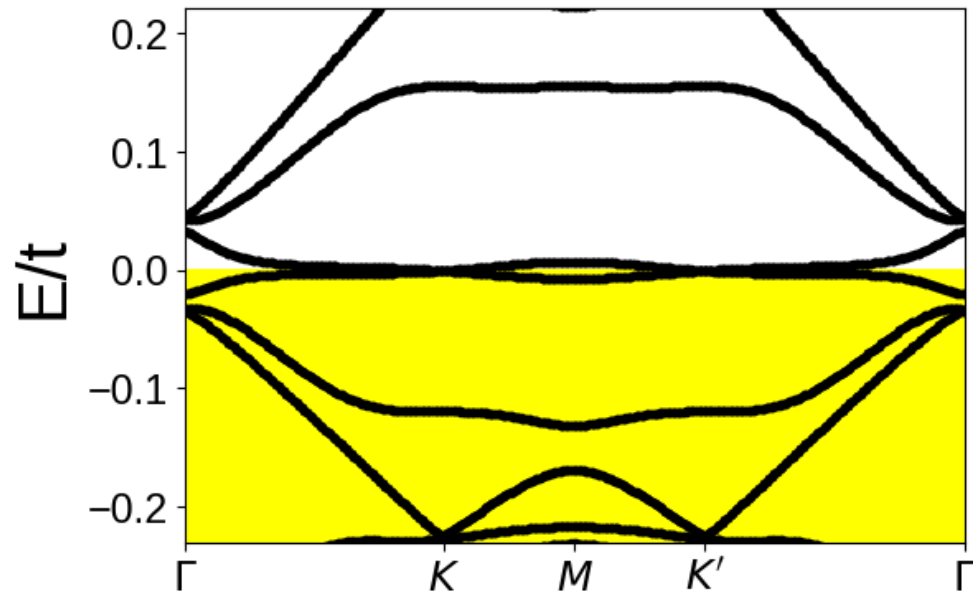
Phys. Rev. Lett. 99, 256802 (2007)  
Phys. Rev. B 82, 121407(R) (2010)  
PNAS 108 (30) 12233-12237 (2011)

# Electronic structure of twisted bilayer

Uncoupled layers



Coupled layers

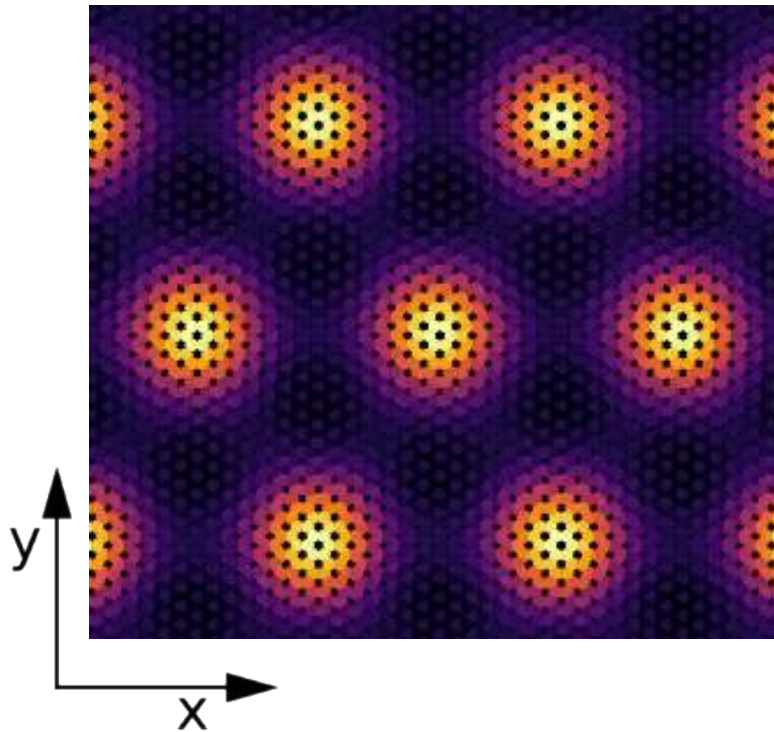


Twisted graphene bilayers feature flat bands driven by the twist

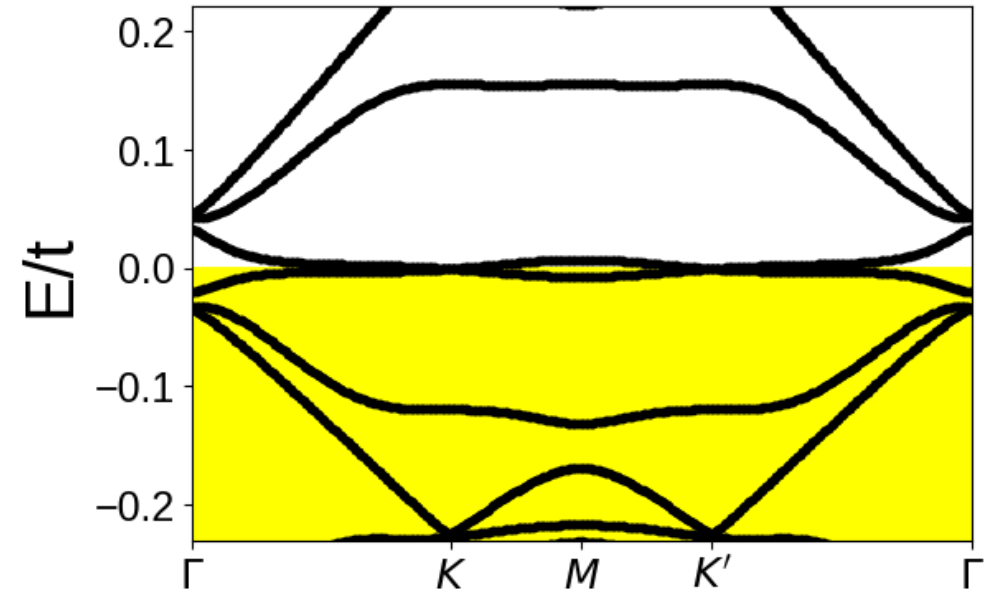
Phys. Rev. Lett. 99, 256802 (2007)  
Phys. Rev. B 82, 121407(R) (2010)  
PNAS 108 (30) 12233-12237 (2011)

# Electronic structure of twisted bilayer

Local density of states



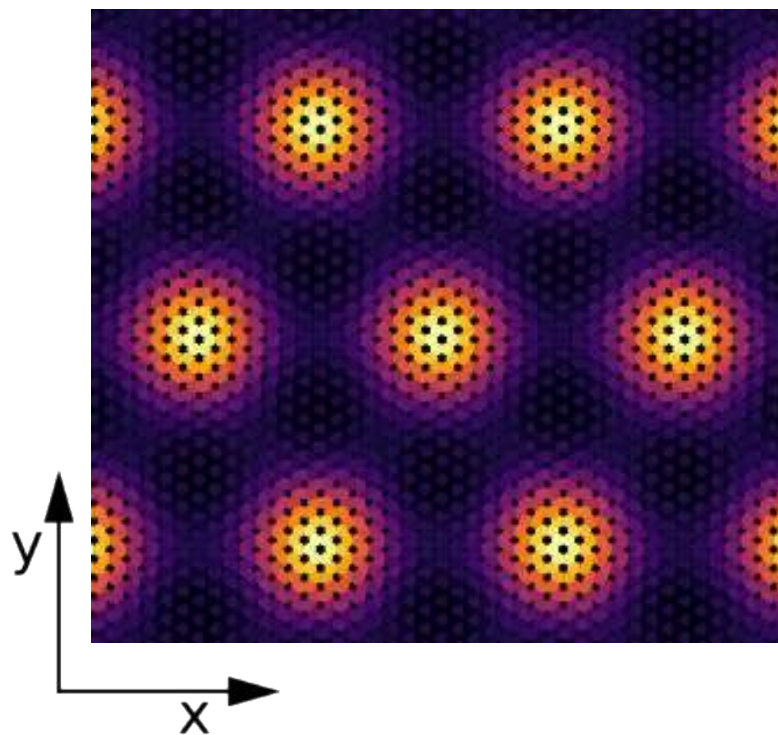
Coupled layers



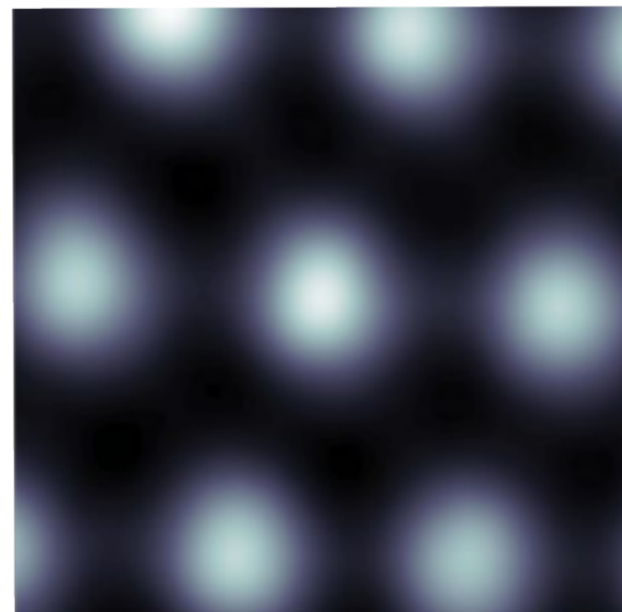
Phys. Rev. Lett. 99, 256802 (2007)  
Phys. Rev. B 82, 121407(R) (2010)  
PNAS 108 (30) 12233-12237 (2011)

# Electronic structure of twisted bilayer

Local density of states



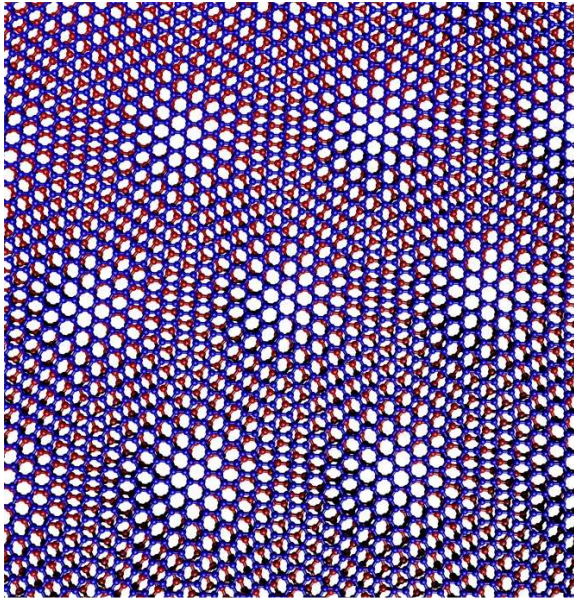
Experimental local density of states



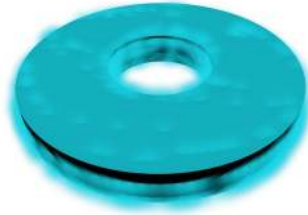
Nature Physics 6, 109–113 (2010)  
Nature 572, 101–105 (2019)  
Nature 572, 95–100 (2019)

# The tunability of twisted van der Waals materials

**Twisted bilayer graphene**



**Superconductivity**



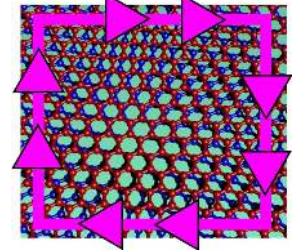
*Nature* 556, 43–50 (2018)

**Topological networks**



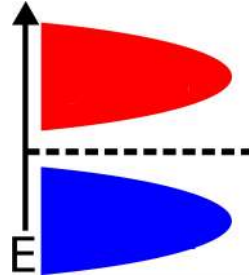
*Nano Lett.* 18, 11, 6725-6730 (2018)

**Chern insulators**



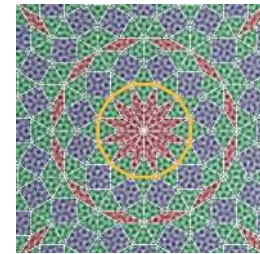
*Science* 365, 605-608 (2019)

**Correlated insulators**



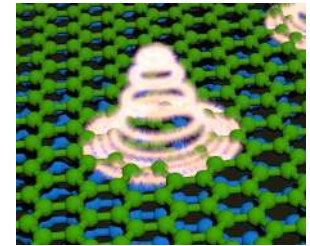
*Nature* 556, 80–84 (2018)

**Quasicrystalline physics**



*Science* 361, 782-786 (2018)

**Proximal fractional Chern insulators**

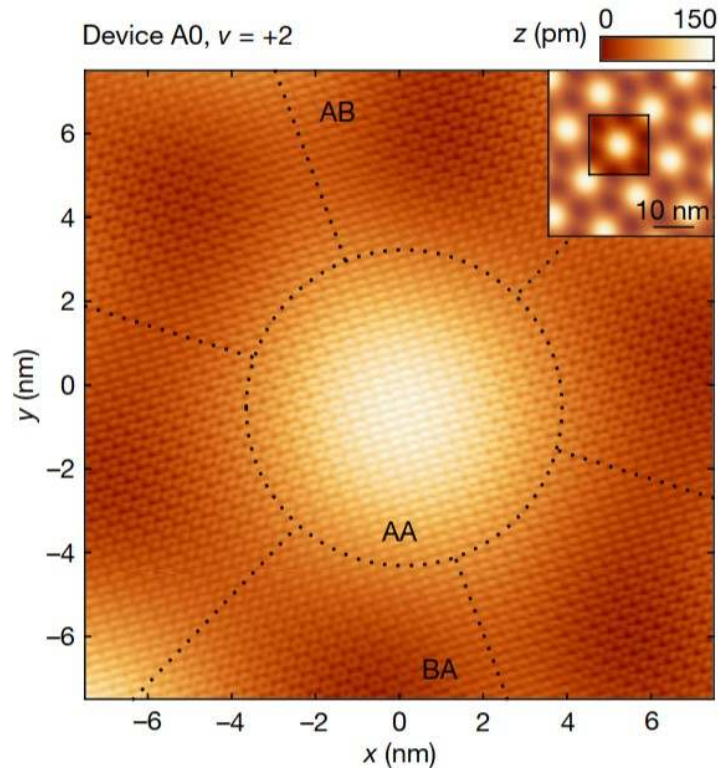


*Nature* 600, 439–443 (2021)

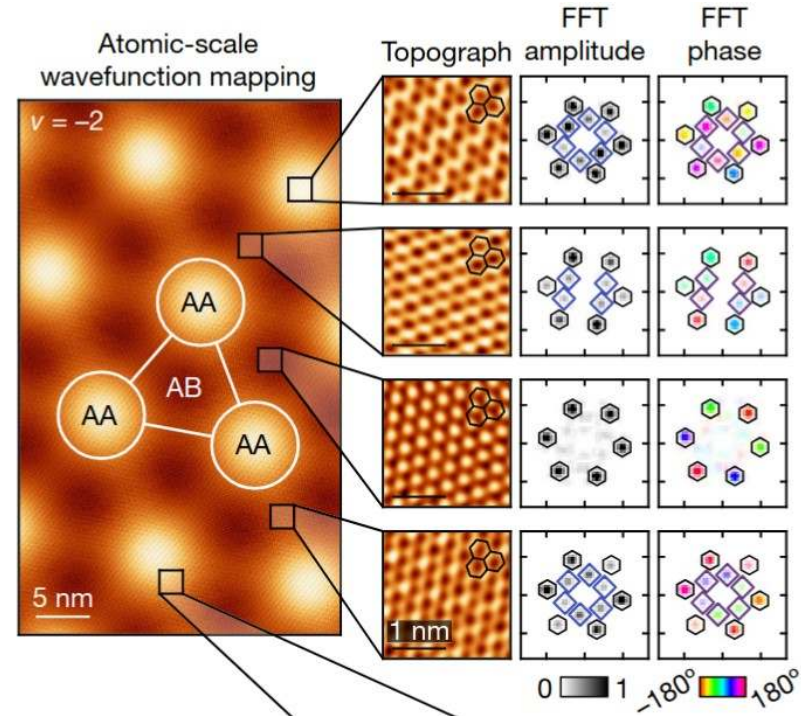
**Twisted multilayers provide a powerful platform for emergent phenomena**

# Experimental observation of multiple scales in twisted bilayers

## Moire and atomic scale with STM

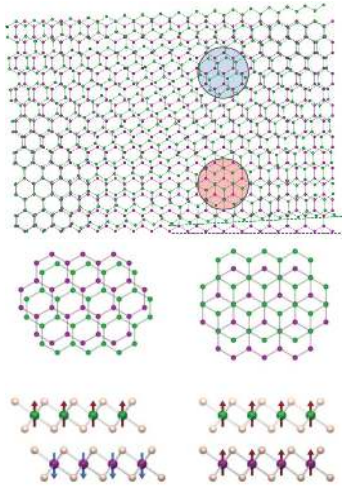


## Atomic-scale symmetry breaking with STM



# Artificial quantum matter in moire van der Waals heterostructures

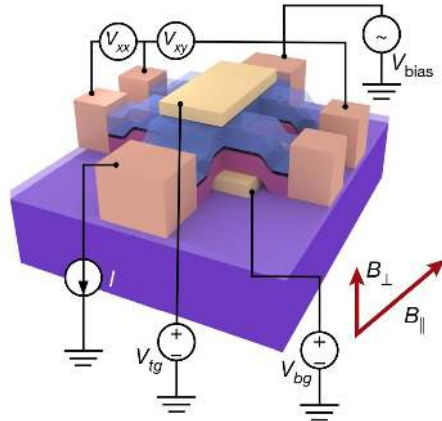
## Unconventional magnets



Twisted  $\text{CrBr}_3$

*Science*, 374(6571),  
1140-1144 (2021)

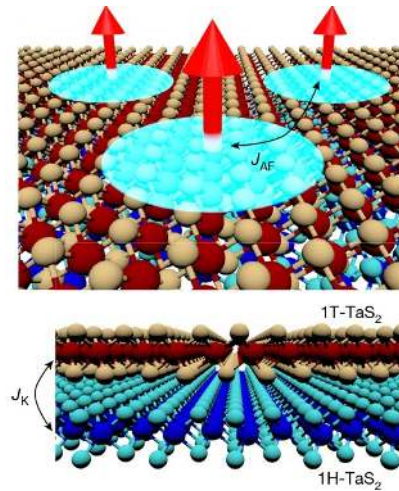
## Unconventional superconductors



Twisted trilayer  
graphene

*Nature* 595,  
526–531 (2021)

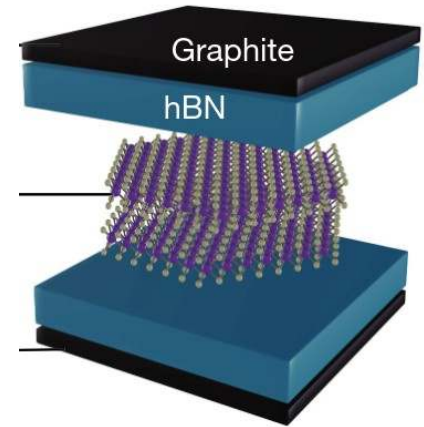
## Heavy-fermion quantum materials



$1\text{H}-1\text{T TaS}_2$

*Nature* 599,  
582–586 (2021)

## Fractional topological matter

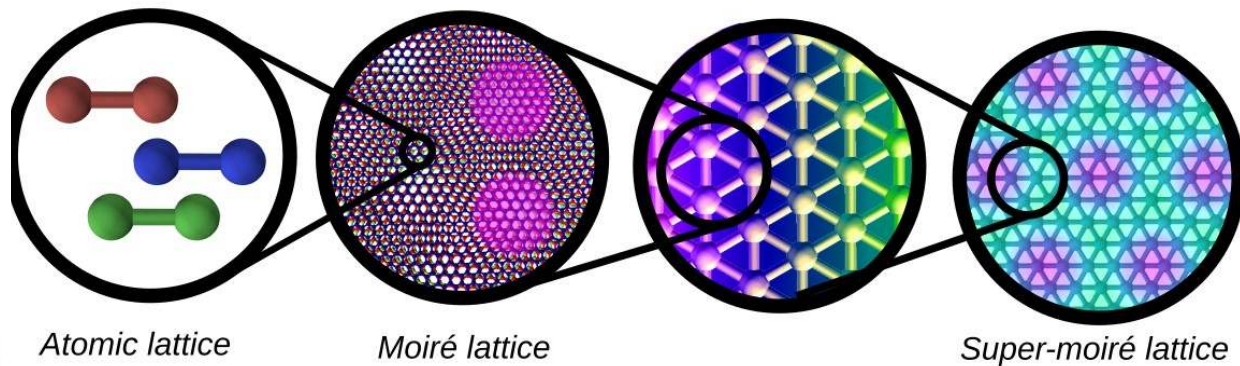
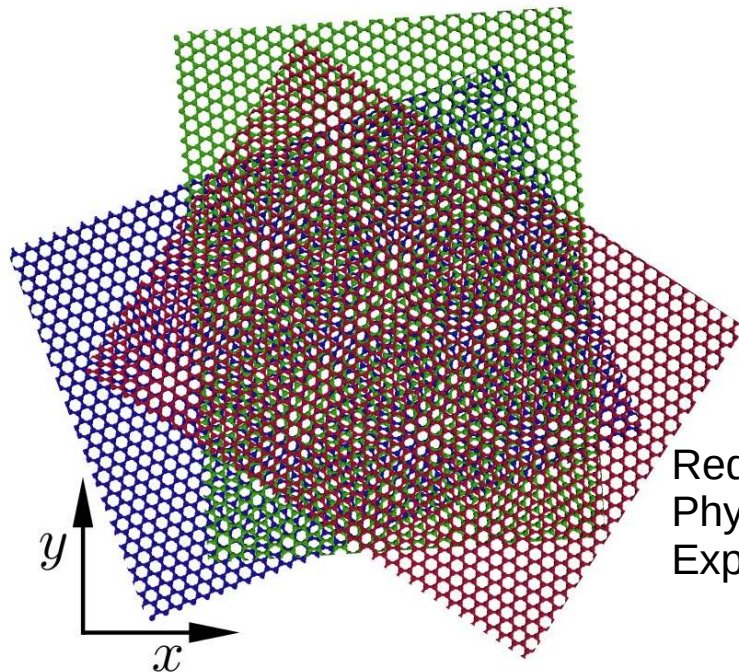


Twisted  $\text{MoTe}_2$

*Nature* 622,  
63–68 (2023)

# Super-moire materials

## Moire-of-moire materials



Requires computing up to a billion sites ( $10^9$ )  
Physics at several length scales (atomic, moire, and super-moire)  
Experiments showing correlations and unconventional superconductivity

*Nature* 620, 762–767 (2023)

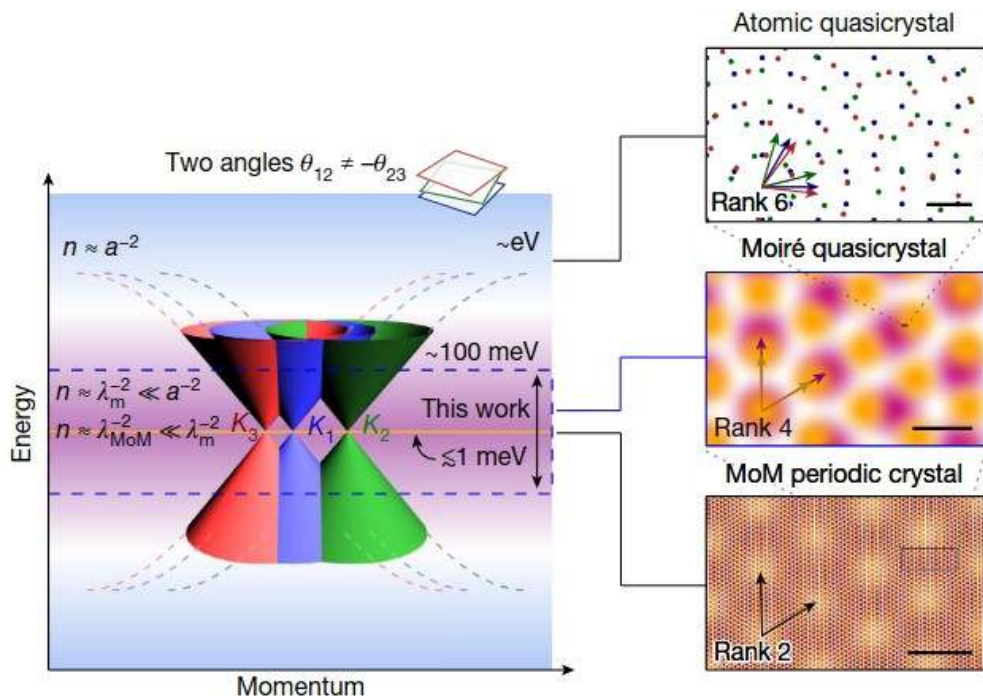
*Nature* 625, 494–499 (2024)

*Nature* 641, 896–903 (2025)

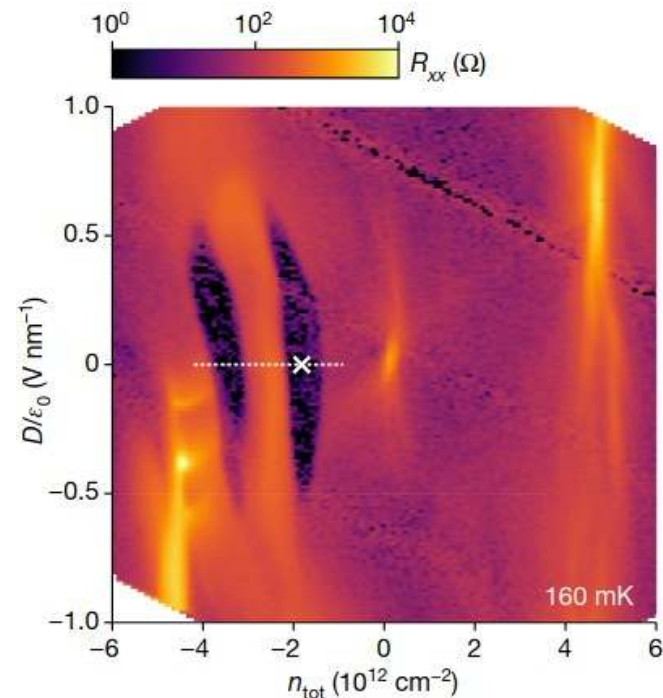
**How can we compute the electronic structure of exceptionally large systems?**

# Super-moire materials, experimentally

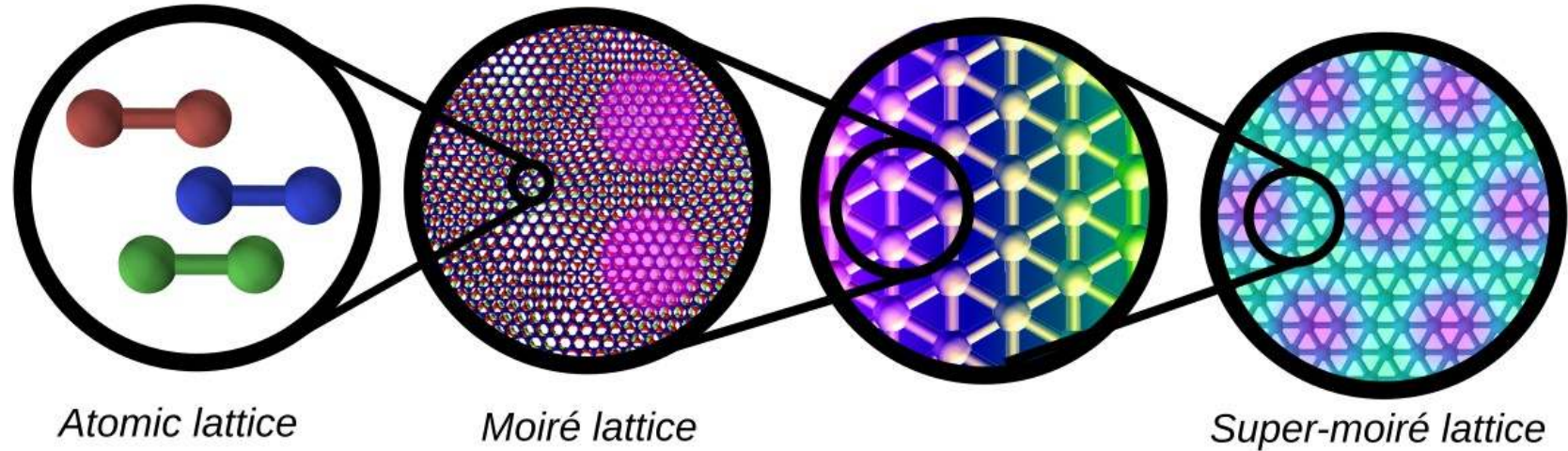
## Moire quasicrystal in twisted trilayer graphene



## Coexisting correlations and superconductivity



# Why super-moire materials are hard (for theorists)



Moiré systems  **$N=10^5$  sites**, calculations taking around 1 minute

Computational cost grows as  **$N$**  (best case) or  **$N^3$**  (worst case)

Super-moiré  **$N=10^9$  sites**, calculations taking 10 days – 1000000 years

**Is there a way to solve exceptionally large electronic structure problems?**

(even going beyond usual memory limitations of conventional methods)

# Behind the scenes

Yitao Sun



Tiago Antão



Marcel Niedermeier



Adolfo Fumega



Correlated states in super-moiré materials with a kernel polynomial quantics tensor cross interpolation algorithm, AO Fumega, M Niedermeier, JL Lado, **2D Materials 12 (1), 015018 (2025)**

Self-consistent tensor network method for correlated super-moire matter beyond one billion sites, Y Sun, M Niedermeier, TVC Antão, AO Fumega, JL Lado, **arXiv:2503.04373 (2025)**

Tensor network method for real-space topology in quasicrystal Chern mosaics, TVC Antão, Y Sun, AO Fumega, JL Lado, **arXiv:2506.05230 (2025)**

# Tensor networks for interacting super-moire materials

# Modeling electrons in a material

Define operators that can create or destroy particles

$c_i$  Annihilation operator, destroys a particle in site  $i$

$c_i^\dagger$  Creation operator, creates a particle in site  $i$

The empty vacuum state  $|\Omega\rangle$  is defined as  $c_i |\Omega\rangle = 0$

The Hamiltonian is written in terms of creation and annihilation operators

$$H = c_0^\dagger c_1 + h.c.$$

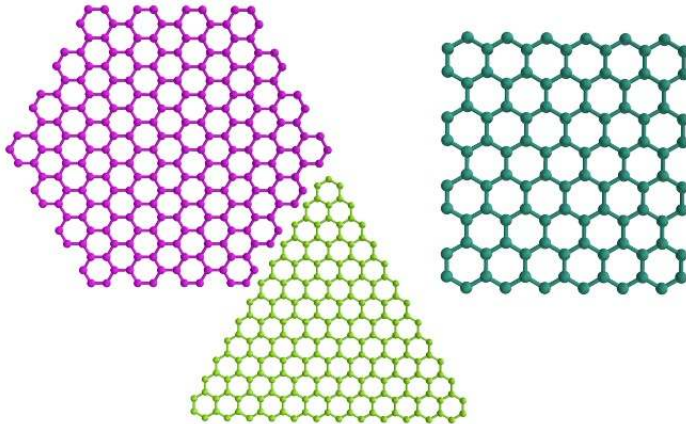
# Modes in a single particle Hamiltonian

To solve a single particle Hamiltonian, we just have to diagonalize the matrix

$$H = \sum_{ij} t_{ij} c_i^\dagger c_j$$

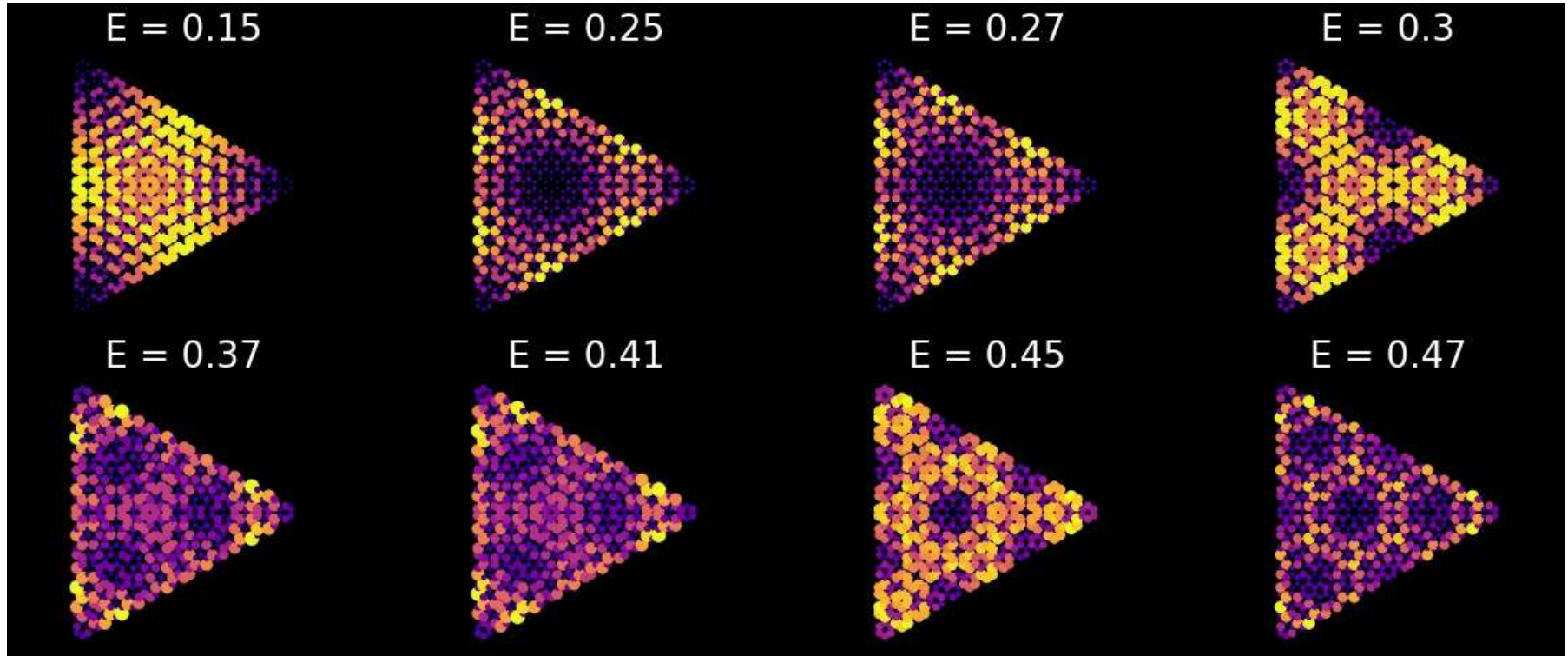
$$H = \sum_{\alpha} \epsilon_{\alpha} \Psi_{\alpha}^{\dagger} \Psi_{\alpha}$$

If take a certain finite geometry, we can find the confined modes

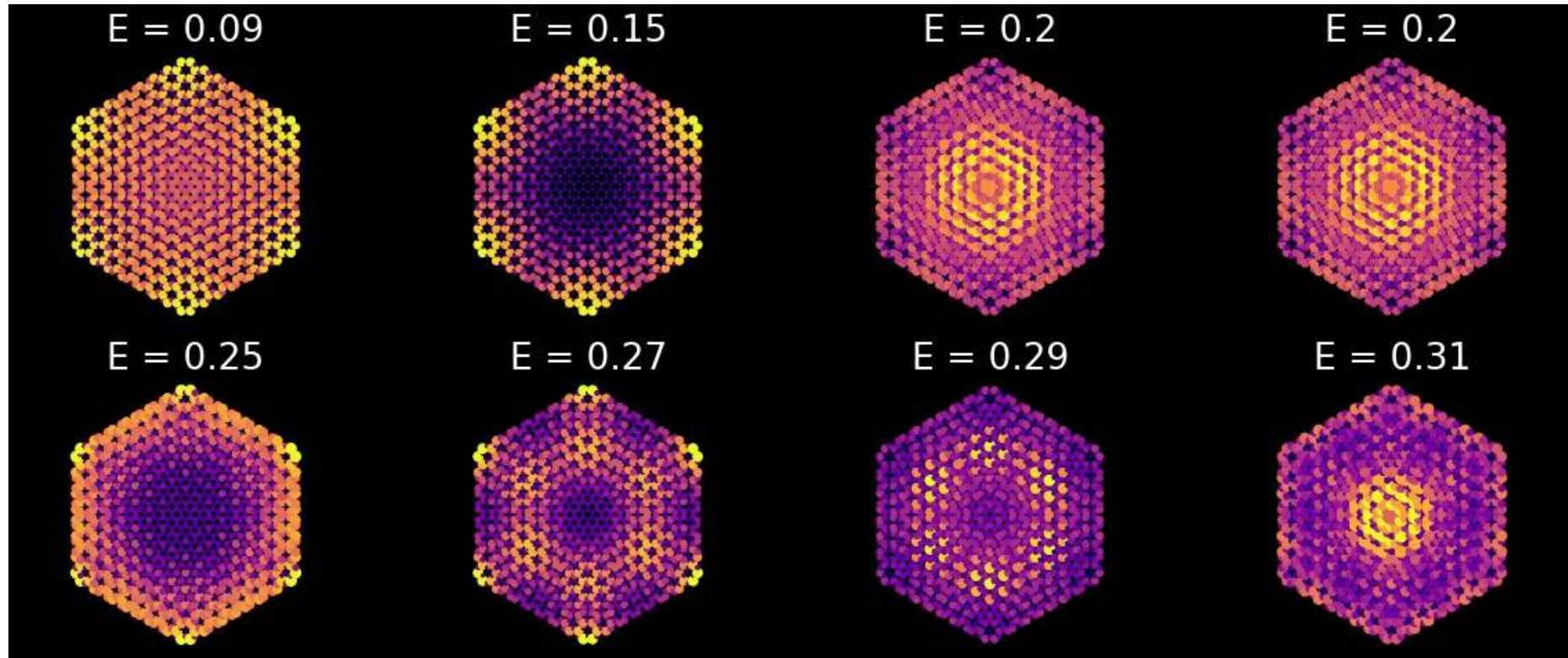


Let us do it for graphene islands

# Confined modes in graphene islands



# Confined modes in graphene islands



# The challenge of correlated super-moire materials

Hamiltonian describing electrons in a super-moire material

$$H = H_0 + H_V = \sum_{\alpha\beta} t_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \sum_{\alpha\beta} V_{\alpha\beta} c_{\alpha}^{\dagger} c_{\alpha} c_{\beta}^{\dagger} c_{\beta}$$

Mean-field treatment of the interacting Hamiltonian

$$H^{MF} = \sum_{\alpha\beta} (t_{\alpha\beta} + \chi_{\alpha\beta}) c_{\alpha}^{\dagger} c_{\beta} = \sum_{\alpha\beta} H_{\alpha\beta}^{MF} c_{\alpha}^{\dagger} c_{\beta}$$

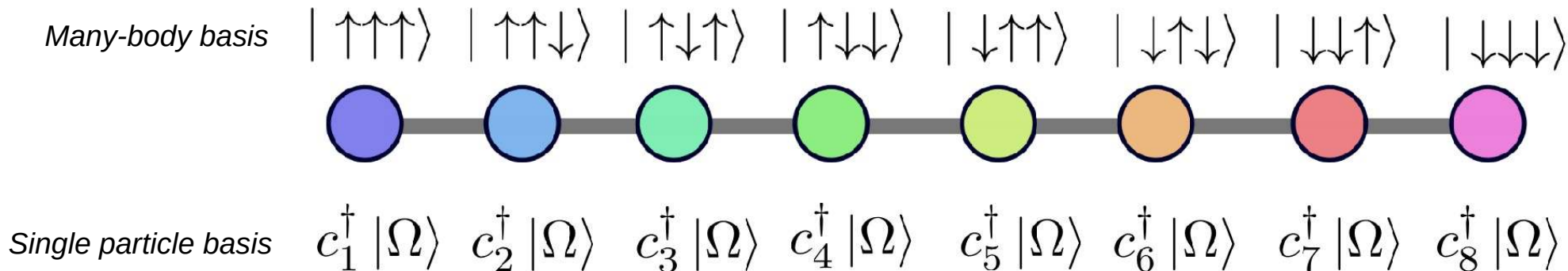
Solving the system requires dealing with matrices proportional to the system size

*In a super-moire system, this requires solving a billion sites*

**How could we solve a system whose Hamiltonian would be too large to store?**  
*(even before considering the time required to solve it)*

# Tensor network for single particle tight binding problems

**We can identify a many-body space with a very large single particle one**



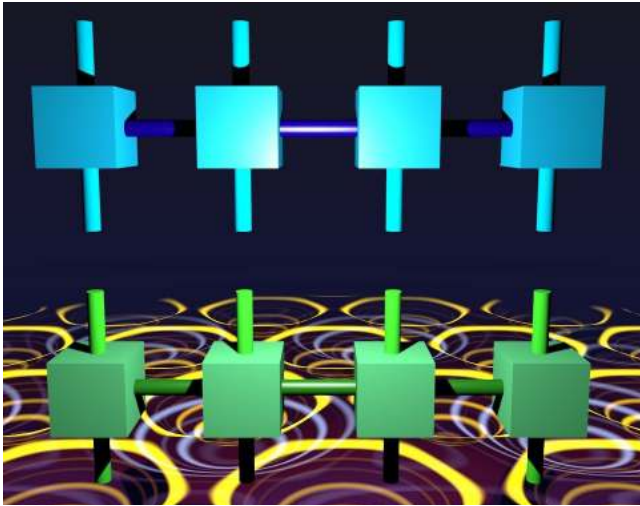
**We can use tensor networks to solve very large tight binding problem**

# Dealing with very large spaces with tensor networks

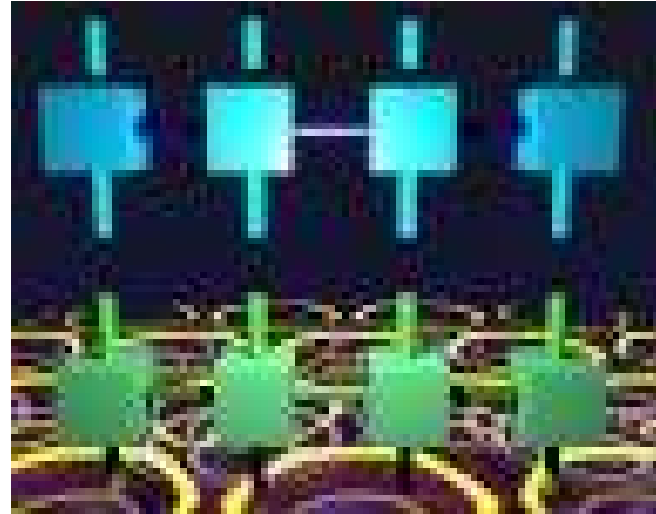
A many-body wavefunction is a very high dimensional object ( $2^L$  coefficients)

$$|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$$

Tensor-networks allow “compressing” exponentially large information with linear resources



“True wavefunction”



“Tensor-network wavefunction”

# Dealing with very large spaces with tensor networks

Typical many-body wavefunction  $|\Psi\rangle = \sum c_{s_1, s_2, \dots, s_L} |s_1, s_2, \dots, s_L\rangle$

We need to determine in total  $2^L$  coefficients

**Is there an efficient way of storing so many coefficients?**

Tensor networks allow parametrizing many-body wavefunctions as

$$|\Psi\rangle = \sum_{\{s\}} \text{Tr} \left[ M_1^{(s_1)} M_2^{(s_2)} \dots M_N^{(s_N)} \right] |s_1 s_2 \dots s_N\rangle$$

$L\chi^2$  parameters

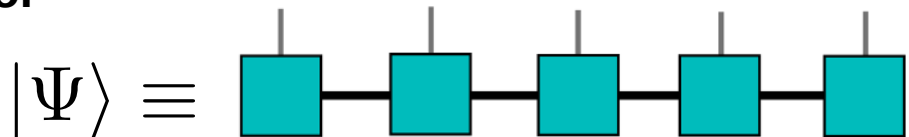


Tensor networks allow to drastically reduce the memory required to store a many-body state

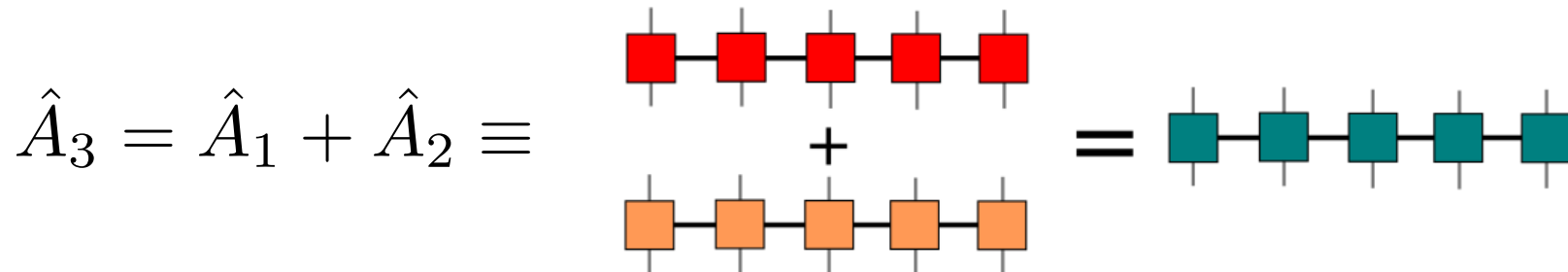
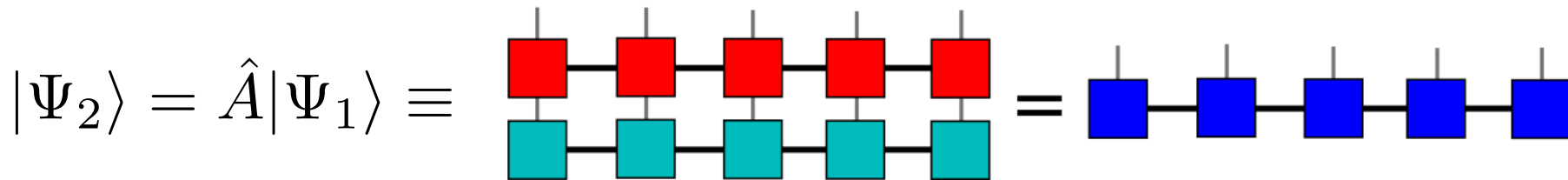
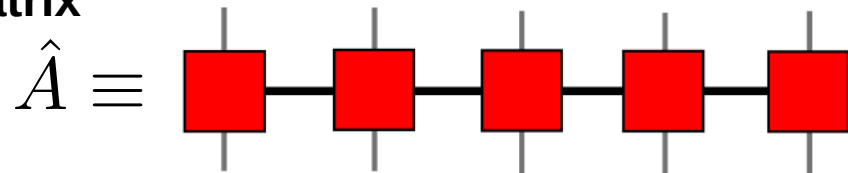
# Exponentially large algebra with tensor networks

Tensor network allow to (approximately) operate in exponentially large vector spaces

**vector**



**matrix**



# Learning the tensor network representation

**We need to represent all the tight binding terms as tensor networks**

- In some cases, we can build the initial tensor network explicitly
- In other cases, we learn the tensor network with quantum tensor cross interpolation

$$\left( \begin{array}{cccccccccccccccc} \bullet & \bullet & \circ & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right) \approx \left( \begin{array}{ccc} \circ & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array} \right) \left( \begin{array}{ccc} \circ & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{array} \right)^{-1} \left( \begin{array}{cccccccccccccccc} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{array} \right)$$

Phys. Rev. Lett. 132, 056501 (2024)

SciPost Phys. 18, 104 (2025)

# Self-consistent electronic interactions with tensor networks

**Super-moire interacting Hamiltonian**

$$H = H_0 + H_V = \sum_{\alpha\beta} t_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \sum_{\alpha\beta} V_{\alpha\beta} c_{\alpha}^{\dagger} c_{\alpha} c_{\beta}^{\dagger} c_{\beta}$$

**Mean-field decoupled Hamiltonian**

$$H^{MF} = \sum_{\alpha\beta} (t_{\alpha\beta} + \chi_{\alpha\beta}) c_{\alpha}^{\dagger} c_{\beta} = \sum_{\alpha\beta} H_{\alpha\beta}^{MF} c_{\alpha}^{\dagger} c_{\beta} \quad \mathcal{H}_{\alpha\beta}^{MF} \equiv \text{[Diagram: A chain of orange squares connected by horizontal lines, representing the mean-field decoupled Hamiltonian tensor network.]}$$

**Tensor network representation of the correlators**

$$\langle c_{\alpha}^{\dagger} c_{\beta} \rangle = \langle \alpha | \Xi(\mathcal{H}^{MF}) | \beta \rangle \equiv \text{[Diagram: A chain of cyan squares connected by horizontal lines, representing the tensor network representation of the correlator.]}$$

# Self-consistent electronic interactions with tensor networks

We represent the super-moire electronic Hamiltonian as a tensor-network

$$\mathcal{H}_{\alpha\beta}^{MF} \equiv \Gamma_{s_1, s'_1}^{(1)} \Gamma_{s_2, s'_2}^{(2)} \Gamma_{s_3, s'_3}^{(3)} \dots \Gamma_{s_L, s'_L}^{(L)} \quad \mathcal{H}_{\alpha\beta}^{MF} \equiv \text{[Diagram: A sequence of orange squares connected horizontally, representing a tensor network.]}$$

**The mean-field problem can be reformulated purely with tensor networks**

Tensor Network SCF loop

$$\begin{aligned} \mathcal{H}_{\alpha\beta}^{MF} &\equiv \text{[Diagram: A sequence of orange squares connected horizontally, representing a tensor network.]} = t_{\alpha\beta} + \chi_{\alpha\beta} \left( \text{[Diagram: A sequence of cyan squares connected horizontally, representing a tensor network.]} \right) \\ \langle c_{\alpha}^{\dagger} c_{\beta} \rangle &\equiv \text{[Diagram: A sequence of cyan squares connected horizontally, representing a tensor network.]} = \sum_n \lambda_n T_n \left( \text{[Diagram: A sequence of orange squares connected horizontally, representing a tensor network.]} \right) \end{aligned}$$

# Spectral functions with the kernel polynomial method

Let us expand a delta function in Chebyshev polynomials

$$\delta(\omega - \mathcal{H}) = \sum_n \lambda_n(\omega) T_n(\mathcal{H})$$

Chebyshev relation

$$T_n(\mathcal{H}) = 2\mathcal{H}T_{n-1}(\mathcal{H}) - T_{n-2}(\mathcal{H})$$

Expansion coefficients for delta function

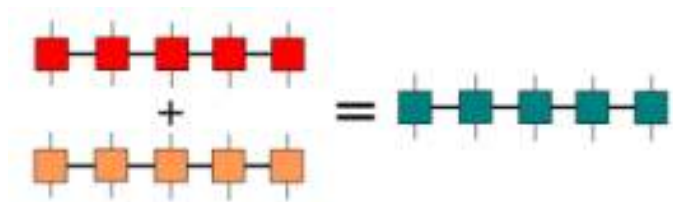
$$\lambda_n(\omega) = \frac{2T_n(\omega)}{\pi\sqrt{1-\omega^2}}$$

Rev. Mod. Phys. 78, 275 (2006)

**KPM can be implemented with tensor networks by using tensor network algebra**

Phys. Rev. B 83, 195115 (2011)

Phys. Rev. Research 1, 033009 (2019)



# Spectral functions with tensor networks

## Local spectral function of the mean-field Hamiltonian

$$D(\omega, \alpha) = \langle \alpha | \left[ \sum_n T_n(\mathcal{H}^{MF}) P_n(\omega) \right] | \alpha \rangle$$

With a tensor network Chebyshev algorithm

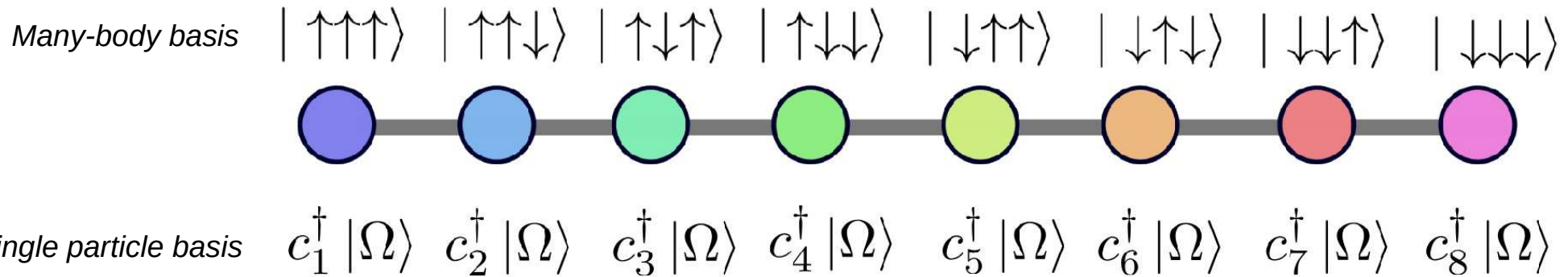
$$P_n(\omega) = \frac{2T_n(\omega)}{\pi\sqrt{1-\omega^2}}$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$

$$\mathcal{H}_{\alpha\beta}^{MF} \equiv \begin{array}{c} | \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} | \\ \text{---} \end{array} \dots \begin{array}{c} | \\ \text{---} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} | \\ \text{---} \end{array}$$

# Tensor network representation of a super-moire Hamiltonian

We can identify a many-body space with a very large single particle one



In the tight binding basis uniform hopping takes the form

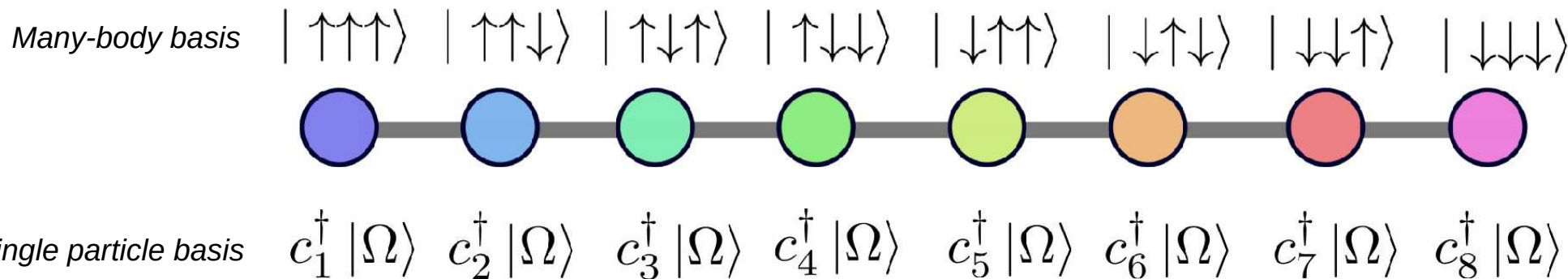
$$H_{0,NN} = \sum_{\alpha,s}^{N-1} t(c_{x_{\alpha+1},s}^\dagger c_{x_{\alpha},s} + h.c.)$$

In the tensor-network pseudospin basis, uniform hopping takes the form

$$\mathcal{H}_{0,NN} = \sum_{l,s}^L t(\sigma_{l,s}^+ \otimes_{m>l} \sigma_{m,s}^- + h.c.)$$

# Tensor network representation of a super-moire Hamiltonian

We can identify a many-body space with a very large single particle one



The tensor-network moire hopping can be built as

Find the MPS representation for the modulation and store in a diagonal MPO  $\mathcal{T}$

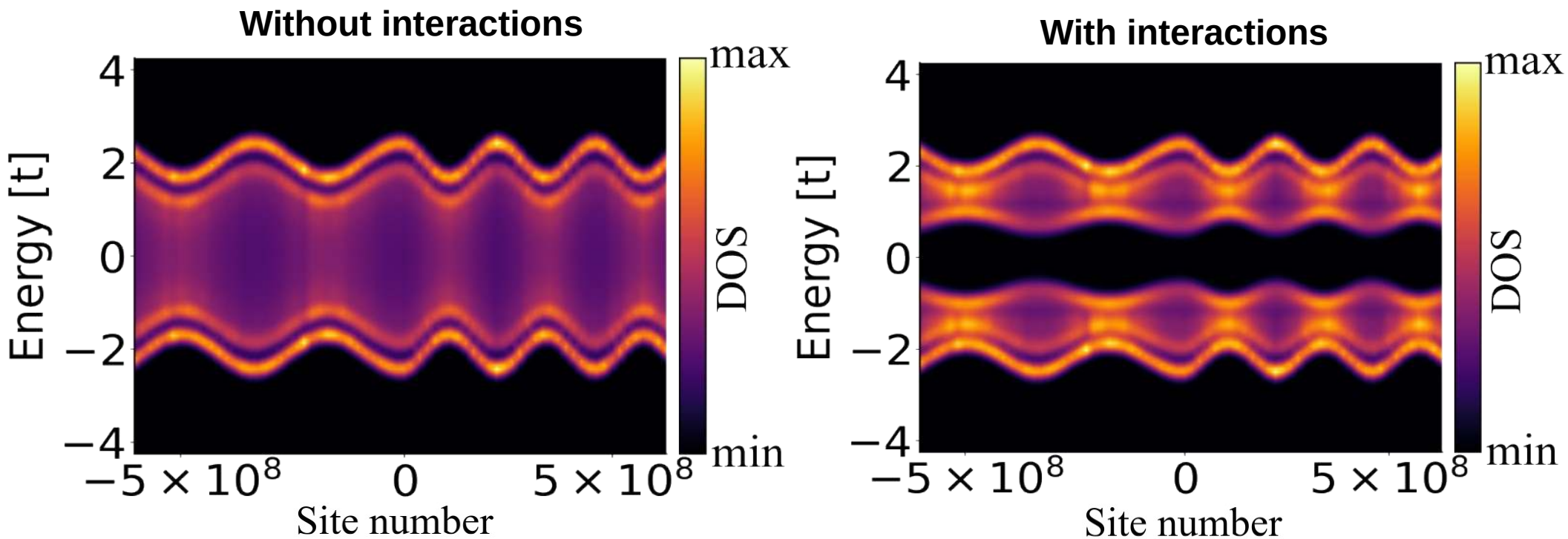
$$|\tau\rangle = \sum M_{s_1}^{(1)} M_{s_2}^{(2)} M_{s_3}^{(3)} \dots M_{s_L}^{(L)} |s_1, s_2, \dots, s_L\rangle$$

**Quantics tensor cross interpolation**

**Modulated super-moire by construction**

$$\mathcal{H}_0 = \{[\mathcal{T} \sum_{l,s}^L (\sigma_{l,s}^+ \otimes_{m>l} \sigma_{m,s}^-)] + h.c.\}$$

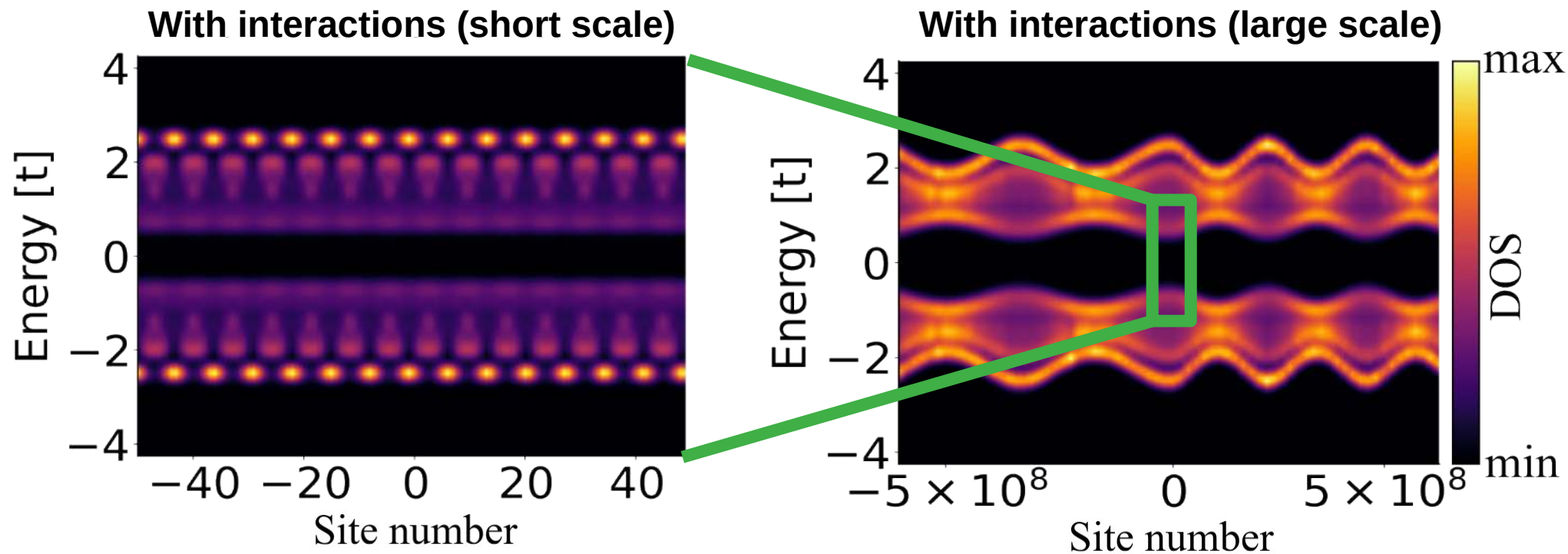
# Solving billion-size super-moire materials



***Tensor networks allow to solve selfconsistently a super-moire with one billion sites***

*arXiv:2503.04373 (2025)*

# Solving billion-size super-moire materials

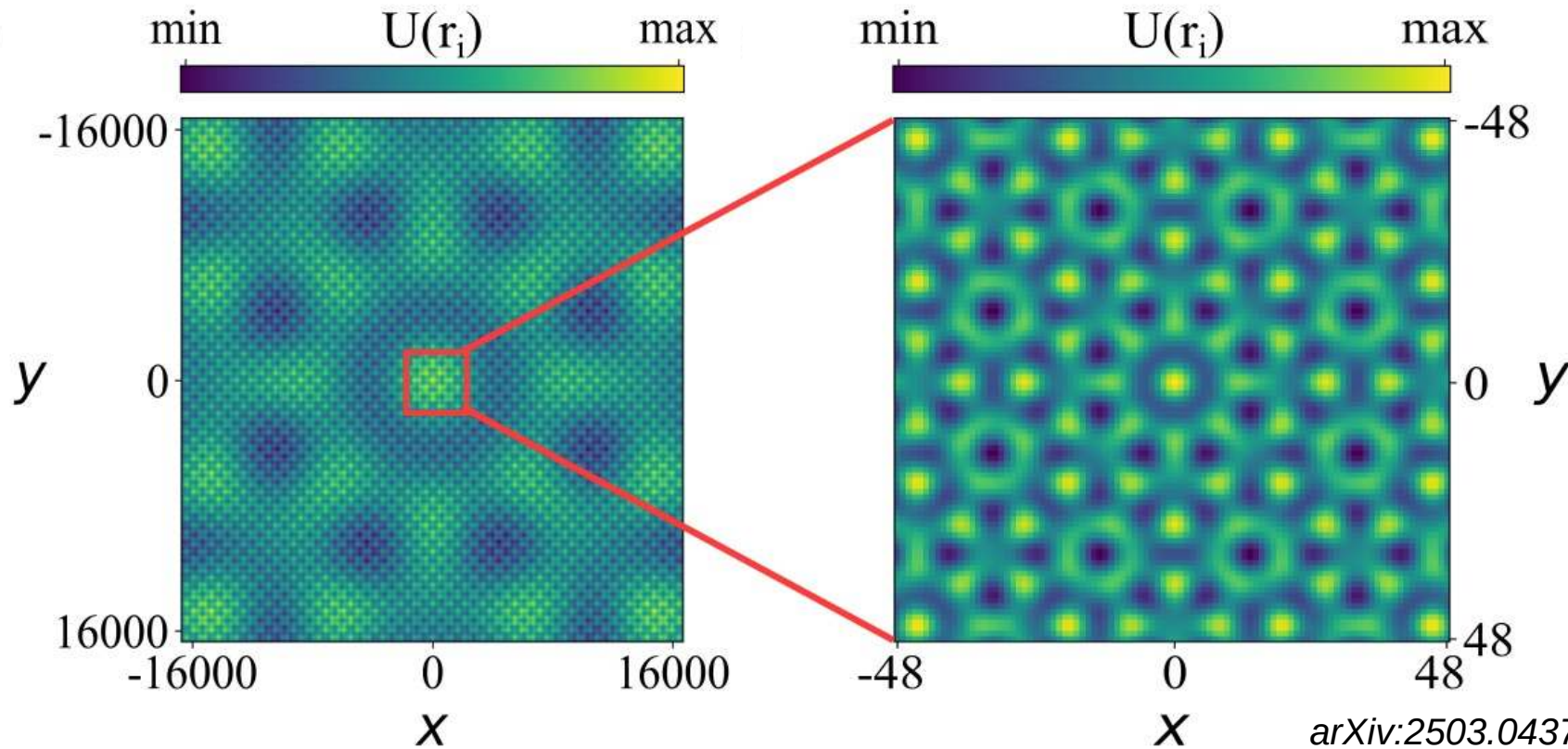


***Tensor networks allow to solve selfconsistently a super-moire with one billion sites***

*arXiv:2503.04373 (2025)*

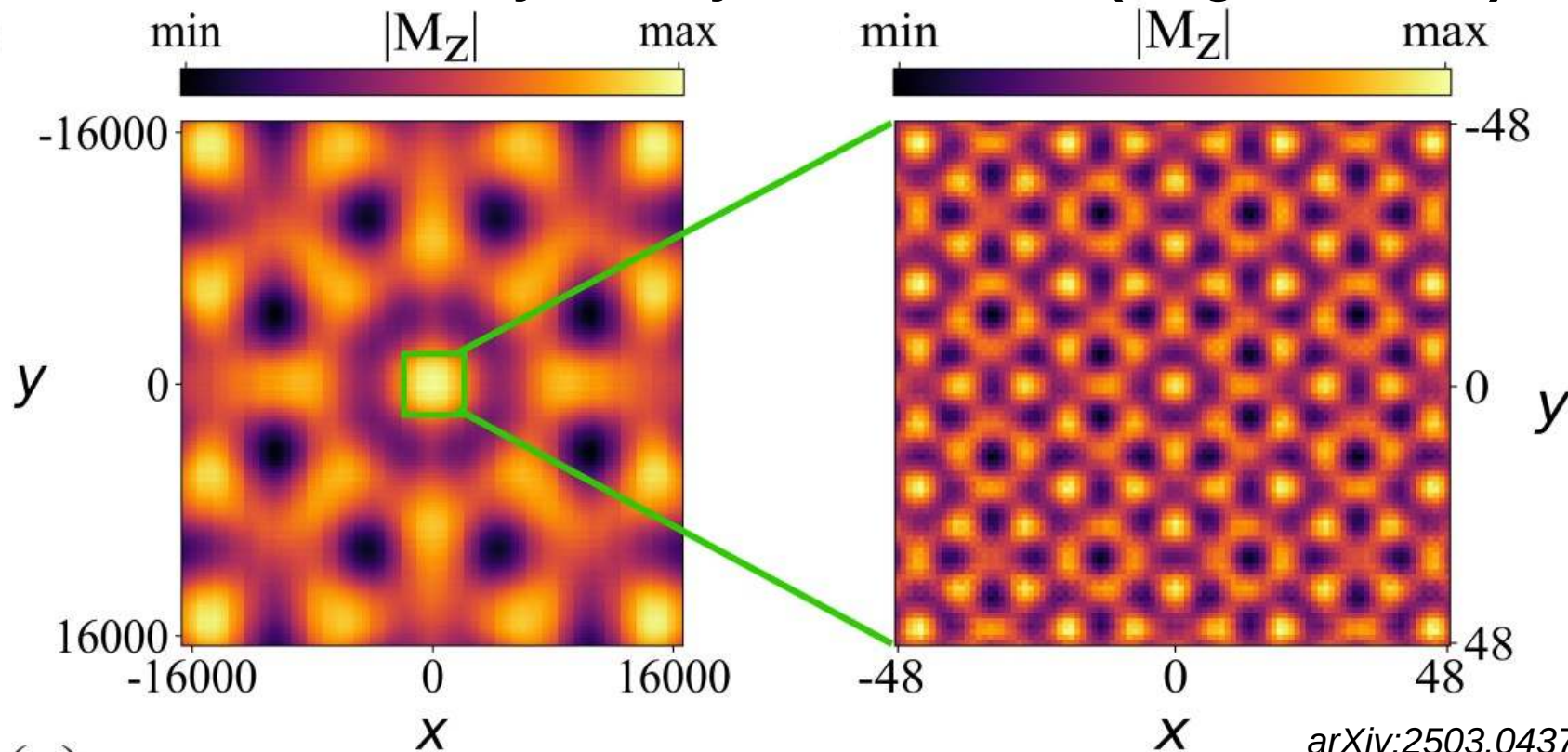
# Two-dimensional billion size interacting super-moire

## Modulation for the Hamiltonian



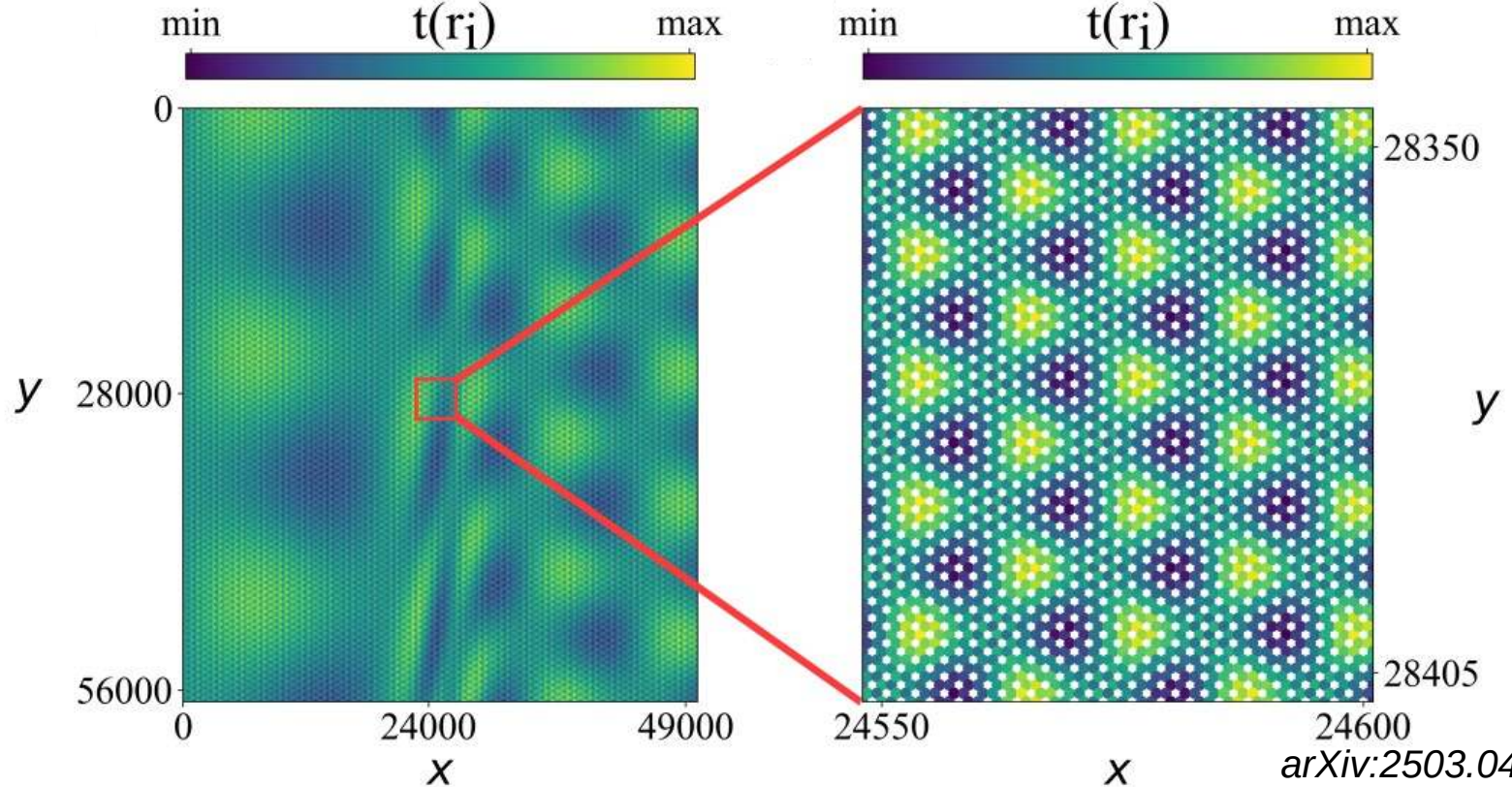
# Two-dimensional billion size interacting super-moire

**Selfconsistent symmetry broken order (magnetization)**



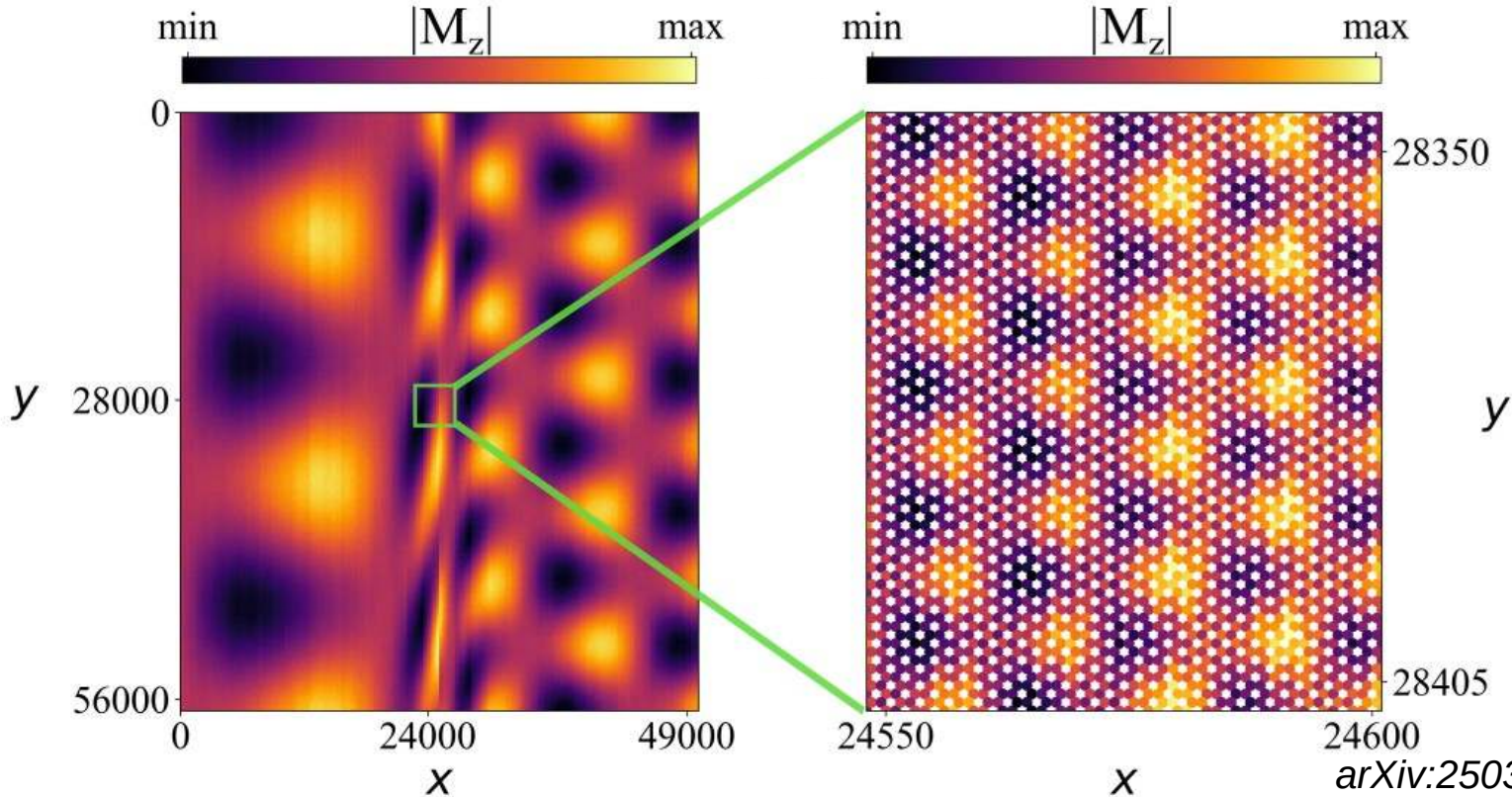
# Two-dimensional billion size interacting super-moire

## Modulation fo the Hamiltonian



# Two-dimensional billion size interacting super-moire

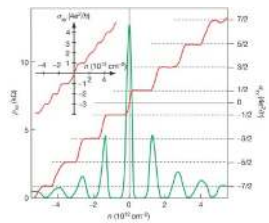
**Selfconsistent symmetry broken order (magnetization)**



# Tensor networks for topological super-moire materials

# Topological quantum matter

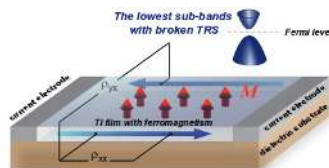
## Quantum Hall effect



graphene

Nature 438, 197-200 (2005)

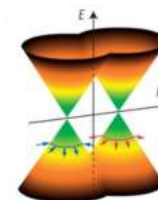
## Quantum Anomalous Hall effect



Cr-doped  
(Bi,Sb)<sub>2</sub>Te<sub>3</sub>

Science 340.6129 (2013): 167-170

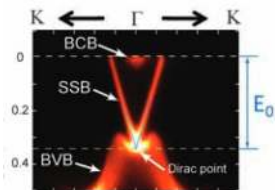
## Weyl semimetal



TaAs

Nature Physics 11, 724–727 (2015)

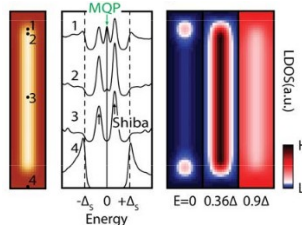
## Quantum Spin Hall effect



Bi<sub>2</sub>Se<sub>3</sub>

Science 325.5937 (2009): 178-181

## Topological superconductor



Fe@Pb

Science 346.6209 (2014): 602-607

## Nodal line semimetals

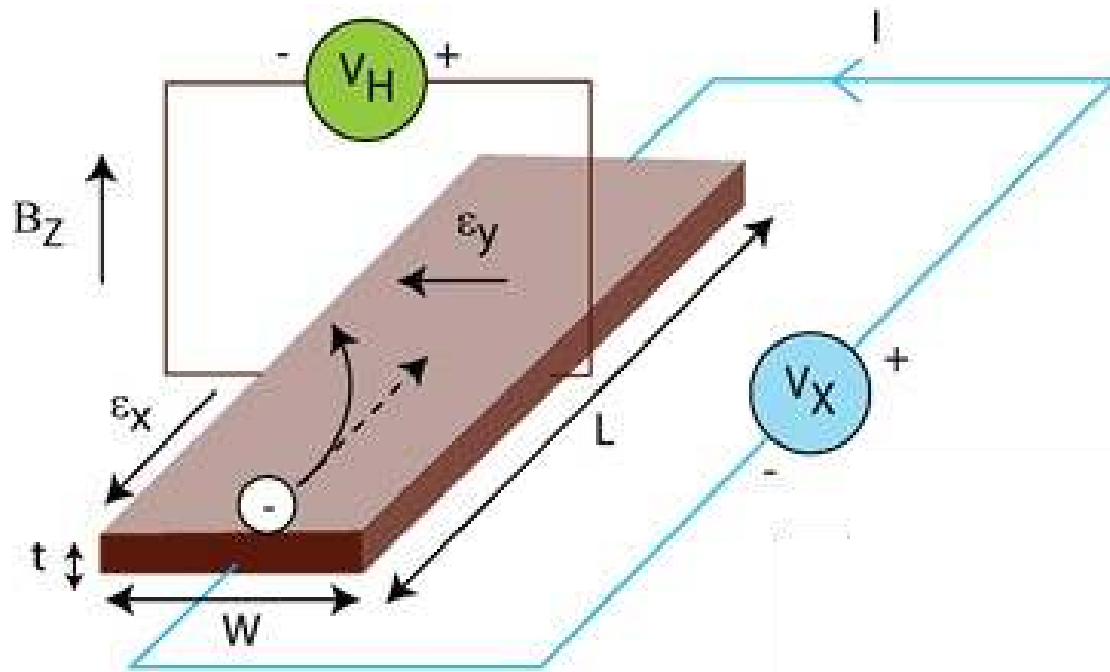


Mg<sub>3</sub>Bi<sub>2</sub>

Advanced Science 6.4 (2019): 1800897

*All these states are described by effective isolated single-particle physics*

# The Hall effect



$$J_x = \sigma_{xy} V_y$$

Hall conductivity

Measure the current perpendicular to a voltage

# The Hall conductivity

The Hall conductivity is obtained as  $\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k}$

**Berry curvature**

$$\Omega_{\alpha} = \partial_{k_x} A_y^{\alpha} - \partial_{k_y} A_x^{\alpha}$$

**Berry connection**

$$A_{\mu}^{\alpha} = i \langle \partial_{k_{\mu}} \Psi_{\alpha} | \Psi_{\alpha} \rangle$$

$$\sigma_{xy} = \sum_{\alpha \in occ} \int \Omega_{\alpha} d^2 \mathbf{k} = \sum_{\alpha} C_{\alpha} = C \longleftarrow \text{Chern number}$$

**The transverse conductivity is a topological invariant**

This means, it must take integer values regardless of defects in an insulator

# Topological invariant in a Hamiltonian

We can classify Hamiltonians according to topological invariants

Hamiltonian

$$H(\mathbf{k})$$

Wavefunction

$$|\Psi(\mathbf{k})\rangle$$

$$C = \int_{BZ} \Omega d^2\mathbf{k}$$

Topological invariant  
(Chern number)

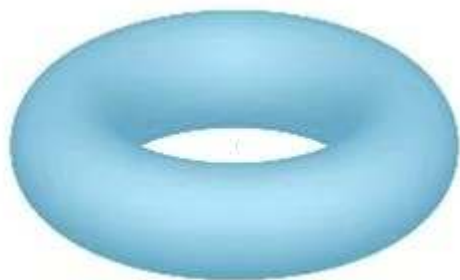
$$\Omega = \nabla \times \mathbf{A}|_z$$

Metric  
(Berry curvature)

# The role of a topological invariant

**Hamiltonians with different topological invariants  
can not be deformed one to another**

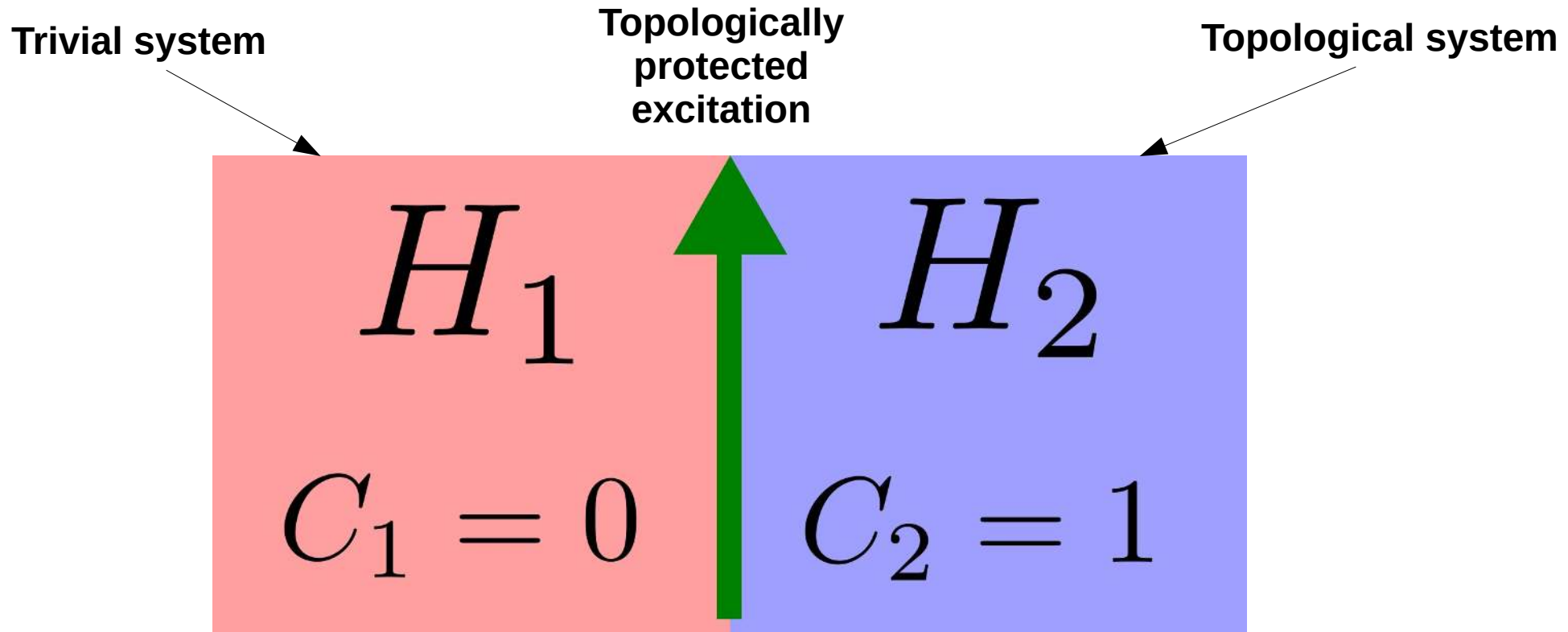
$$C = 1$$



$$C = 2$$



# The consequence of different topological invariants



*Topological excitations appear between topologically different systems*

# Real space topological invariants

Topological invariants are often defined in reciprocal space

$$C = \int_{BZ} \Omega d^2 \mathbf{k}$$

$$\Omega_{\alpha} = \partial_{k_x} A_y^{\alpha} - \partial_{k_y} A_x^{\alpha}$$

$$A_{\mu}^{\alpha} = i \langle \partial_{k_{\mu}} \Psi_{\alpha} | \Psi_{\alpha} \rangle$$

Is it possible to define them in real space?

Real-space Chern number

$$C_{\alpha} = 2\pi i \langle \alpha | \hat{Q} \hat{X} \hat{P} \hat{Y} \hat{Q} - \hat{P} \hat{X} \hat{Q} \hat{Y} \hat{P} | \alpha \rangle$$

$$\hat{P} = \int_{-\infty}^{\epsilon_F} \delta(\omega - H) d\omega$$

$$\hat{Q} = 1 - \hat{P}$$

# Kernel polynomial for topological invariants in real-space

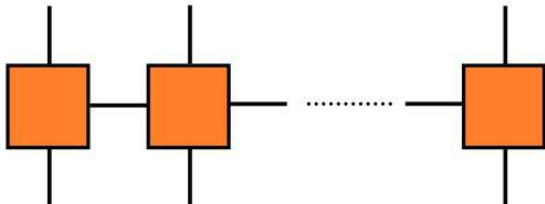
The density matrix of a super-moire system can be expressed as a tensor network

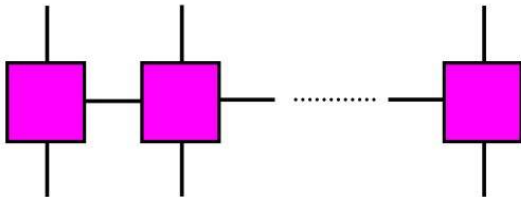
$$\hat{\mathcal{P}} = \int_{-\infty}^{\varepsilon_F} \delta(\omega - \hat{H}) d\omega = \sum \Xi_{s_1, s'_1}^{(1)} \Xi_{s_2, s'_2}^{(2)} \cdots \Xi_{s_L, s'_L}^{(L)} |s\rangle \langle s'|$$

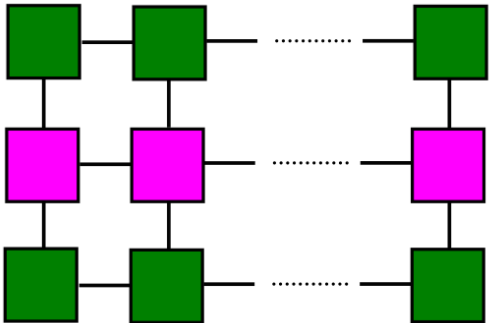
With a kernel polynomial tensor network algorithm

$$\hat{\mathcal{P}} = \sum_n T_n(\hat{\mathcal{H}}) \int_{-\infty}^{\varepsilon_F} d\omega \frac{T_n(\omega)}{\sqrt{1 - \omega^2}},$$
$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$
$$T_0 = 1 \text{ and } T_1(x) = x$$

# Computing Chern numbers with tensor networks

Density matrix as tensor network  $\hat{P} = \int_{-\infty}^{\varepsilon_F} \delta(\omega - \hat{H}) d\omega =$  

Topological marker as tensor network  $\hat{\Gamma} = \hat{Q}\hat{X}\hat{P}\hat{Y}\hat{Q} - \hat{P}\hat{X}\hat{Q}\hat{Y}\hat{P} =$  

Chern number from tensor network contraction  $C_\alpha = 2\pi i \langle \alpha | \hat{\Gamma} | \alpha \rangle =$  

# Tensor network topological marker

Real-space Chern number (Chern marker)

$$C_{\alpha} = 2\pi i \langle \alpha | \hat{Q} \hat{X} \hat{P} \hat{Y} \hat{Q} - \hat{P} \hat{X} \hat{Q} \hat{Y} \hat{P} | \alpha \rangle$$

Tensor-network Chern marker

$$\hat{C}_{\alpha} = 2\pi i \left( \begin{array}{c} \hat{Q} \\ \hat{y}_{\alpha}^{\Lambda} \\ \hat{P} \\ \hat{x}_{\alpha}^{\Lambda} \\ \hat{Q} \end{array} \begin{array}{c} \text{---} \end{array} \begin{array}{c} \hat{P} \\ \hat{y}_{\alpha}^{\Lambda} \\ \hat{Q} \\ \hat{x}_{\alpha}^{\Lambda} \\ \hat{P} \end{array} \right)$$

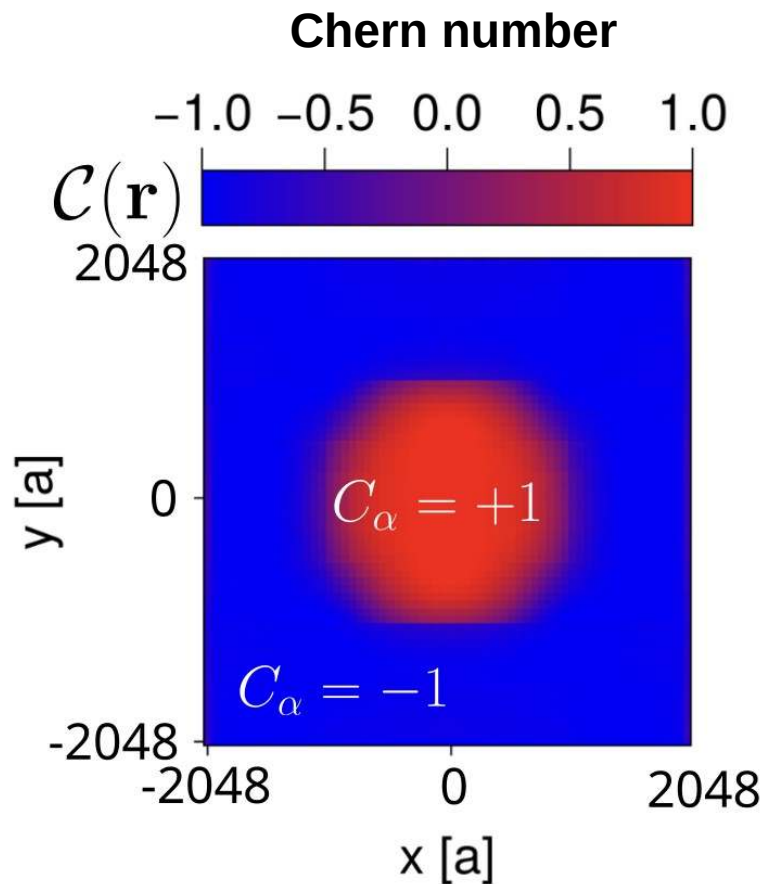
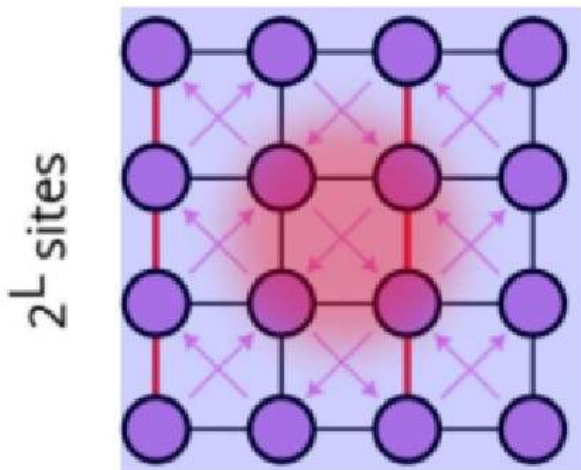
# Topological domain with tensor networks

arXiv:2506.05230 (2025)

**Topological marker**

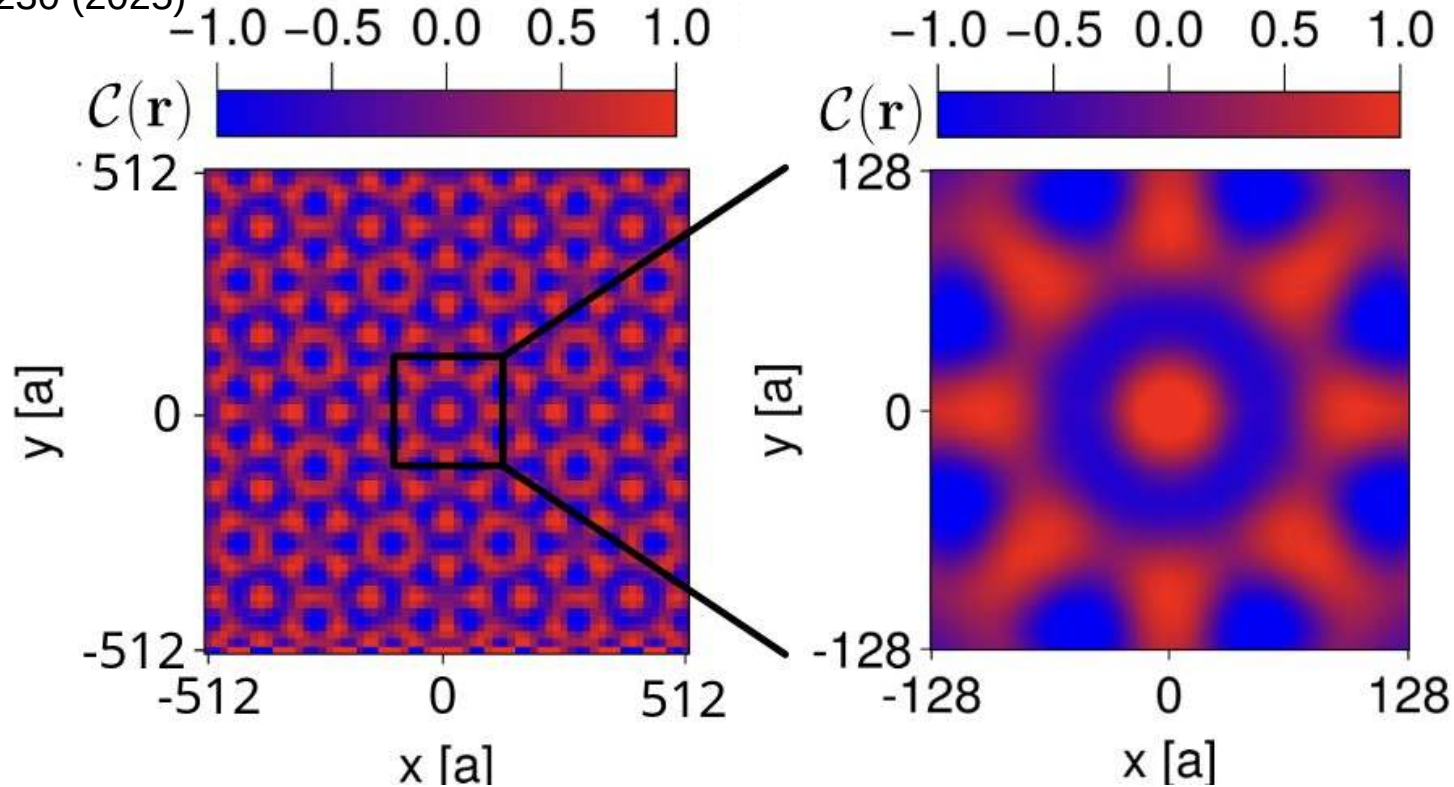
$$C_{\alpha} = 2\pi i \langle \alpha | \hat{Q} \hat{X} \hat{P} \hat{Y} \hat{Q} - \hat{P} \hat{X} \hat{Q} \hat{Y} \hat{P} | \alpha \rangle$$

**In a spatially modulated  
topological Hamiltonian**



# Chern mosaic in a 8-fold topological quasicrystal

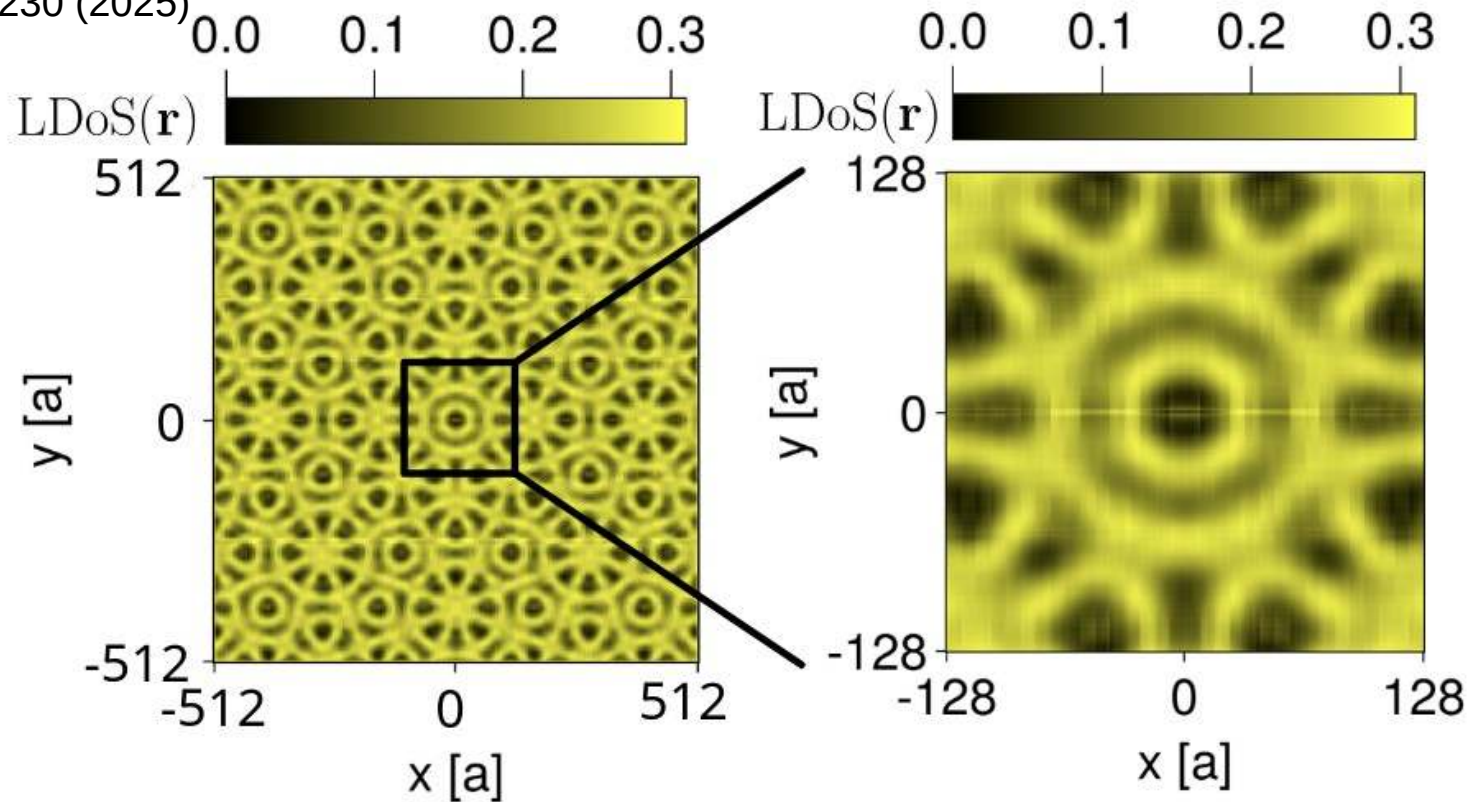
arXiv:2506.05230 (2025)



Domains with different topological invariants can be directly computed in real space

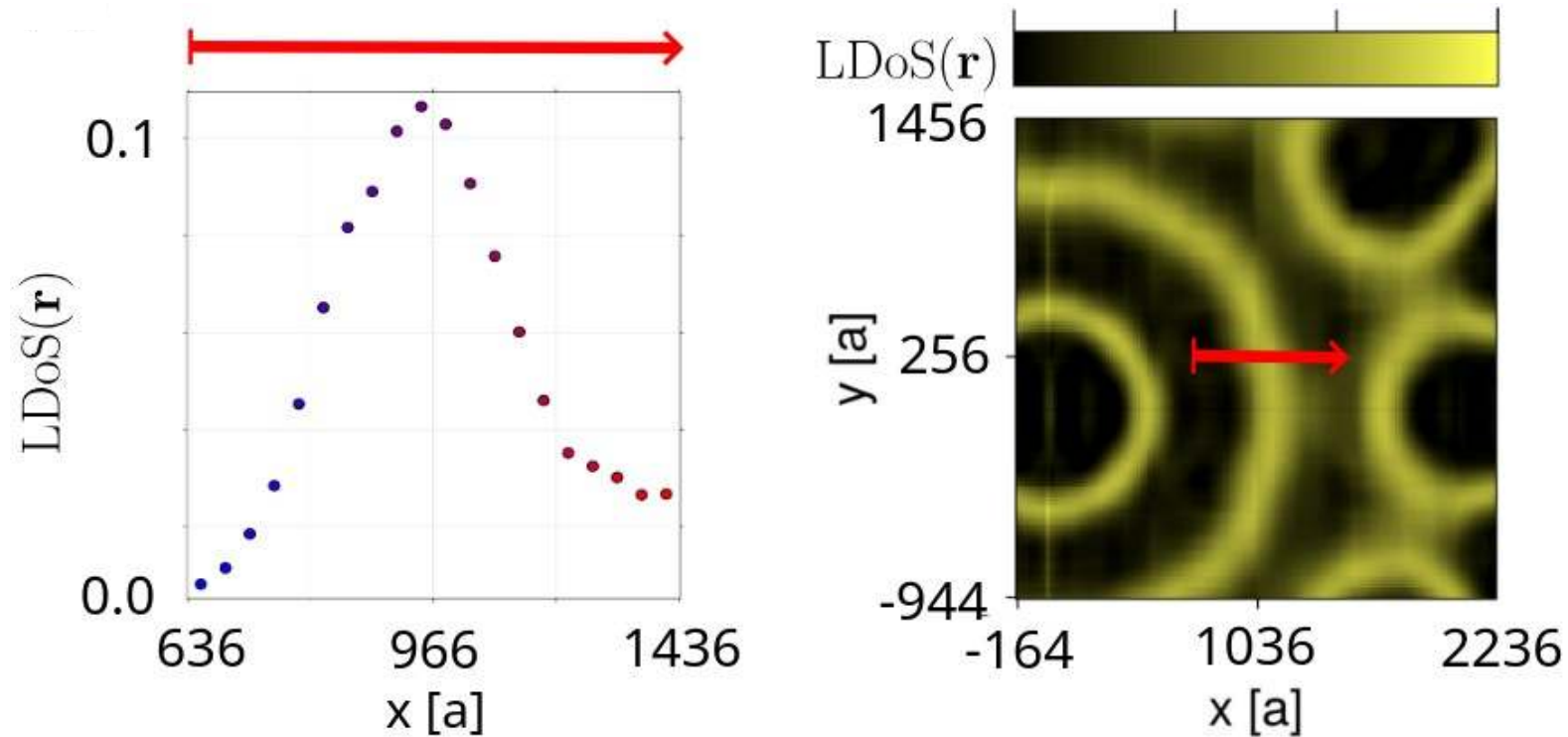
# Topological modes in a 8-fold topological quasicrystal

arXiv:2506.05230 (2025)



Topological modes appear between topological domains in real space

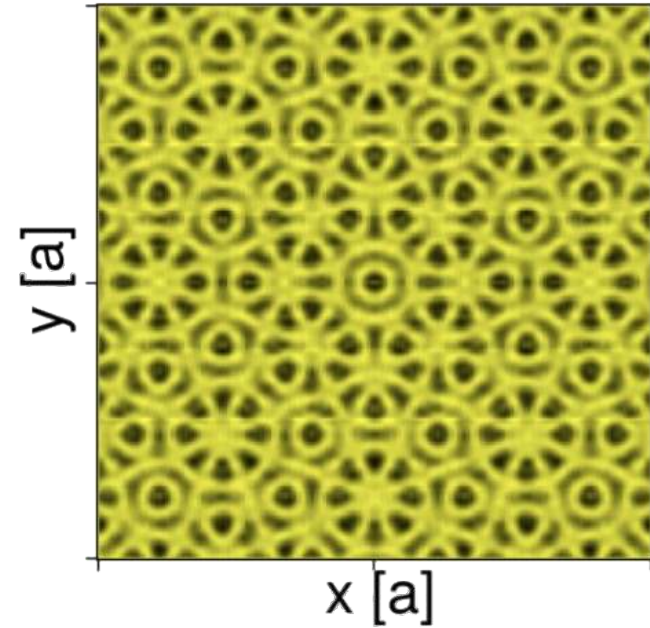
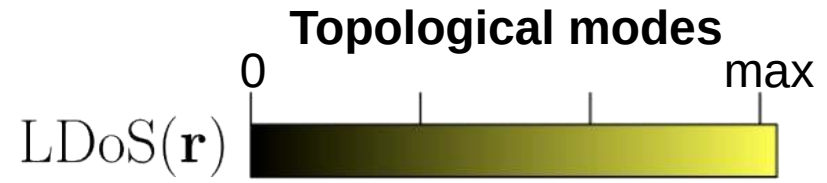
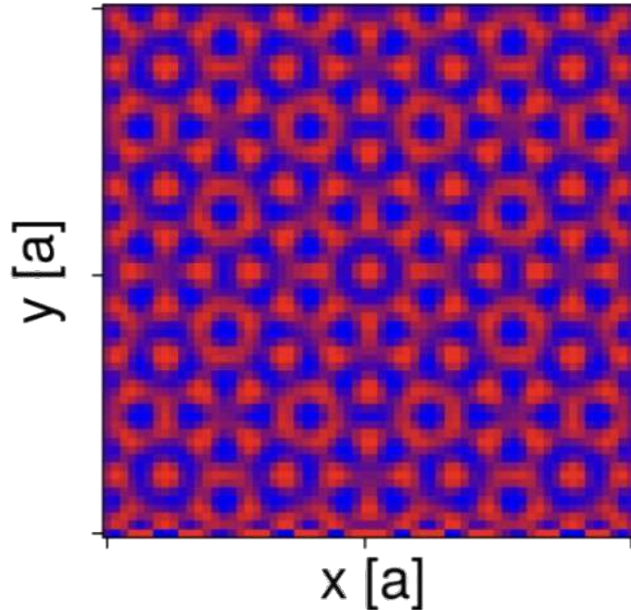
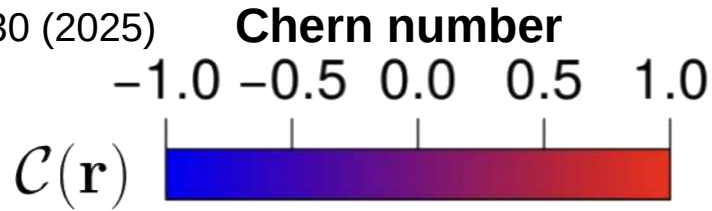
# Zero modes across topological quasicrystalline domain



The real space distribution of topological states can be mapped in real space

# Chern mosaic and topological modes in a quasicrystal

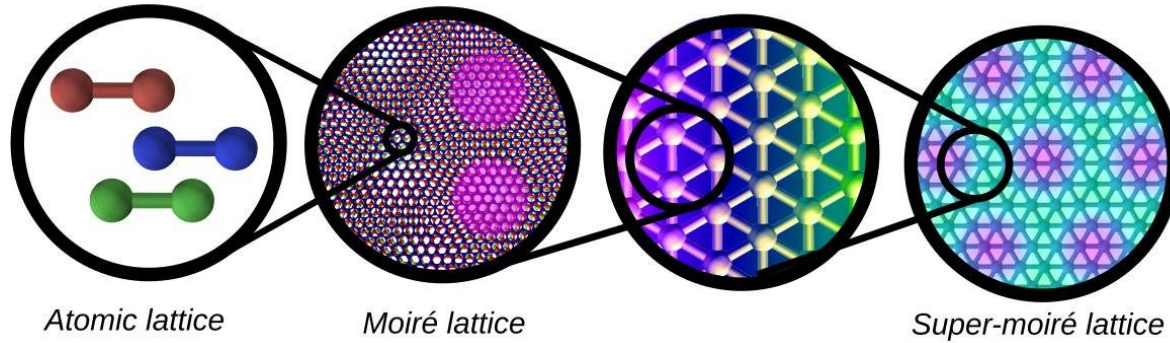
arXiv:2506.05230 (2025)



Tensor networks allow computing topology in exceptionally large super-moire systems

# Summary

Tensor networks allows solving large electronic structure problems, reaching the regime required for super-moire materials



2D Materials 12 (1), 015018 (2025)  
arXiv:2503.04373 (2025)  
arXiv:2506.05230 (2025)

$$\mathcal{H}_{\alpha\beta}^{MF} \equiv \text{[orange squares]} = t_{\alpha\beta} + \chi_{\alpha\beta} \left( \text{[cyan squares]} \right)$$

$$\langle c_{\alpha}^{\dagger} c_{\beta} \rangle \equiv \text{[cyan squares]} = \sum_n \lambda_n T_n \left( \text{[orange squares]} \right)$$

$$\hat{C}_{\alpha} = 2\pi i \left( \begin{array}{c} \hat{Q} \\ \hat{y}_{\alpha}^{\Lambda} \\ \hat{p} \\ \hat{x}_{\alpha}^{\Lambda} \\ \hat{Q} \end{array} \text{[cyan grid]} - \begin{array}{c} \hat{p} \\ \hat{y}_{\alpha}^{\Lambda} \\ \hat{Q} \\ \hat{x}_{\alpha}^{\Lambda} \\ \hat{p} \end{array} \text{[green grid]} \right)$$